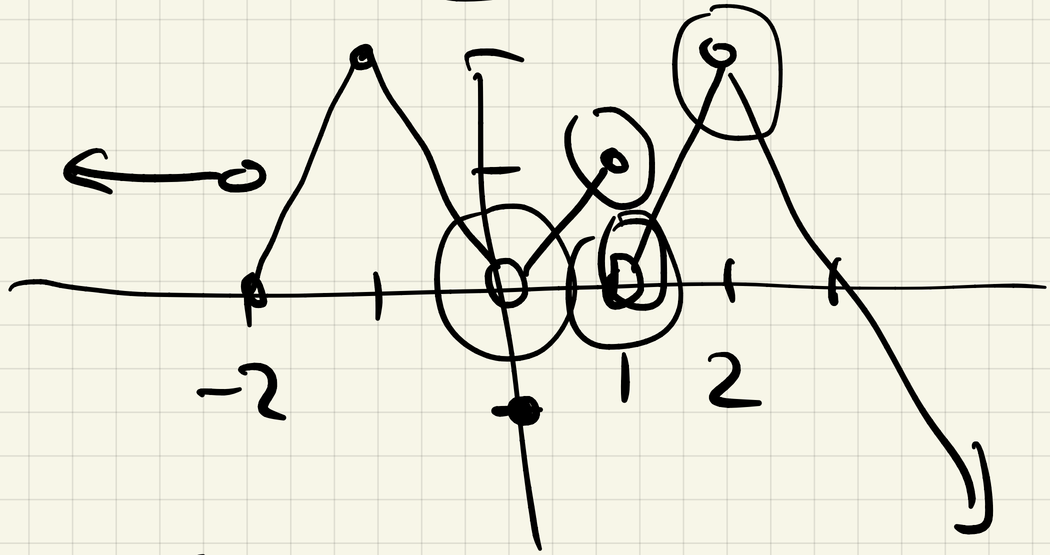
2/13/ Calc I Exam I → Thursday

Quiz 7

$\boxed{1}$



$\lim_{x \to 0^-} f = 0$ $\lim_{x \to 1^-} = 1$ $\lim_{2^-} f = ?$

$\lim_{x \to 0^+} f = 0$ $\lim_{x \to 1^+} f = 0$ $2^+ = 2$

$\lim_{x \to 0} f = 0$ $\lim_{x \to 1} f = DNE$ $2 = 2$

(j) $x = -2, 0, 1, 2$

$\boxed{2}$

$$g(x) = \begin{cases} -3\dfrac{\sin x}{x} & x < 0 \\ 1 - 4\cos x & x \geq 0 \end{cases}$$

