2/27/ Calc I Exam 1 $\longrightarrow$ 3/21 ?

Quiz 9

1. (a) $f = x^9 - 7x^3$

$f' = 9x^8 - 21x^2$

(b) $f = \frac{8}{x^2} - 12\sqrt[3]{x}$

$= \underline{8x^{-2} - 12x^{4/3}}$

$f' = -16x^{-3} - 4x^{-2/3}$

(c) $f = \underbrace{(3x^3 + 2x^2)}_{u} \cdot \underbrace{e^x}_{v}$

$\overset{u'}{(9x^2 + 4x)}e^x + (3x^3 + 2x^2)e^x$

$(3x^3 + 11x^2 + 4x)e^x$

(d) $f = \dfrac{(3x^3+2x^2)}{e^x}$ $\qquad \dfrac{u}{v}$

$$\left(\frac{vu' - uv'}{v^2}\right)$$

$$f' = \frac{e^x(9x^2+4x) - (3x^3+2x^2)e^x}{(e^x)^2}$$

$$= \frac{\cancel{e^x}(-3x^3+7x^2+4x)}{(\cancel{e^x})(e^x)} =$$

$$\frac{-3x^3+7x^2+4x}{e^x}$$

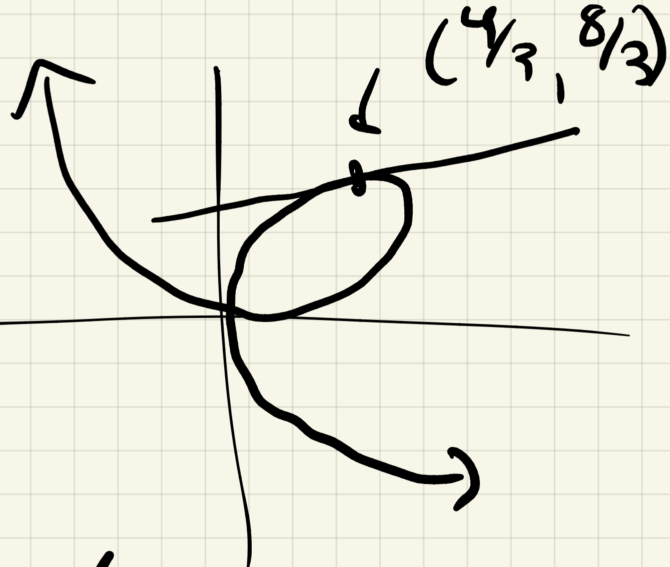
<u>Last time</u> Chain Rule

Implicit differentiation

($\frac{d}{dx}$ of an equation)

<u>Ex)</u> $x^3+y^3-6xy=0$

$$\left(\frac{4}{3}\right)^3 + \left(\frac{8}{3}\right)^3 - 6\left(\frac{4}{3}\right)\left(\frac{8}{3}\right)$$



$$\underbrace{\frac{64}{27} + \frac{512}{27}} - \frac{192}{9}$$

$$\frac{576}{27} - \frac{576}{27} = 0 \checkmark$$

$$(f(x))^3$$

$$3(f(x))^2 \cdot f'(x)$$

Idea

$$\frac{d}{dx}\left(x^3 + y^3 - 6\underline{xy} = 0\right)$$

(think $y = f(x)$)

$$3x^2 + \underline{3y^2\frac{dy}{dx}} - 6\left(1 \cdot y + \underline{x \cdot \frac{dy}{dx}}\right) = 0$$

$$3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$$

$$(3y^2 - 6x)\frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y-3x^2}{3y^2-6x}$$

Answer: involves both x/y.

at $(x,y) = \left(\frac{4}{3}, \frac{8}{3}\right)$,

$$m = \frac{6\left(\frac{8}{3}\right) - 3\left(\frac{4}{3}\right)^2}{3\left(\frac{8}{3}\right)^2 - 6\left(\frac{4}{3}\right)} =$$

$$\frac{\frac{6\cdot 8}{3} - \frac{4^2}{3}}{\frac{8^2}{3} - \frac{24}{3}} = \frac{\frac{32}{3}}{\frac{40}{3}} = \frac{32}{40} = \frac{4}{5}$$

