2/29 | Calc I

Quiz 10

1. $f = -3\cos x + 5\boxed{\sec x}$

$f' = -3(-\sin x) + 5\sec x\tan x$

$= 3\sin x + 5\sec x\tan x$

2. $y = (5x^3+4x^2-1)^6$

$y = u^6$

$u = \underbrace{5x^3+4x^2-1}$

$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

$= 6u^5\cdot(15x^2+8x)$

$= 6(5x^3+4x^2-1)^5\cdot(15x^2+8x)$

3. $f(x) = \sin(7x+4)$

$\cos(7x+4)\cdot 7$

4. $e^{(3x^2+5)}$

$$y = e^{u}$$
$$u = 3x^2+5$$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$

$$e^{u} \cdot 6x$$

$$= e^{(3x^2+5)} \cdot 6x$$

$$= 6x \cdot e^{(3x^2+5)}$$

5. $y = (\underbrace{5x + \tan(3x)})^7$

$$\frac{dy}{dx} = 7(\underbrace{5x + \tan(3x)})^6 \cdot$$

$$\left[5 + \sec^2(3x) \cdot 3 \right]$$

Last time derivatives of inverses:

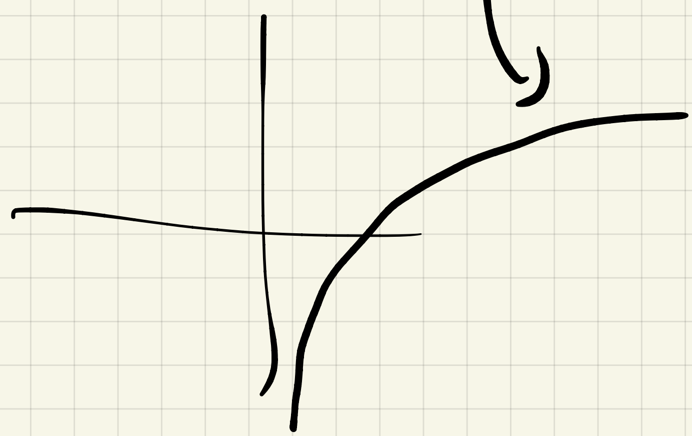
$$g = f^{-1}(x) \rightarrow$$
$$f(g(x)) = x \xrightarrow{d/dx}$$