$$\lim_{x \to 0^-} g = \lim_{x \to 0^-} -3 \boxed{\frac{\sin x}{x}} = -3$$

$$\frac{\sin x}{x} = 1$$

$$\lim_{x \to 0^+} g = \lim_{x \to 0} 1 - 4\cos x = -3$$

$$1 - 4\cos 0 = 1 - 4 = -3$$

$$g(0) = 1 - 4\cos 0 = -3$$

Last week: Slope to a curve

$y = f(x)$ at $P = (x_0, f(x_0))$ is

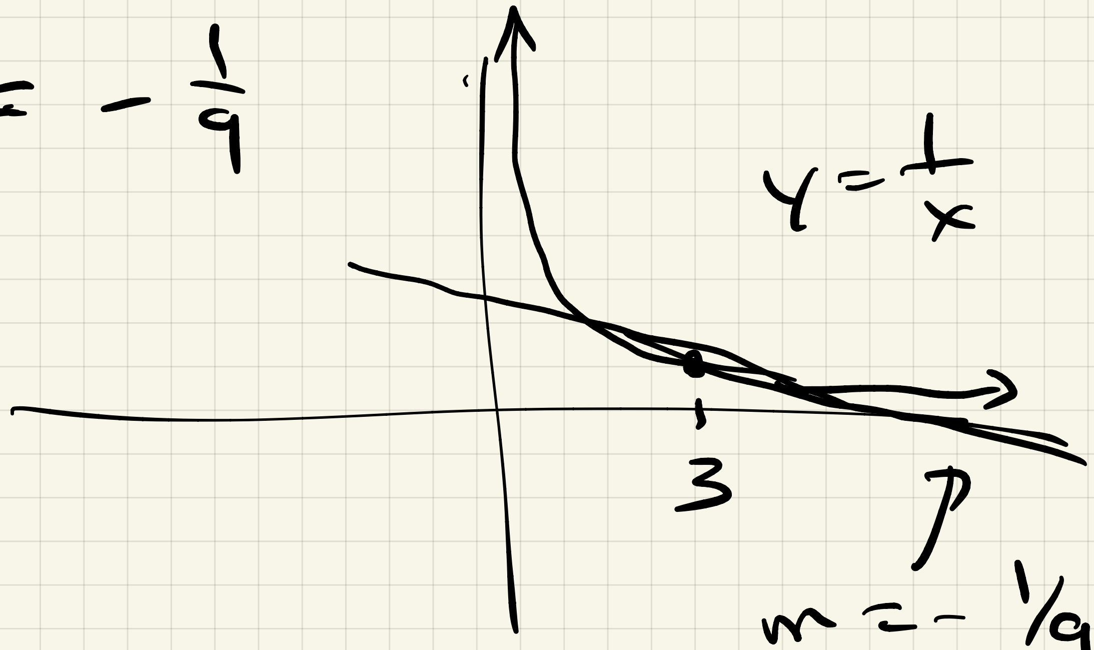
$$m = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

= slope of tangent line

Last Thursday, we found

for $f(x) = \frac{1}{x}$, $x_0 = 3$

$\Rightarrow m = -\frac{1}{9}$



<u>Ex1</u>: Find value of x_0 so that the slope of the curve is $-1/10$.

<u>OK</u>: let x_0 be fixed, but unknown.

$$m = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h} =$$

$$\lim_{h \to 0} \frac{\frac{1}{x_0+h} - \frac{1}{x_0}}{h} =$$

$$\lim_{h \to 0} \frac{\dfrac{\cancel{x_0} - (\cancel{x_0} + h)}{x_0(x_0+h)}}{h} =$$

$$\lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}\, x_0(x_0+h)} =$$

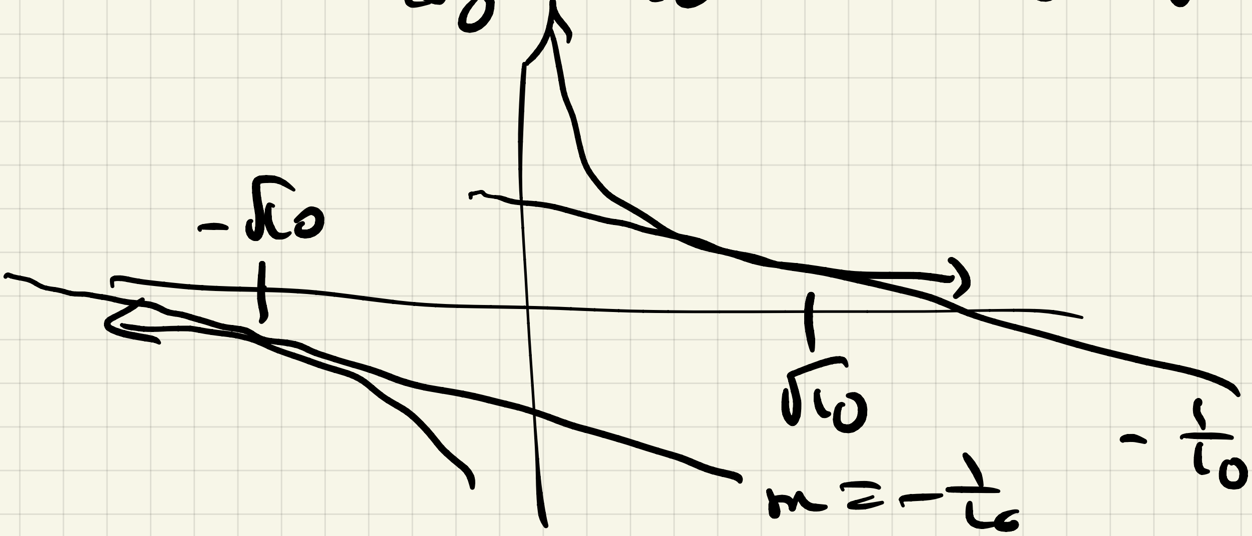
$$\lim_{h \to 0} \frac{-1}{x_0(x_0+h)} = -\frac{1}{x_0^2}$$