Ex2 Find the tangent line and normal line to ($\perp$ to tangent)

$$2(x^2+y^2)^2 = -25xy$$

at $(-1,2)$

Apply d/dx to both sides

$$2(x^2+y^2)^{2} = -25xy$$

2.4

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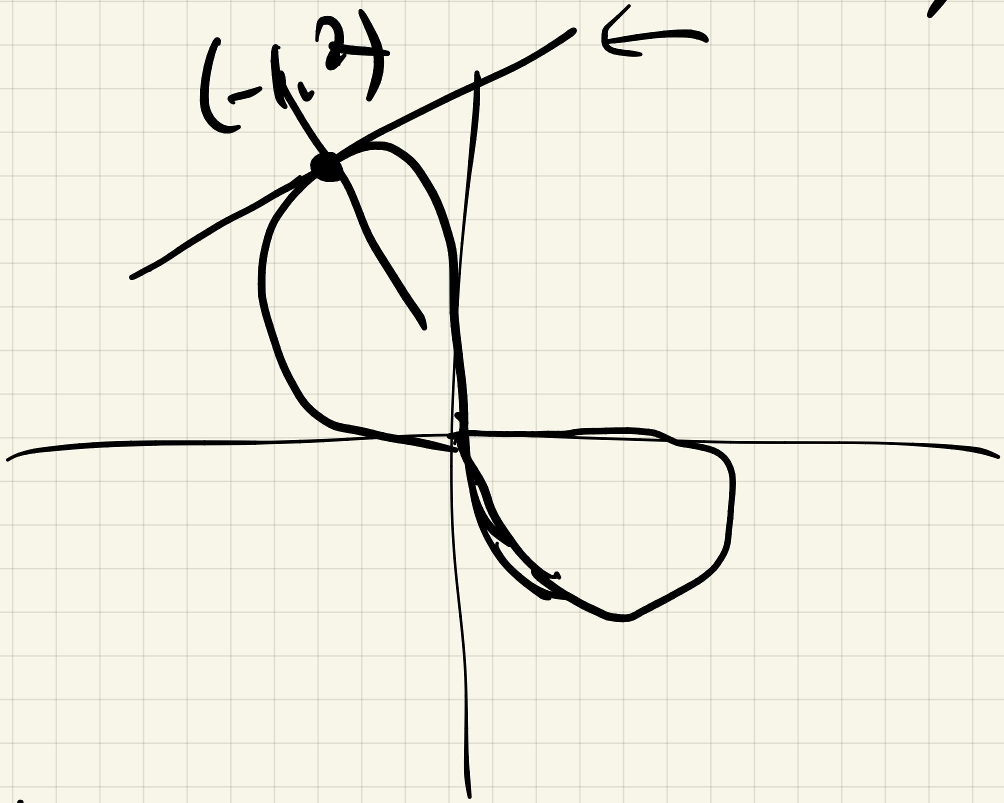
$$2\cdot 2\boxed{(x^2+y^2)^1(2x+2y\frac{dy}{dx})} = -25(1\cdot y + x\frac{dy}{dx})$$

