2/22/ Calc

Quiz 8

$f(x) = \frac{3}{x} + 4$

$f(x+h$

1(a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} =$

$$\lim_{h \to 0} \frac{\left(\frac{3}{x+h} + 4\right) - \left(\frac{3}{x} + 4\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{x+h} + \cancel{4} - \frac{3}{x} - \cancel{4}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3x}{x(x+h)} - \frac{3(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\left[\frac{\cancel{3x} - \cancel{3x} - 3h}{x(x+h)}\right]\left(\frac{1}{h}\right)}{h \left]\left(\frac{1}{h}\right)\right.}$$

$$= \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}\,x(x+h)} =$$

$$\lim_{h \to 0} \frac{-3}{x(x+h)} = \frac{-3}{x^2}$$

(<u>note</u>: $f = \frac{3}{x} + 4 = 3x^{-1} + 4$

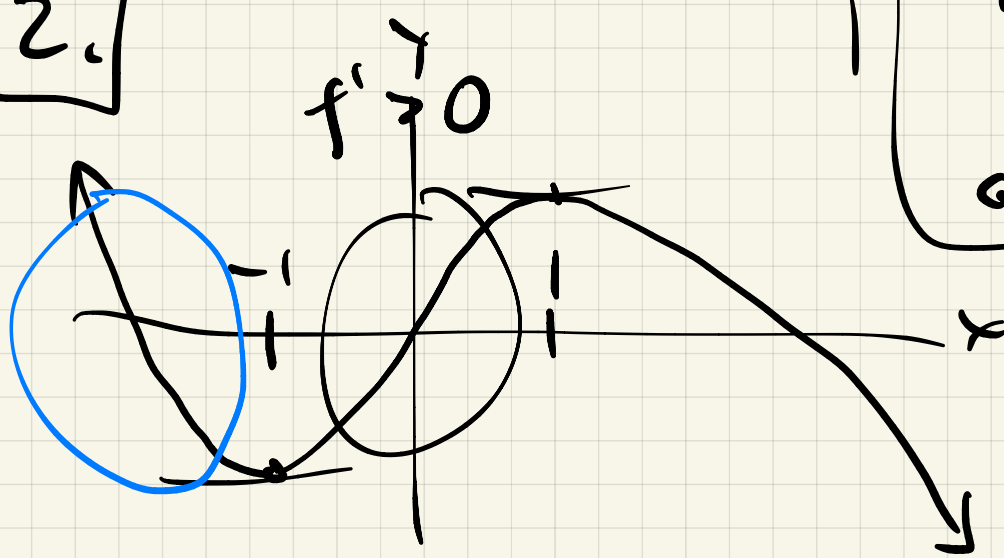
$$f' = -3x^{-2} + 0$$

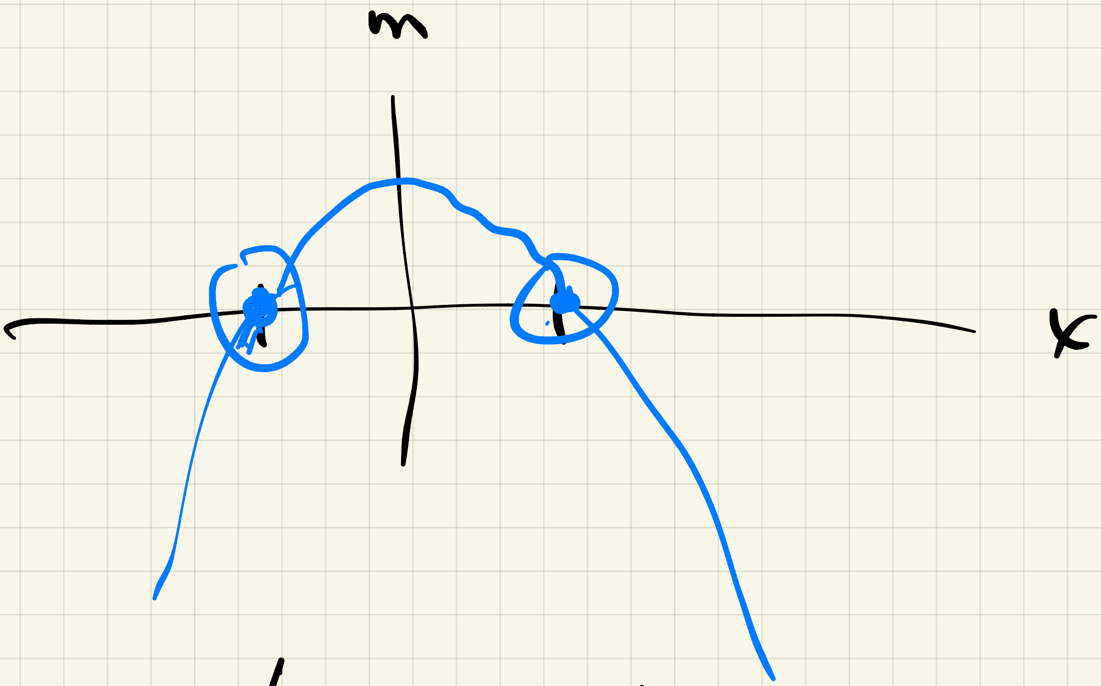
$$= \frac{-3}{x^2}$$)

(1) $f'(1) = \frac{-3}{1^2}$