$\left(\underline{\text{Notice}} \quad x_0 = 3 \Rightarrow m = -\frac{1}{9} \right)$

we need

$$-\frac{1}{x_0^2} = m = -\frac{1}{10} \Rightarrow$$

$$x_0^2 = 10 \Rightarrow x_0 = \pm\sqrt{10}$$



Notice: $m = \boxed{f'(x_0) = -\frac{1}{x_0^2}}$

Defn: The **derivative of $y = f(x)$** (with respect to x) is

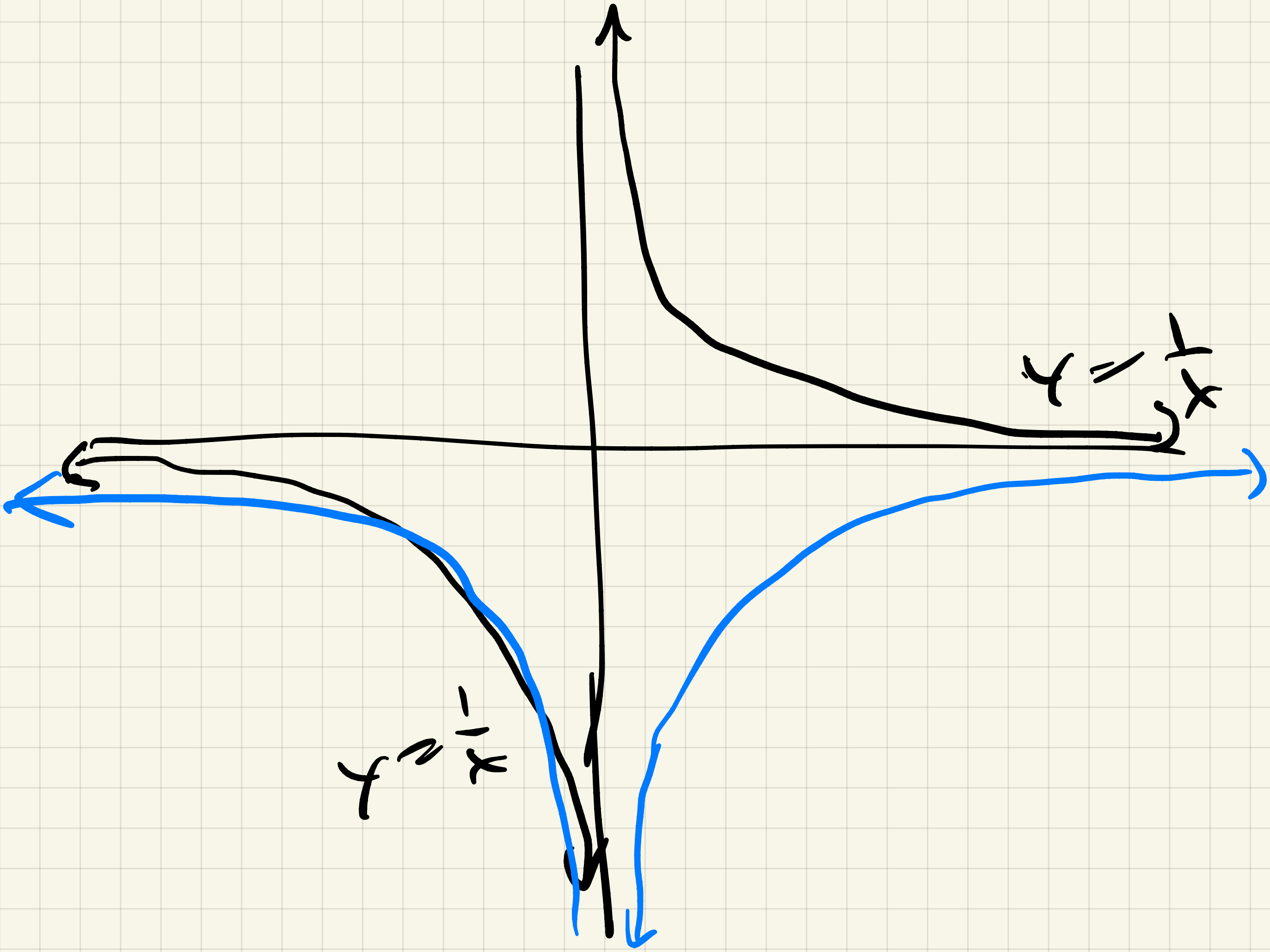
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(if it exists)