$$8(x^2+y^2)x + 8(x^2+y^2)y\frac{dy}{dx} = -25y - 25x\frac{dy}{dx}$$

$$\left(8(x^2+y^2)y + 25x\right)\frac{dy}{dx} = -25y - 8(x^2+y^2)x$$

so $\dfrac{dy}{dx} = \dfrac{-25y - 8(x^2+y^2)x}{25x + 8(x^2+y^2)y}$

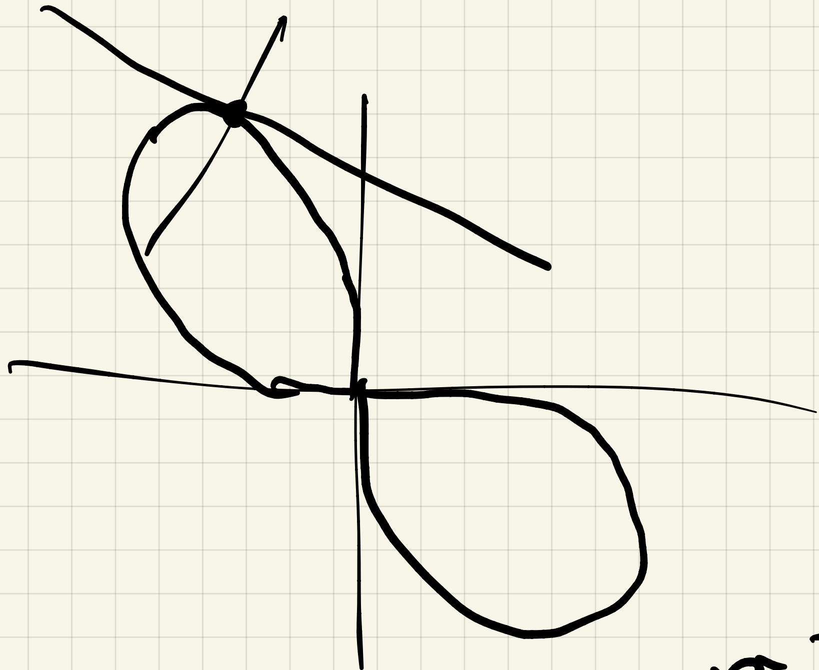
$m \overset{?}{=} ??$



At $(-1,2)$

$$\frac{dy}{dx} = \frac{-25(2) - 8(5)(-1)}{25(-1) + 8(5)(2)} =$$

$$\frac{-10}{55} = -\frac{2}{11}$$



$m = -2/11$

(a) <u>Tangent</u>: $P = (-1, 2)$

$$y - 2 = -\frac{2}{11}(x+1)$$

$$y = -\frac{2}{11}x + \frac{20}{11}$$

(b) <u>Normal</u> $m = 11/2$

$$y - 2 = \frac{11}{2}(x+1)$$

$$y = \frac{11}{2}x + \frac{15}{2}$$

Ex3 Find slopes of tangent lines at $(1,1)$ and $(1,\sqrt{3})$ to curve

$$y^4 - 4\boxed{y^2x^2} + 3x^4 = 0$$

$\frac{d}{dx}$

$$4y^3 \cdot \frac{dy}{dx} - 4\left(2y\frac{dy}{dx}\cdot x^2 + y^2 \cdot 2x\right) + 12x^3 = 0$$

$$\frac{dy}{dx}\left(4y^3 - 8x^2y\right) = -12x^3 + 8xy^2$$

$$\frac{dy}{dx} = \frac{-12x^3 + 8xy^2}{4y^3 - 8x^2y}$$

at $(1,1)$ $m = \frac{-4}{-4} = 1$

at $(1, \sqrt{3})$

$$\frac{-12 + 8(\sqrt{3})^2}{4(\sqrt{3})^3 - 8\sqrt{3}} =$$

$$\frac{12}{12\sqrt{3} - 8\sqrt{3}} = \frac{12}{4\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Note: Can solve for y:

$$y^4 - 4y^2x^2 + 3x^4 = 0$$

quad form →

$$(y^2)^2 - 4x^2(y^2) + 3x^4 = 0$$

$$\frac{4x^2 \pm \sqrt{16x^4 - 12x^4}}{2}$$

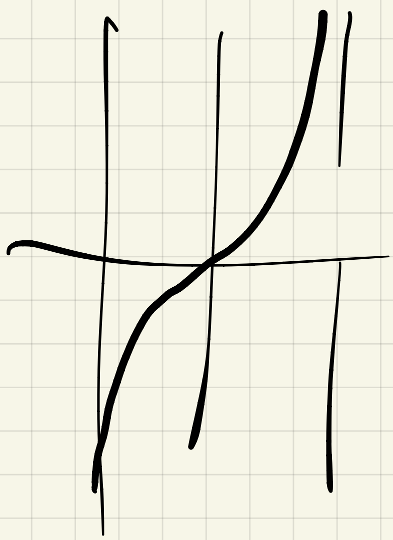
$$\frac{4x^2 \pm \sqrt{4x^4}}{2} =$$

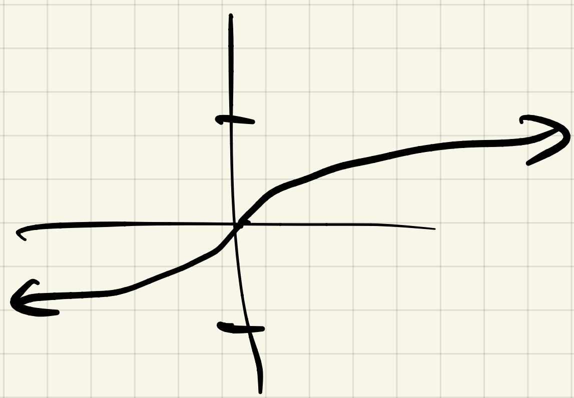
$$\frac{4x^2 \pm 2x^2}{2} = 2x^2 \pm x^2$$

$$3x^2, x^2$$

Ex 4 $\boxed{\tan y = x}$ find $\frac{dy}{dx}$

$y = \tan x$





$$\frac{d}{dx}(\tan y) = \frac{d}{dx}(x)$$

$$\sec^2 y \cdot \frac{dy}{dx} = 1$$

$$= \boxed{\cos^2 y}$$

so $\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{(\tan y)^2 + 1}$

$\sec^2 y = \tan^2 y + 1$

$$\frac{1}{x^2+1}$$

§3.8 / 3.9

derivatives of inverses;

Recall $g(x)$ is the inverse to $f(x)$ if

$$\begin{cases} f(g(x)) = x & \text{all } x \text{ in dom } g \\ g(f(x)) = x & \text{all } x \text{ dom } f \end{cases}$$