$$f'(g(x))\cdot g'(x) = 1$$

$$\boxed{g'(x) = \frac{1}{f'(g(x))}}$$

Special case: $f(x) = e^x \Rightarrow f' = e^x$

$g(x) = \ln x$



$$\frac{d}{dx}(g(x)) = \frac{1}{f'(g(x))}$$

$$\|$$

$$\frac{d}{dx}(\ln x) = \frac{1}{e^{(\ln x)}} = \frac{1}{x}$$

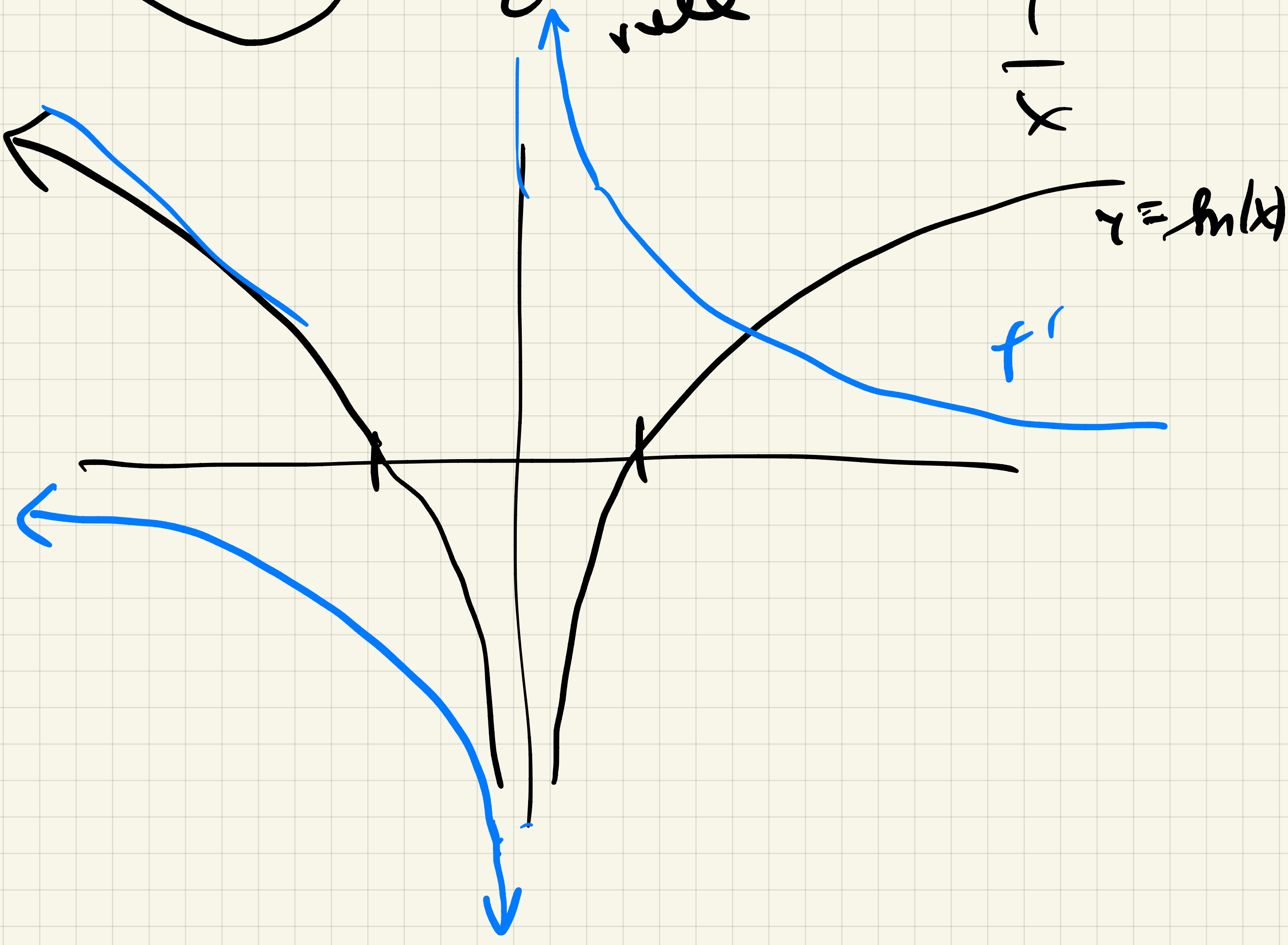
Thm $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$ (for $x \neq 0$)

<u>$x > 0$</u> : $|x| = x$

<u>$x < 0$</u> : $|x| = -x$ $\qquad \frac{d}{dx}(-x)$

$$\frac{d}{dx}(\ln(-x)) = \frac{1}{(-x)} \cdot (-1) = \frac{1}{x}$$

chain rule



Ex] Differentiate

(a) $y = \ln(\underline{\sin x})$

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x =$$

$$\frac{\cos x}{\sin x} = \cot x$$

$(y = \ln|\sin x| \Rightarrow y' = \cot x$

(b) $y = \ln(x^9)$

$$y' = \frac{1}{x^9} \cdot 9x^8 = \frac{9}{x}$$

Why?

$$y = \ln(x^9) = \underline{9\ln x}$$

$$y' = 9 \cdot \frac{1}{x} \checkmark$$

(c) $y = \boxed{\ln \frac{(2x+1)^4}{(7x+10)^9}}$

<u>One possibility:</u>

$$y' = \frac{1}{\left(\frac{(2x+1)^4}{(7x+10)^9}\right)} \cdot \frac{(7x+10)^9 \cdot 4(2x+1)^3 \cdot 2 - 9(7x+10)^8 \cdot 7 \cdot (2x+1)^4}{(7x+10)^{18}}$$