Note: $f'(x)$ is a **function**

In Ex1: we found that

$$f(x) = \frac{1}{x}, \qquad f'(x) = -\frac{1}{x^2}$$



Language: f is **differentiable** at x if $f'(x)$ exists

Notation: $f'(x)$, $\frac{d}{dx}(f(x))$

$\frac{dy}{dx}$ $y'(x)$ $Df(x)$

Ex2 $f(x) = x^2 - 4x$

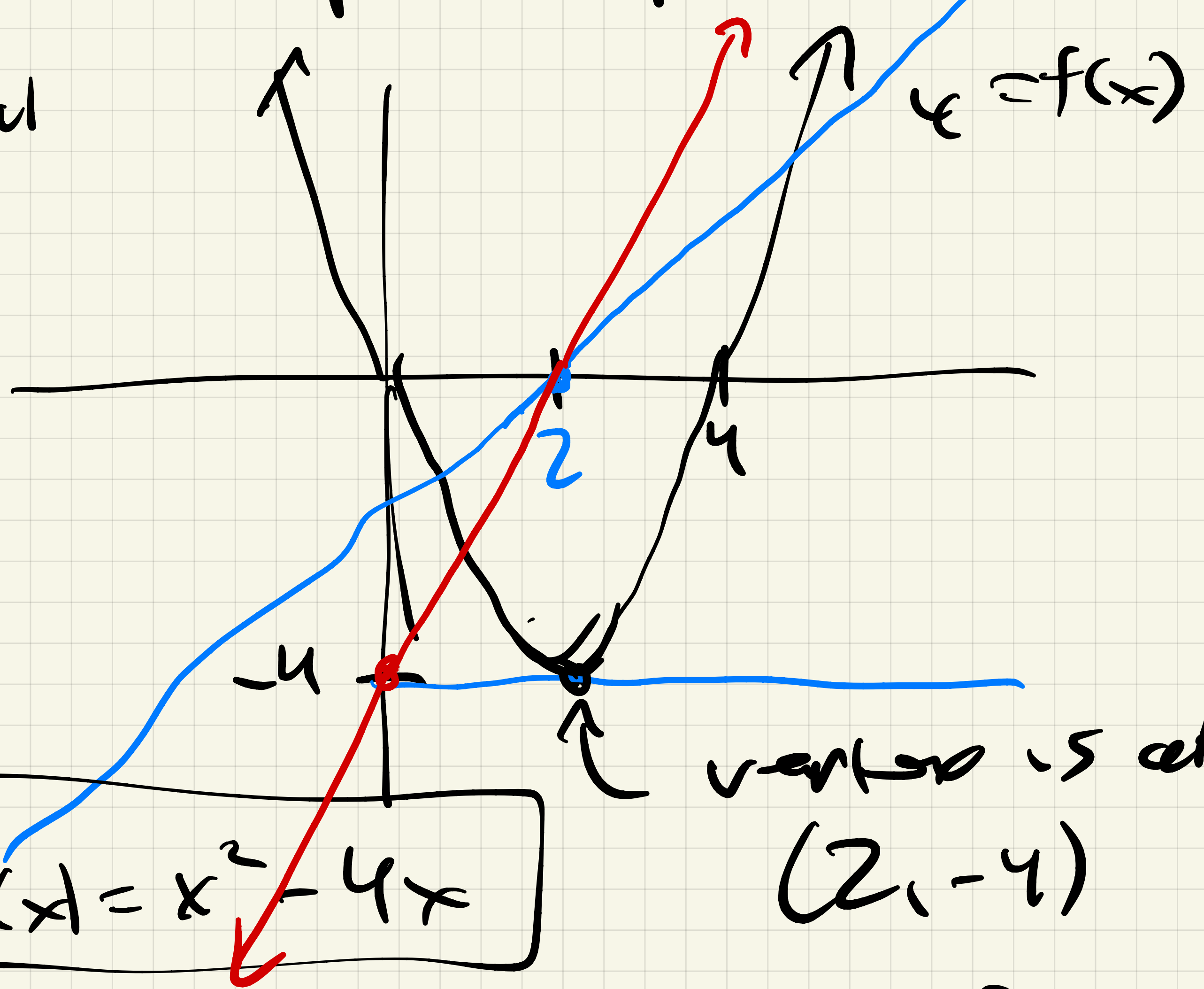
(a) make a sketch of $f(x)$

(b) sketch $y = f'(x)$

(c) Compute $f'(x)$

(d) Compare to part (b)

