2/16/ Calc I : U grades

Exam I

avg 123

12

82%

150 —

8

135 —

8

120 —

1

105 —

3

1(e) $\dfrac{\boxed{\ln(3x+6)}}{x-5}$

domain

$x \neq 5$

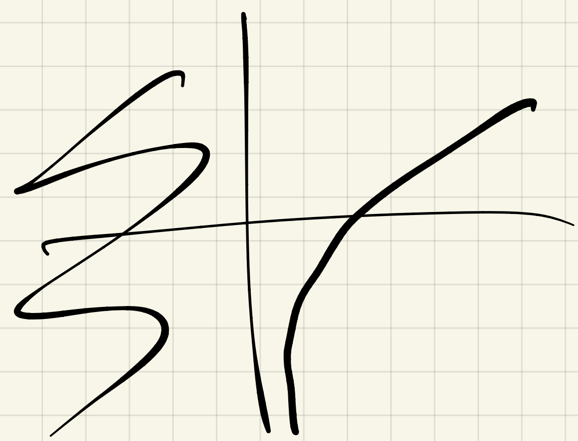
$x > -2$

$3x+6>0$

$\Downarrow$

$x>-2$

$(-2,5)\cup(5,\infty)$



y int $\dfrac{\ln 6}{-5}$

x int $\ln(3x+6)=0$

$3x+6=e^0=1$

$3x=-5$ $x=-5/3$

#2 (c) $\dfrac{x^2+x-6}{x^2-9} = \dfrac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}(x-3)}$ $\dfrac{1}{0^+}$

$x \to 3^+$ $x > 3$ $\boxed{+\infty}$

(d) $\lim\limits_{x \to 5} \dfrac{\sqrt{x+11} - 4}{x-5} \dfrac{(\sqrt{x+11} + 4)}{(\sqrt{x+11} + 4)} =$

$\lim\limits_{x \to 5} \dfrac{x+11-16}{\cancel{(x-5)}(\sqrt{x+11}+4)} = \cancel{x-5}$ $= \dfrac{1}{8}$

(e) $\lim\limits_{x \to 0} \dfrac{3 \sin 4x}{7x} = \lim\limits_{x \to 0} \dfrac{4}{7} \cdot \boxed{\dfrac{3 \sin 4x}{4x}}$

$\lim\limits_{u \to 0} \dfrac{\sin u}{u} = 1$ $= \dfrac{12}{7}$ 1

#3 $x \sim 1$ $x = .99$ $.999$

$x = 1.01$ 1.001

#4 $\lim\limits_{x \to 3^-} f(x) = \dfrac{x}{x^2+2x}\Big|_{x=3}$ $\dfrac{1}{5}$

$$\lim_{x \to 3^+} f(x) = \frac{1}{8-x} \qquad \frac{1}{5}$$

$$f(3) \qquad \frac{1}{8}$$

§3.3

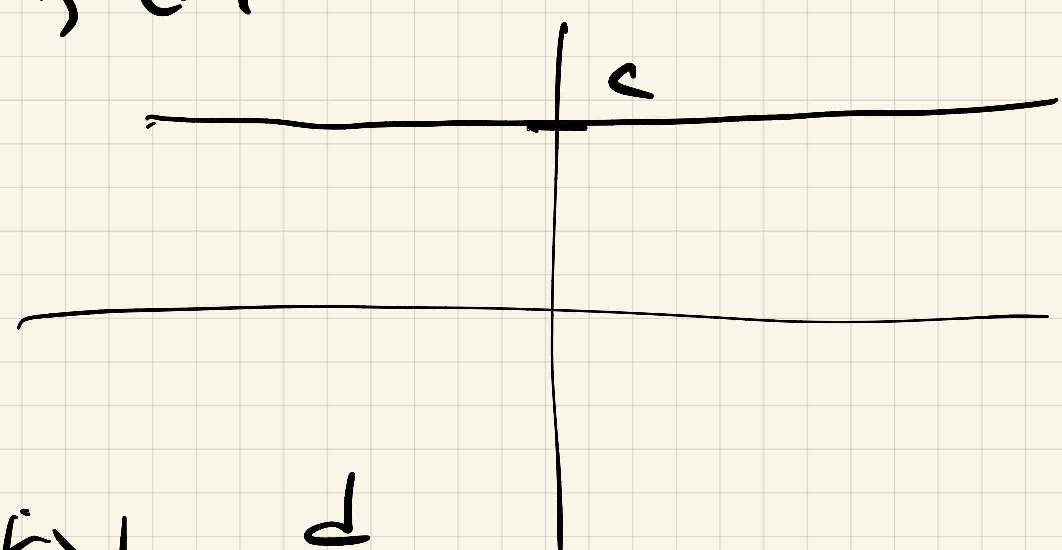
~~3.2~~ Derivative $y = f(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