$\boxed{f'(a) = \text{slope of tangent line to curve at at } x = a}$

$\boxed{2.}$





§3.3/3.5 derivative rules

$$\frac{d}{dx}(u\cdot v) = \frac{du}{dx}\cdot v + \frac{dv}{dx}\cdot u$$

$$\boxed{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\,dv/dx}{v^2}}$$

Aside: Reciprocal rule

$$\boxed{\frac{d}{dx}\left(\frac{1}{v}\right) = \frac{-dv/dx}{v^2}}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{d}{dx}\left(u\cdot\frac{1}{v}\right)$$

$$\frac{du}{dx}\cdot\frac{1}{v} + u\,\underbrace{\frac{d}{dx}\left(\frac{1}{v}\right)}$$

$$\frac{v}{v}\frac{du}{dx}\cdot\frac{1}{v} + u\left(\frac{-\,dv/dx}{v^2}\right)$$

$$\frac{v\cdot\frac{du}{dx} - u\,\frac{dv}{dx}}{v^2}$$

Last time:

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

Ex1 $\quad y = \frac{5\cos x}{x+1} = 5\cos x\cdot\left(\frac{1}{x+1}\right)$

$\cancel{\frac{dy}{dx}} =$

$$\frac{dy}{dx} = 5(-\sin x)\left(\frac{1}{x+1}\right) + 5\cos x\left(\frac{-1}{(x+1)^2}\right)$$