(a)



(c) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h}$$

$$\parallel$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h - \cancel{x^2} + \cancel{4x}}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 - 4h}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h - 4)}{\cancel{h}}$$

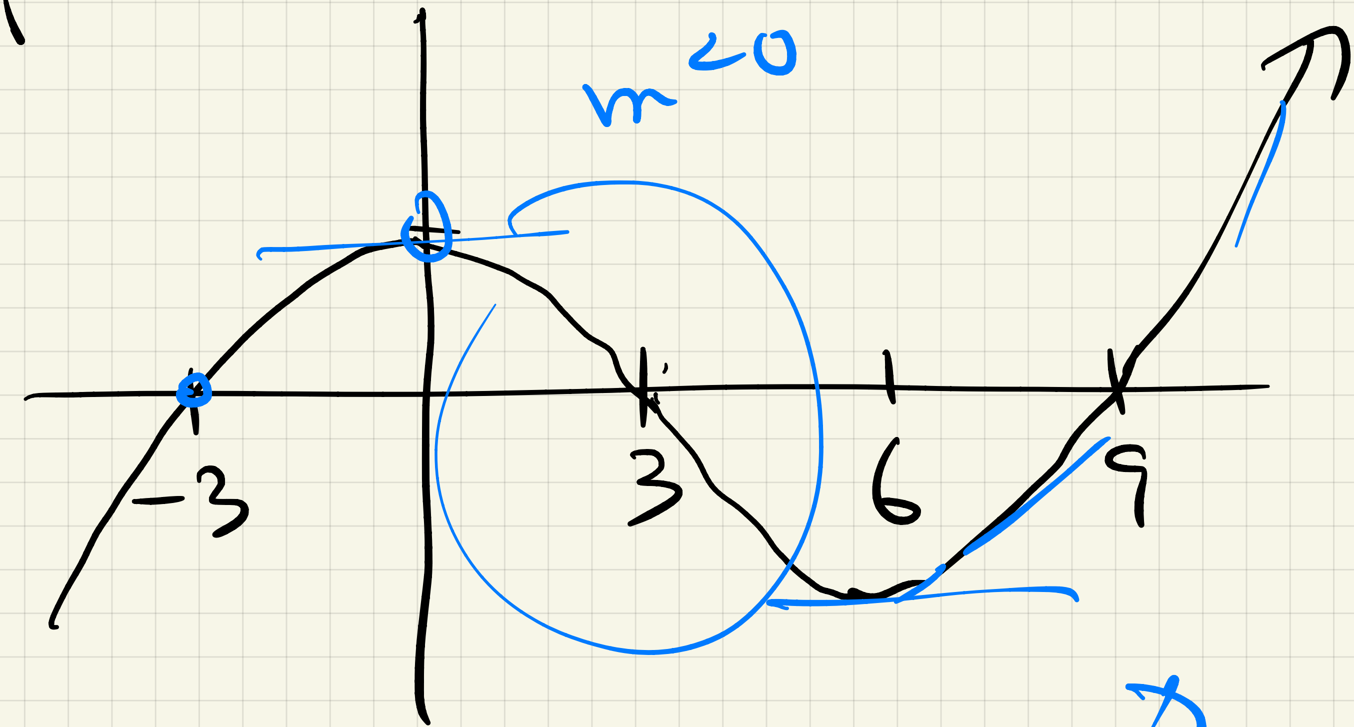
$$= \lim_{h \to 0} 2x + h - 4 = \underline{\underline{2x - 4}}$$