§3.3 Derivative Rules

① **Constant Rule** If $f(x) = c$ is constant, then

$$f'(x) = 0$$



Notations: $f'(x)$ $\quad y'(x)$

$\frac{d}{dx}(f)$ $\quad \frac{df}{dx}$ $\quad \frac{dy}{dx}$

$\frac{d}{dx}(5) = 0$ $\quad \boxed{\frac{d}{dx}(e) = 0}$

$\frac{d}{dx}(\pi + 7) = 0$

② Power Rule: $n > 0$ integer,

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

Why?

$$\frac{d}{dx}[x^n] = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{x^n} + n x^{n-1} h + \frac{n(n-1)}{2} x^{n-2} h^2 + \frac{n(n-1)(n-2)}{3 \cdot 2 \cdot 1} x^{n-3} h^3 + \ldots + h^n - \cancel{x^n}}{h}$$

i.e. $(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$$\lim_{h \to 0} \frac{nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \dots + h^n}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}\left(nx^{n-1} + \frac{n(n-1)}{2}x^{n-2}h + \dots h\right)}{\cancel{h}}$$

nx^{n-1} ✓ $\qquad 0$

Ex 2

$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(x^{50}) = 50x^{49}$$

constant

$$\frac{d}{dx}(e^5) = 0$$

$$\frac{d}{dx}\left(\frac{1}{x^{50}}\right) = \frac{d}{dx}\left(x^{-50}\right) = -50x^{-51}$$

General Power Rule:

If n is a real number and $\boxed{x^n / x^{n-1}}$ are defined

Then $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\frac{d}{dx}(\sqrt[3]{x}) = \frac{d}{dx}(x^{1/3})$$

$$= \tfrac{1}{3} x^{-2/3}$$

$$= \frac{1}{3x^{2/3}} = \frac{1}{3\sqrt[3]{x^2}}$$

$$\frac{d}{dx}(\sqrt[4]{x}) = \frac{1}{4} x^{-3/4} \quad x \neq 0$$

valid for $x \geq 0$

Combinations

(A) c constant, then

$$\frac{d}{dx}\left(c\,f(x)\right) = c\,\frac{d}{dx}\left(f(x)\right)$$

(13) $\frac{d}{dx}\left(f(x) \pm g(x)\right) =$

$$\frac{d}{dx}f(x) \pm \frac{d}{dx}(g(x))$$

(a) $\frac{d}{dx}\left(\frac{20}{x^{50}}\right) = \frac{d}{dx}\left(20\,x^{-50}\right) =$

$$20\left(\frac{d}{dx}\,\underline{\underline{x^{-50}}}\right)$$

$$\underline{20 \cdot -50}\,x^{-51} \qquad 7(-1)x^{-2}$$

$$-1000\,x^{-51} \qquad 7x^{-1}$$

(b) $\frac{d}{dx}\left(\underbrace{3x^4} + \boxed{\frac{7}{x}} - 9\sqrt[3]{x}\right) =$

$$9x^{1/3}$$

$$12x^3 - \frac{7}{x^2} - 3x^{-2/3}$$

(c) $\frac{d}{dx}(x^2-4x) = 2x-4$

(C) <u>Product Rule</u>:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) =$$

$$\left[\frac{d}{dx}(f(x))\right]\cdot g(x) + f(x)\frac{d}{dx}(g(x))$$

<u>Ex</u> $y = (\underbrace{x^4-x^2}_{f})(\underbrace{x^4+x^2}_{g})$

$$\frac{dy}{dx} = \underline{(4x^3-2x)(x^4+x^2)} +$$

$$\underline{(x^4-x^2)(4x^3+2x)}$$

$\parallel$ simplify

$$4x^7-2x^5+4x^5-2x^3$$

$$4x^7+2x^5-4x^5-2x^3 =$$

$8x^7 - 4x^3$

Check: $y = (x^4 - x^2)(x^4 + x^2)$

$= x^8 - x^4$

$\frac{dy}{dx} = 8x^7 - 4x^3$ ✓

(b) $y = \underbrace{5}_{f}\underbrace{(x^3 - x^{10})}_{g}$

$y' = \cancel{0 \cdot (x^3 - x^{10})} + 5(3x^2 - 10x^9)$

$15x^2 - 50x^9$

D $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - g'(x)f(x)}{g(x)^2}$

$\frac{d}{dx}\left(\frac{1}{x^3}\right) = \frac{(x^3)(0) - (3x^2)(1)}{(x^3)^2} =$

0

$$\frac{-3x^2}{x^6} = \frac{-3}{x^4} \checkmark$$

<u>check</u> $\frac{d}{dx}(x^{-3}) = -3x^{-4}$

$$= \frac{-3}{x^4} \checkmark$$