$$= \frac{-5(x+1)\sin x - 5\cos x}{(x+1)^2}$$

<u>Ex2</u> (a) $y = \tan x = \dfrac{\sin x}{\cos x}$

$$\frac{dy}{dx} = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} =$$

$$\frac{1}{\cos^2 x} = \sec^2 x$$

(b) $y = \sec x = \dfrac{1}{\cos x}$

<u>remk</u> : $\dfrac{a \cdot b}{a} = \dfrac{b}{1} = b$

$\dfrac{a+b}{a} \neq \dfrac{b}{1}$

$$\frac{dy}{dx} = \frac{-\frac{d}{dx}(\cos x)}{\cos^2 x} =$$

$$= -\frac{(-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$$

$$\parallel$$

$$\sec x \tan x$$

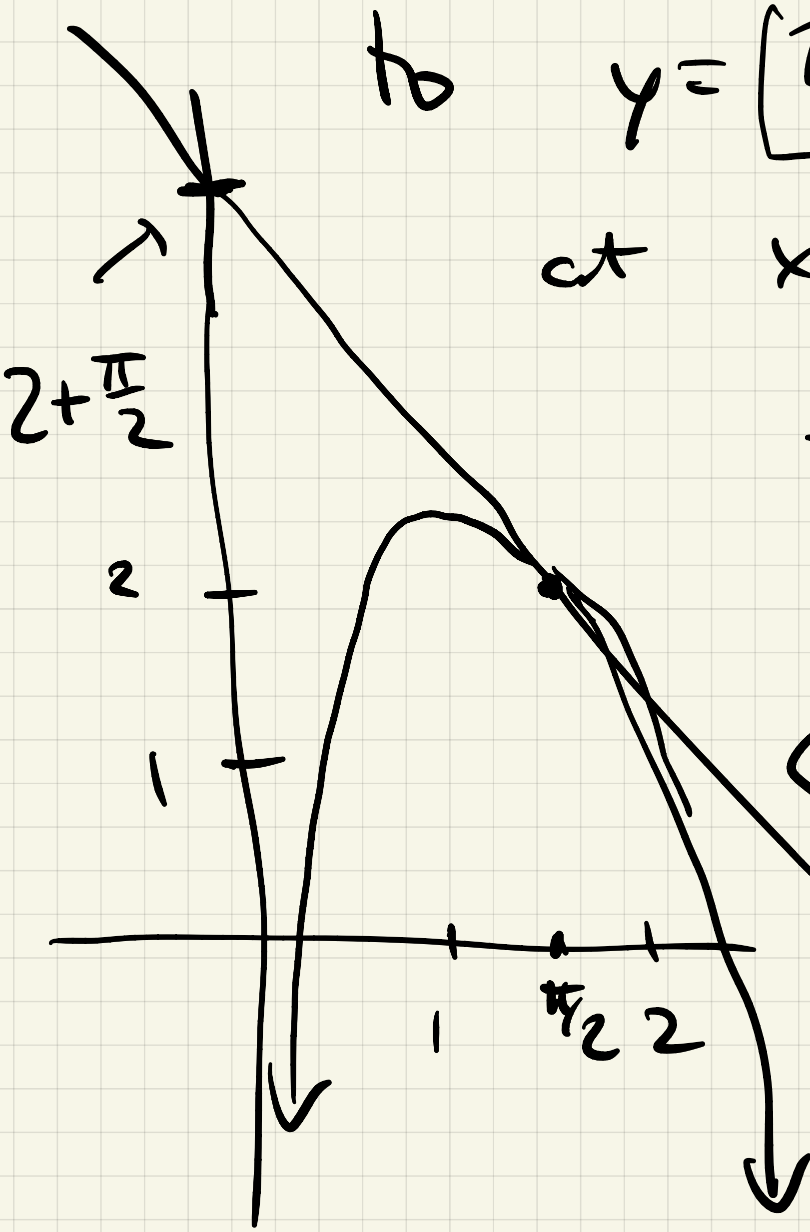
Similar for $\cot x / \csc x$

y	y'
$\sin x$	$\cos x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\cos x$	$-\sin x$
$\cot x$	$-\csc^2 x$
$\csc x$	$-\csc x \cot x$

Ex) Find the tangent line

to $y = \boxed{4 + \cot x - 2\csc x}$

at $x = \pi/2$ "$f(x)$"



$f(\pi/2) = 2$

find (

$m = -1$

$$\frac{dy}{dx} = \frac{d}{dx}\left((4) + \cot x - 2\csc x\right)$$

$$= \underbrace{0 - \csc^2 x - 2(-\csc x \cot x)}_{-\csc^2 x + 2\csc x \cot x}$$

at $x = \pi/2$, $\quad -1 \quad + 0$

So $\boxed{m = -1}$ $P = (\pi/2, 2)$

$$y - 2 = -1(x - \pi/2)$$

$$\boxed{y = -x + \boxed{\pi/2 + 2}}$$

Ex2 $y = \sec^2 x - \tan^2 x$ "1"

$$= \underbrace{(\sec x)(\sec x)} - \boxed{(\tan x)(\tan x)}$$

$$\frac{dy}{dx} = \underbrace{(\sec x \tan x)(\sec x) + (\sec x)(\sec x \tan x)}_{2\sec^2 x \tan x}$$

$$- \Big(\underbrace{\sec^2 x \tan x + \tan x \sec^2 x}_{2\sec^2 x \tan x} \Big)$$

$$= 0 \;!$$

Why??