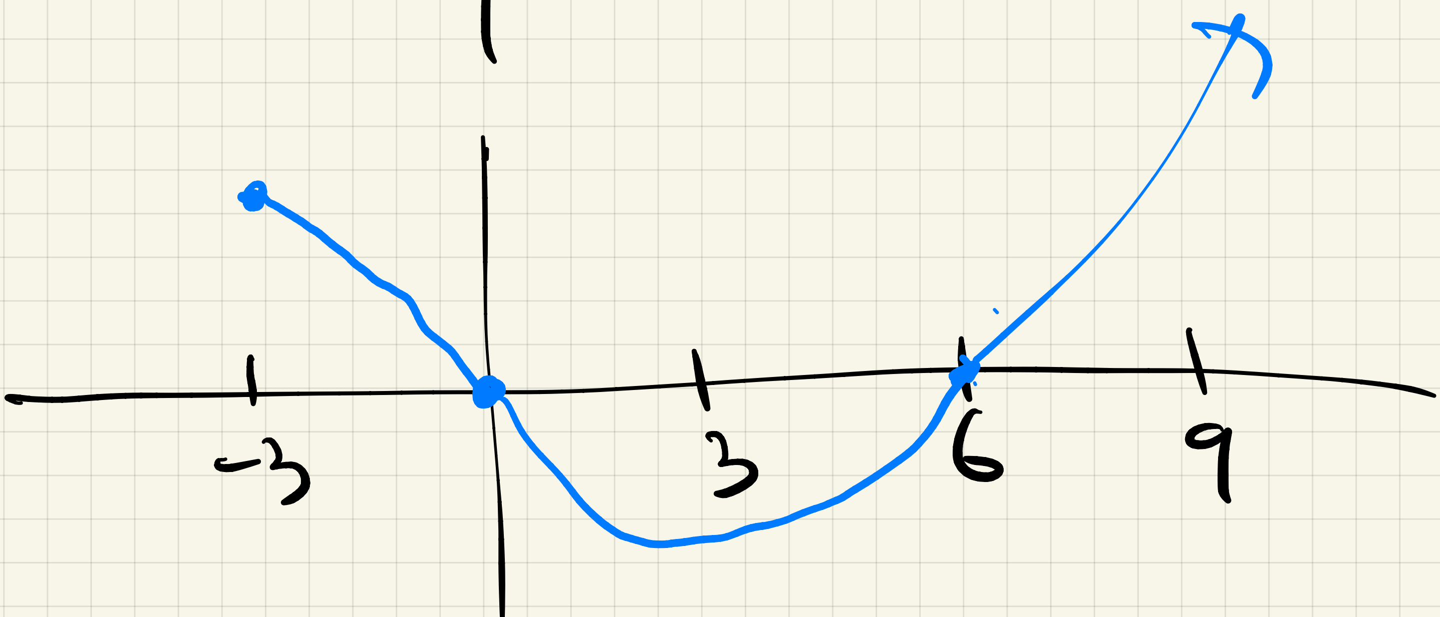
(d) $y = 2x - 4$

<u>Ex3</u> Given $y = f(x)$,

sketch $y = f'(x)$

(a)