Ex) $f(x) = 4x^5 - 2$

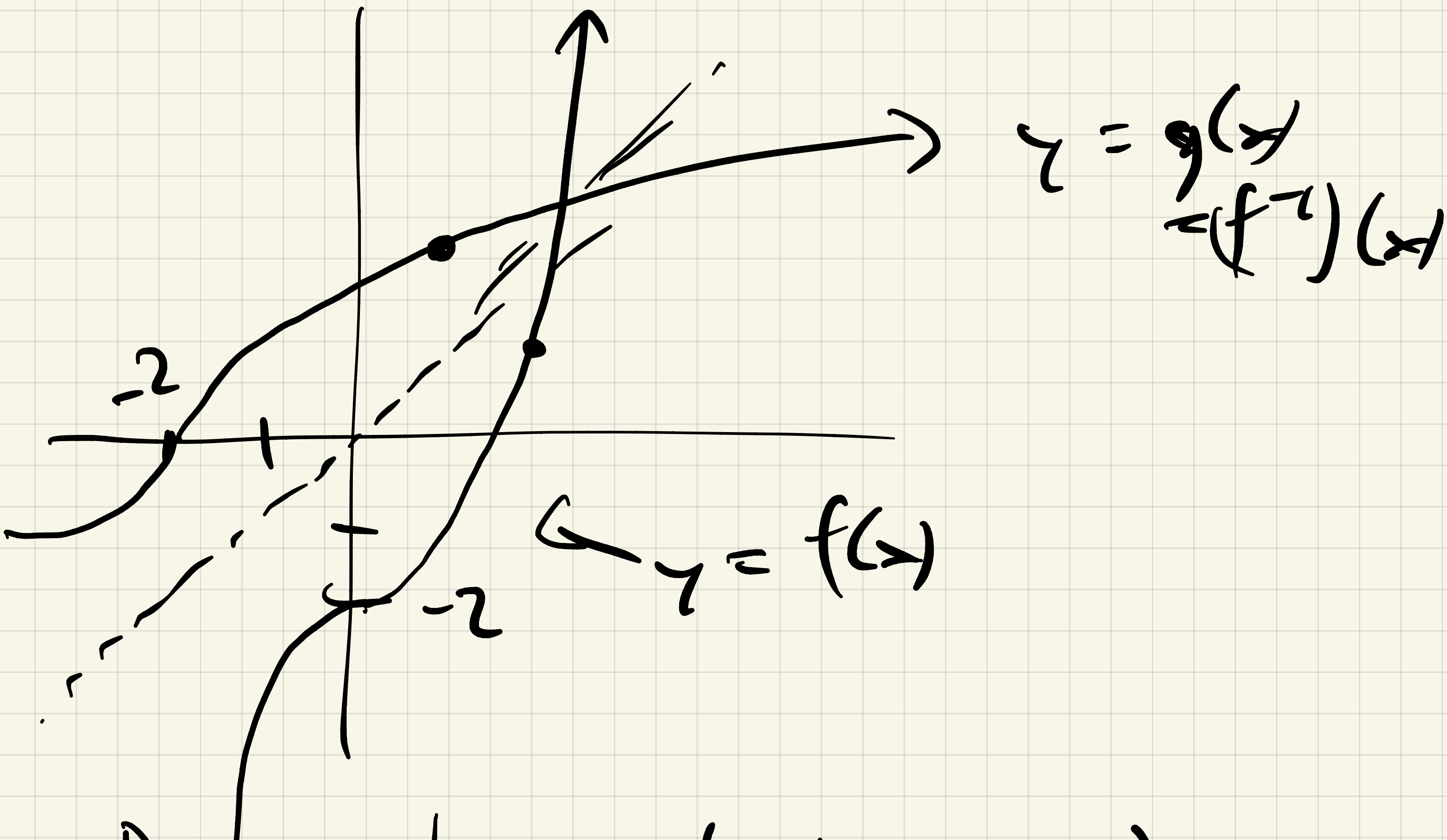
$$y = 4x^5 - 2$$

$$x = 4y^5 - 2$$

$$x+2 = 4y^5$$

$$\frac{x+2}{4} = y^5$$

$$y = \sqrt[5]{\frac{x+2}{4}}$$



Derivatives of inverse:

If $g(x) = f^{-1}(x)$, then

$$f(g(x)) = x$$

$$\frac{d}{dx} \quad f'(g(x)) \cdot g'(x) = 1$$

$$\boxed{g'(x) = \frac{1}{f'(g(x))}}$$

Test: on $\boxed{f = 4x^5 - 2}$
at $(1,2)$

$(2,1)$ is on graph of g

$f(1) = 2 \Rightarrow g(2) = 1$ ←

check: $g'(2) = \dfrac{1}{f'(g(2))} = \dfrac{1}{f'(1)}$

$f' = 20x^4$, $f'(1) = 20$,

$g'(2) = \dfrac{1}{20}$

Other way: $g(x) = \sqrt[5]{\dfrac{x+2}{4}}$

$g(x) = \left(\dfrac{x+2}{4}\right)^{1/5}$ $\qquad \dfrac{x}{4} + \dfrac{2}{4}$

$g'(x) = \dfrac{1}{5}\left(\dfrac{x+2}{4}\right)^{-4/5} \cdot \dfrac{1}{4}$ ←

$$g'(2) = \frac{1}{5} \cdot 1 \cdot \frac{1}{4} = \frac{1}{20} \checkmark$$

<u>Important example</u> !

$$y = f(x) = e^x \longleftarrow$$

$$g(x) = \ln x$$

$$\frac{d}{dx}(g(x)) = \frac{1}{f'(g(x))}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{e^{g(x)}} = \frac{1}{e^{\ln x}} =$$

$$\frac{1}{e^{\ln x}} = \frac{1}{x}, \quad \boxed{x > 0}$$