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 = \sec^2 x - \tan^2 x$$

§3.6 Chain Rule:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot \boxed{g'(x)}$$

$(f \circ g)(x)$

composition

Alternate statement:

If $y = f(u)$ and $\boxed{u = g(x)}$

then $\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$

Ex (a) $y = h(x) = \sin(x^2+2)$

$$= f(g(x))$$

$$f(x) = \sin x$$

$$g(x) = x^2+2$$

$$\frac{dy}{dx} = f'(\underline{g(x)}) \cdot g'(x)$$

$$f'(x) = \cos x$$

$$g'(x) = 2x$$

$$\cos(x^2+2) \cdot 2x$$

The other way: $f = x^2+2 \quad f' = 2x$

$g = \sin x \quad g' = \cos x$

$$f'(g(x)) \cdot g'(x) =$$

$$2\sin x \cdot \cos x$$

but, $f(g(x)) = \sin^2 x + 2$

Comparison $\boxed{y = \sin u}$ $\boxed{u = x^2+2}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos(u) \cdot 2x$$

$$= \cos(x^2+2) \cdot 2x \checkmark$$

$$= 2x\cos(x^2+2)$$

(b) $y = \sqrt{x^3+2}$

$= f(g(x))$

$f(x) = \sqrt{x} = x^{1/2}$ $\quad f' = \frac{1}{2}x^{-1/2}$

$g(x) = x^3+2$ $\quad g' = 3x^2$

$$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$$

$$\frac{1}{2}(x^3+2)^{-1/2} \cdot 3x^2$$

$$= \frac{3x^2}{2\sqrt{x^3+2}}$$

(c) $y = \cos^{50} x = (\cos x)^{50}$

$= f(g(x))$

$f(x) = x^{50}$ | $f' = 50x^{49}$

$g(x) = \cos x$ | $g' = -\sin x$

$\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

$= 50(\cos x)^{49}(-\sin x)$

$= -50\cos^{49} x \sin x$

<u>Chain</u>: $y = f(\square)$

$y' = f'(\square) \cdot \square'$

$$\boxed{\begin{array}{l}\underline{\text{Generalized power rule}}: \\ y = u^n \quad (u = \text{function of } x).\end{array}}$$

then $\frac{dy}{dx} = \boxed{n u^{n-1}} \cdot \frac{du}{dx}$

Ex) $y = (\underline{x + \sin x})^{11}$

(a) $\frac{dy}{dx} = 11(x+\sin x)^{10} \cdot (1 + \cos x)$

(b) $y = \sqrt[8]{3e^x + 4x}$

$= (3e^x + 4x)^{1/8}$

$y' = \frac{1}{8}(3e^x + 4x)^{-7/8} \cdot (3e^x + 4)$

(c) $y = 2(x^2+1)^{-3} = \frac{2}{(x^2+1)^3}$

$y' = 2(-3)(\underline{\underline{x^2+1}})^{-4} \cdot (2x)$

$= \frac{-12x}{(x^2+1)^4}$

Trig/ transcendental :

$u =$ function of x

$$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\sin u) = \cos u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \cdot \frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u \cdot \frac{du}{dx}$$

$$y = \left(e^{x^3}\right)' = e^{x^3} \cdot 3x^2$$

$$y = \tan(x^2 + \sin x)$$

$$y' = \sec^2(x^2 + \sin x)\left[2x + \cos x\right]$$