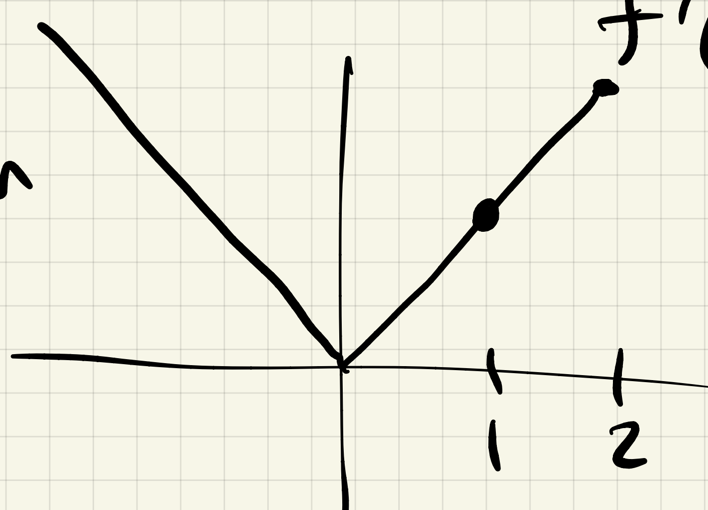
Key questions:

$f'(x) > 0$

$f'(x) < 0$

$f'(x) = 0$

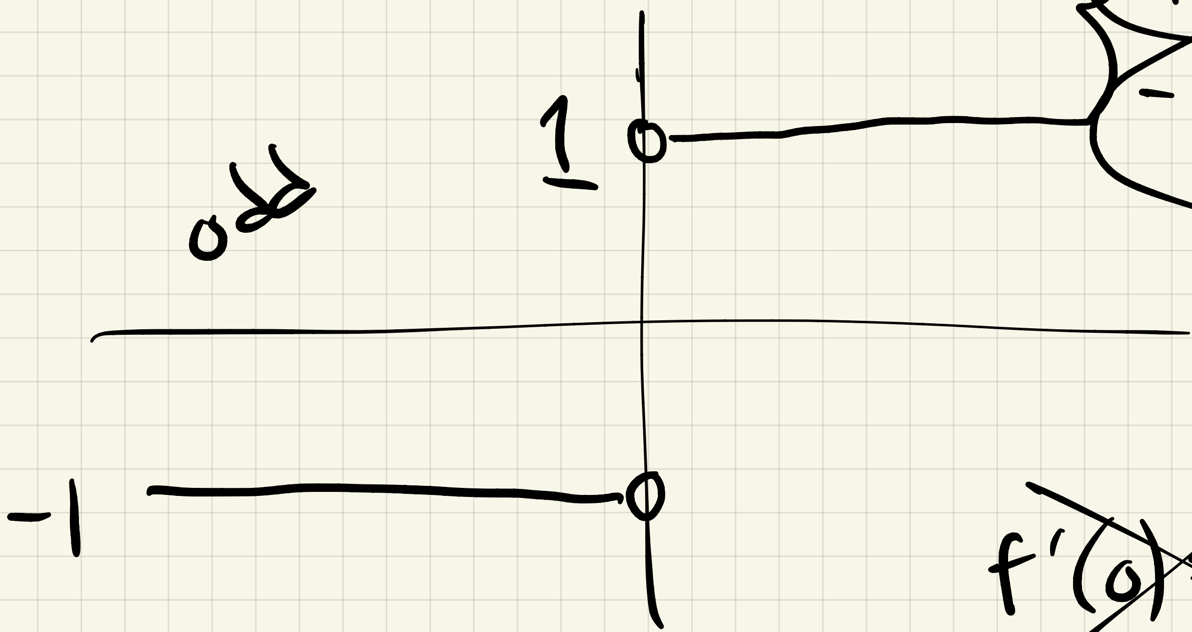
(b) even



$f(x) = |x|$

||

$$\begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