<u>Easy way:</u> $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$$y = \ln \frac{(2x+1)^4}{(7x+10)^9} =$$

$$= \ln(2x+1)^4 - \ln(7x+10)^9$$

$$= 4\boxed{\ln(2x+1)} - 9\ln(7x+10)$$

$$y' = 4\,\frac{2}{2x+1} - 9\,\frac{7}{7x+10}$$

$$= \frac{8}{2x+1} - \frac{63}{7x+10}$$

(d) $y = \tan(\underline{\ln(5x^2+3)})$

$$y' = \sec^2(\boxed{\ln(\boxed{5x^2+3})}) \cdot$$

$$\frac{1}{5x^2+3} \cdot 10x =$$

$$\frac{\sec^2(\ln(5x^2+3)) \cdot 10x}{5x^2+3}$$

What about other bases?

$\underline{\underline{y = a^x}}$ $\qquad$ $y = \underline{\underline{\log_a x}}$

Trick know $a = \left(e^{\ln a}\right)$,

so $y = a^x = \left(e^{\ln a}\right)^x$

$= e^{\boxed{x(\ln a)}}$

$\frac{dy}{dx} = \boxed{e^{x(\ln a)}} \ln a$ ✓

$\frac{d}{dx}(a^x) = a^x \cdot \ln a$

Applications

Thm

Ⓐ $\boxed{\frac{d}{dx}(a^x) = a^x \cdot \ln a}$

Ⓑ $\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{x \cdot \ln a}}$

Why Ⓑ? $\log_a x = \frac{\ln x}{\ln a}$

(Ch1 formula)

$$\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}$$

Ex 2 Differentiate:

(a) $$y = 3^{\tan(7x+3)}$$

$$\frac{dy}{dx} = 3^{\boxed{\tan(7x+3)}} \cdot \ln 3 \cdot \sec^2(7x+3) \cdot 7$$

(b) $$y = \ln\left(2^{\boxed{5x^2+8}}\right)$$

$$y' = \frac{1}{\cancel{2^{5x^2+8}}} \cdot \cancel{2^{(5x^2+8)}} \cdot \ln 2 \cdot 10x$$

$$= \ln 2 \cdot 10x$$

$$y = (5x^2+8)(\ln 2)$$

$$y' = 10x \cdot \ln 2 \checkmark$$

(c) $y = \log_{10}(\sec x)$

$$y' = \frac{1}{(\cancel{\sec x}) \cdot \ln 10} \cancel{\sec x} \tan x$$

$$= \frac{\tan x}{\ln 10}$$

(d) $y = \log_7\left(\frac{\sin\theta \cos\theta}{e^\theta 2^\theta}\right)$

COULD write

$$\boxed{y' = \frac{1}{\left(\frac{\sin\theta \cos\theta}{e^\theta \cdot 2^\theta}\right)\cdot \ln 7} \cdot \frac{\cancel{e^\theta} 2^\theta - - - \text{———}}{(\cancel{e^\theta} \cdot 2^\theta)^2}}$$

Instead: use log properties

$$y = \log_7\left(\frac{\sin\theta \cos\theta}{e^\theta \cdot 2^\theta}\right) =$$

$$\log_7(\sin\theta \cdot \cos\theta) - \log_7(e^\theta \cdot 2^\theta)$$

$$\parallel$$

$$\log_7(\sin\theta) + \log_7(\cos\theta) - \log_7(e^\theta) - \log_7(2^\theta)$$

$$\parallel$$

$$\log_7(\sin\theta) + \log_7(\cos\theta) - \theta \underline{\log_7(e)} - \theta\, \underline{\log_7 2}$$

$$y' = \frac{\cos\theta}{\sin\theta \cdot \ln 7} + \frac{-\sin\theta}{\cos\theta \cdot \ln 7} - \log_7 e - \log_7 2$$

