2/23/ Calc I :

Derivatives of all trig functions

y	y'
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cos x$	$-\sin x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

Chain Rule : $\frac{d}{dx}\left(f(\underset{u}{g}(x))\right)$

$$f'(\underbrace{g(x)}) \cdot g'(x)$$

OR $y = f(u)$ and $u = g(x)$ then

$$\frac{dy}{dx} = \frac{dy}{\underbrace{du}} \cdot \frac{du}{dx}$$

Special cases:

$u = g(x) =$ function of x

$$\frac{d}{dx}(u^n) = n u^{n-1} \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx} \quad \longleftarrow$$

$$\frac{d}{dx}(\sin(u)) = \cos u \cdot \frac{du}{dx}$$

$\vdots$

Ex)

f	f'
$\sin(x^3+5)$	$\underbrace{(\cos(x^3+5)) \cdot 3x^2}_{\parallel}$ $3x^2\cos(x^3+5)$
$e^{\tan x}$	$e^{\tan x} \cdot \sec^2 x$

$$\tan(e^x) \quad \Big| \quad \sec^2(e^x) \cdot e^x$$

$$\cos(\sqrt{x+1}) \quad \Big| \quad -\sin(\sqrt{x+1}) \cdot$$

$$(x+1)^{1/2} \qquad \frac{1}{2}(x+1)^{-1/2} \cdot 1$$

$$= \frac{-\sin(\sqrt{x+1})}{2\sqrt{x+1}}$$

Ex $\quad y = \sqrt{\cos(x^3 + e^{\sin x})}$

[w]

$$\left(\cos(x^3 + e^{\sin x})\right)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\cos(x^3 + e^{\sin x})\right)^{-1/2} \cdot$$

$$-\sin(x^3 + e^{\sin x}) \cdot$$

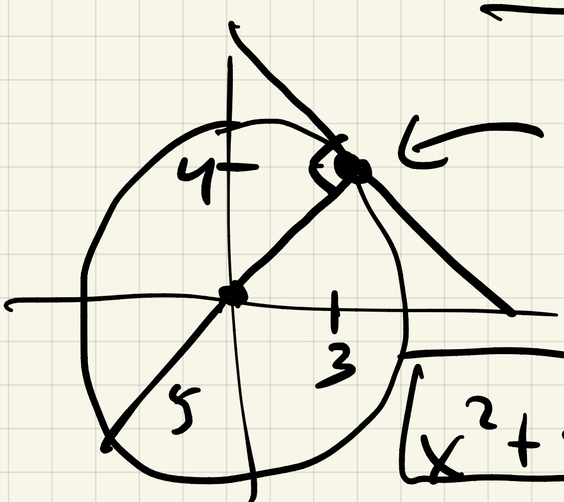
$$\left[3x^2 + e^{\sin x} \cdot \cos x\right]$$

(b) $y = e^{\tan\left(\sqrt{\cos(4x^8+x^5)}\right)}$

$$\left(\begin{array}{l} f(\square)' \\ f'(\square)\cdot\square' \end{array} \right)$$

$$\frac{dy}{dx} = e^{\tan\left(\sqrt{\cos(4x^8+x^5)}\right)} \cdot \sec^2\left(\sqrt{\cos(4x^8+x^5)}\right) \cdot \frac{1}{2}\left(\cos(4x^8+x^5)\right)^{-1/2} \cdot -\sin(4x^8+x^5) \cdot (32x^7+5x^4)$$

§3.7 Implicit differentiation

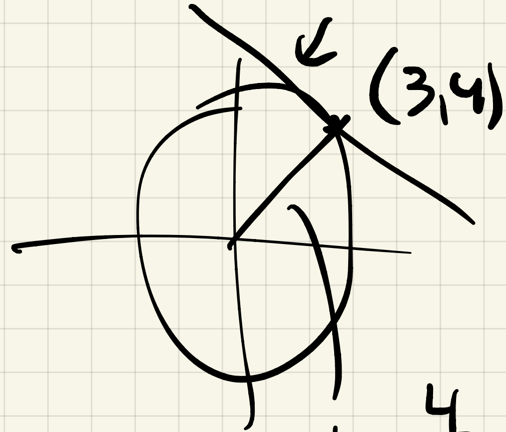


slope of tangent $= m$
$= ??$

$\boxed{x^2+y^2=25}$

$m = -3/4$

① Geometry:

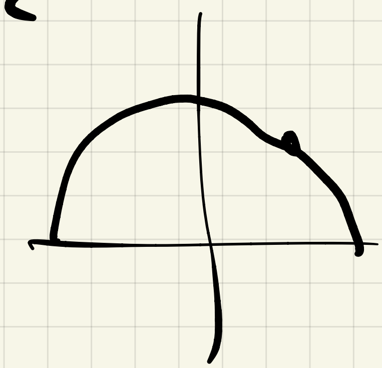


② Calculus: derivative: $m' = \frac{4}{3}$

$$y^2 = 25 - x^2$$

$$y = \pm\sqrt{25 - x^2}$$

$$y = \sqrt{25 - x^2}$$



$$\frac{dy}{dx} = \frac{1}{2}(25 - x^2)^{-1/2} \cdot (-2x)$$

at $x = 3$:

so $m = \left.\frac{dy}{dx}\right|_{x=3}$

$$\frac{1}{2}(16)^{-1/2}(-2 \cdot 3)$$

$$= \frac{-\cancel{x}\cdot 3}{\cancel{2}\sqrt{16}} = \frac{-3}{4}$$

(3) $x^2 + y^2 = 25$

<u>Idea</u>: Think of $y = f(x)$, as a function, differentiate equation

$$\frac{d}{dx}\quad x^2 + \boxed{(f(x))^2} = 25$$

$$2x + 2f(x)\cdot f'(x) = 0$$

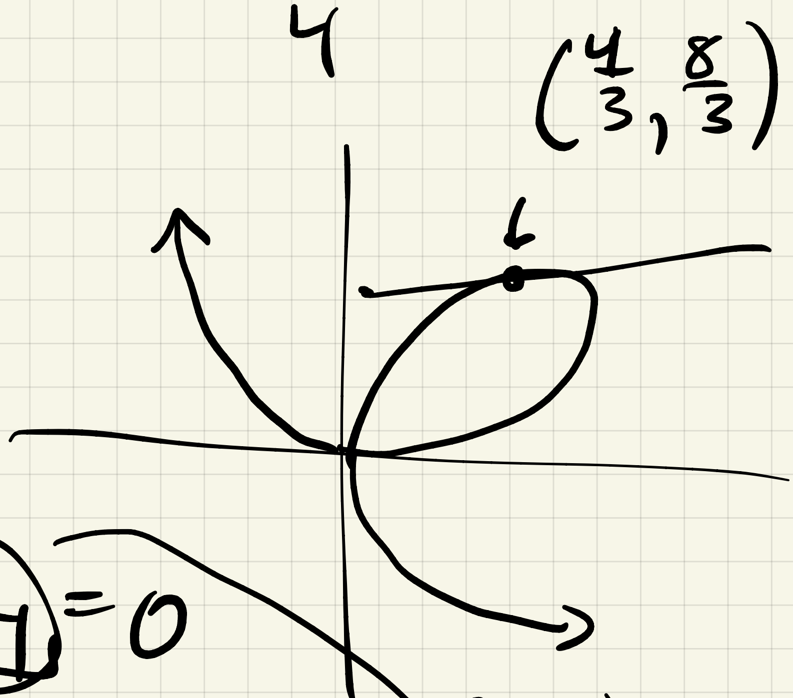
$$2f(x)\, f'(x) = -2x$$

$$f'(x) = \frac{-2x}{2f(x)}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}}$$

at $(x,y) = (3,4)$

$$m = \frac{-3}{4} \checkmark$$



Ex)

$$x^3 + \boxed{y^3} - 6xy = 0$$

(y = function of x)

d/dx

$$0 = 3x^2 + 3y^2 \cdot \frac{dy}{dx} - 6\left[1 \cdot y + x \cdot \frac{dy}{dx}\right]$$