Application

General power rule:

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

easier way: $X = e^{\ln x}$

$$X^n = \left(e^{\ln x}\right)^n = e^{n \ln x}$$

$$\frac{d}{dx}(x^n) = \frac{d}{dx}\left(\boxed{e}^{\boxed{n \ln x}}\right) =$$

$$\underbrace{e^{n \ln x}} \cdot n \cdot \frac{1}{x} =$$

x^{-1}

$$X^n \cdot n \cdot \frac{1}{x} =$$

$$n x^{n-1}$$

§ 3.9 Inverse trig functions:

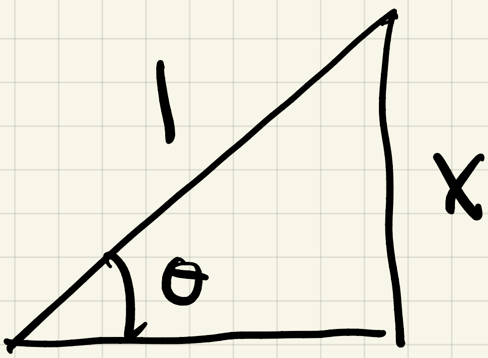
$g = f^{-1} \Rightarrow g'(x) = \dfrac{1}{f'(g(x))}$

Find $\frac{d}{dx}(\arcsin x) = \frac{d}{dx}(\sin^{-1} x)$

better test

arcsin x inverse to $\sin x$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\cos(\underbrace{\arcsin x}_{\theta})}$$



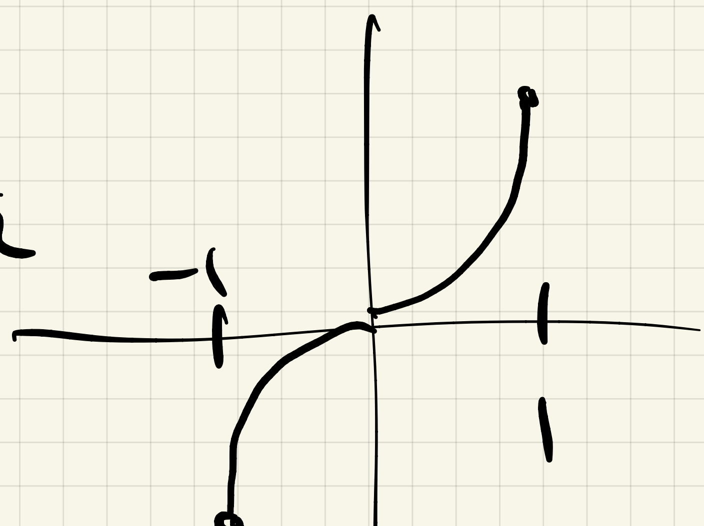
$a = \sqrt{1-x^2}$

$$\frac{1}{\sqrt{1-x^2}}$$

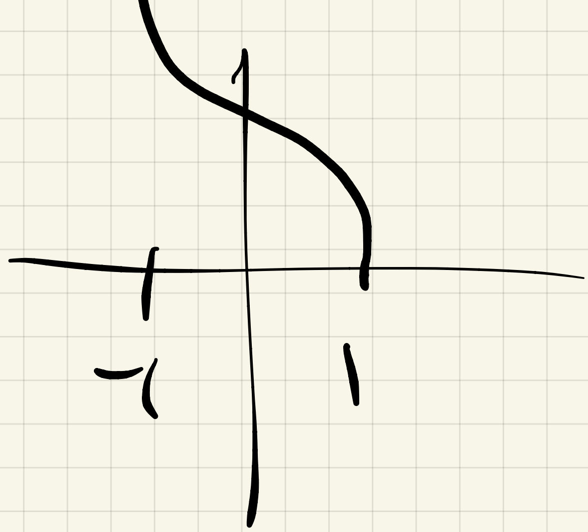
$a^2 + x^2 = 1$

$a^2 = 1 - x^2$

$a = \pm\sqrt{1-x^2}$



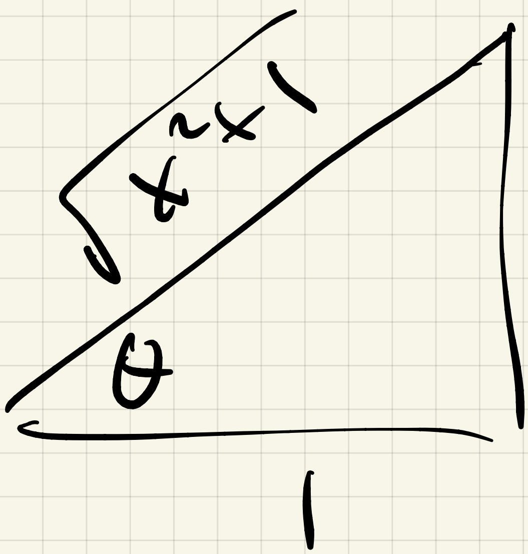
$$\boxed{\frac{d}{dx}(\arccos x) = \frac{-1}{\sqrt{1-x^2}}} \checkmark$$



Another way:

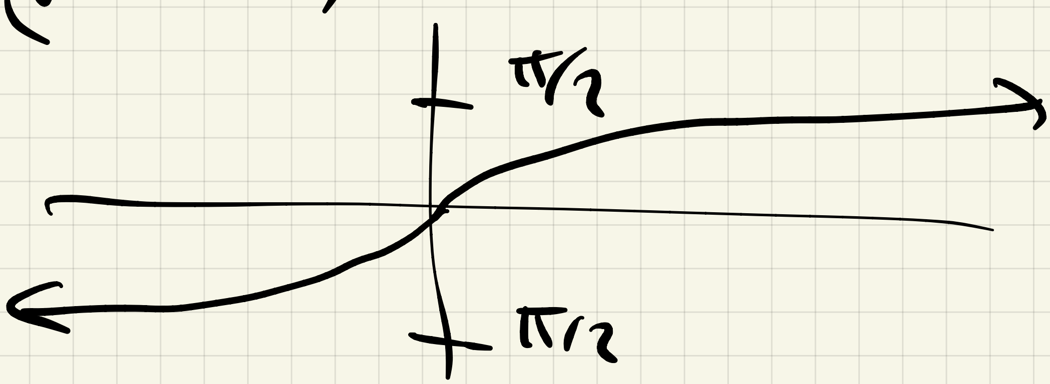
$\arcsin x + \arccos x = \pi/2$

$$\frac{d}{dx}(\arctan x) = \frac{1}{\sec^{2}(\underbrace{\arctan x}_{\theta})}$$



$\Rightarrow \sec\theta = \sqrt{x^2+1}$

$$\frac{1}{\left(\sqrt{x^2+1}\right)^2} = \frac{1}{x^2+1}$$

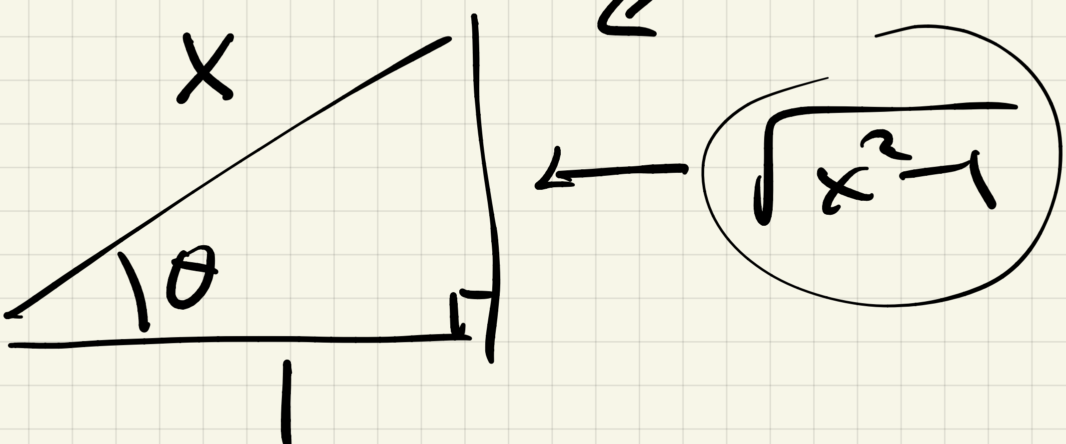


Similarly: $\frac{d}{dx}(\operatorname{arccot} x) = \frac{-1}{x^2+1}$

$$\frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{\underbrace{\sec(\operatorname{arcsec} x)}_{x} \cdot \tan(\operatorname{arcsec} x)}$$

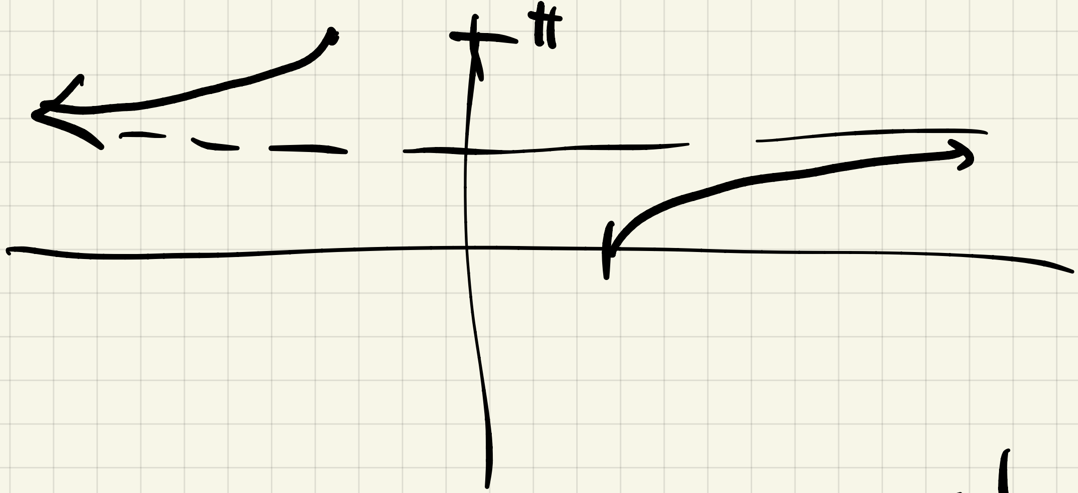
$g = \operatorname{arcsec} x$

$f = \sec x$



$$\tan(\operatorname{arcsec} x) = \sqrt{x^2-1}$$

$$\frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{|x|\sqrt{x^2-1}}$$



$$\frac{d}{dx}(\operatorname{arccsc} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$\frac{|x|}{\cancel{q = \sqrt{1-s^2}}}$$

$$y = \boxed{s\sqrt{1-s^2}} + \arccos s$$

$$y' = \sqrt{1-s^2} + s\cdot\frac{1}{2}(1-s^2)^{-1/2}(-2s) - \frac{1}{\sqrt{1-s^2}}$$

$$(1-s^2)^{1/2}$$

$$\sqrt{1-s^2} + \frac{-s^2}{\sqrt{1-s^2}} - \frac{1}{\sqrt{1-s^2}}$$

$$\frac{\cancel{1}-s^2 - s^2 - \cancel{1}}{\sqrt{1-s^2}} = \frac{-2s^2}{\sqrt{1-s^2}}$$