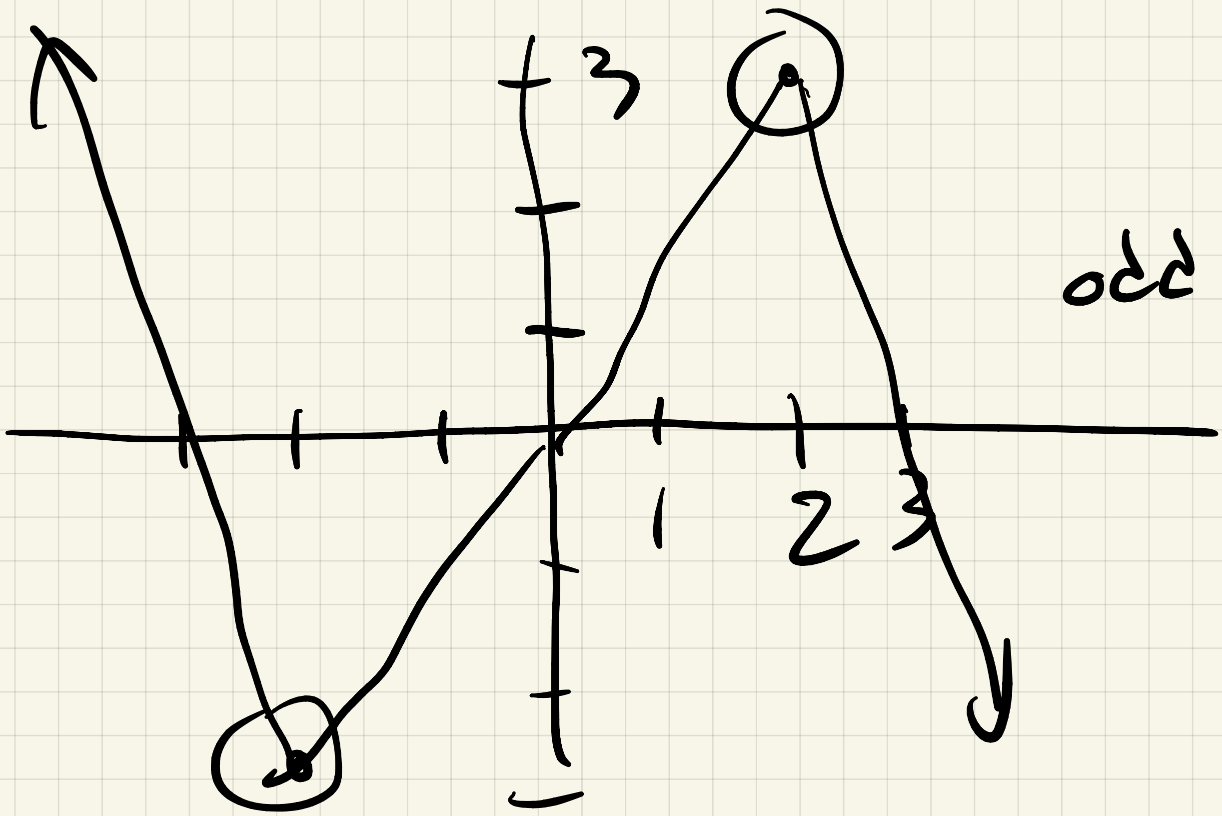


$\cancel{f'(0) = 0}$???

$$\begin{cases} \lim_{h\to 0} \frac{f(h) - f(0)}{h} = \lim_{h\to 0} \frac{|h|}{h} \\ \lim_{h\to 0^+} \frac{|h|}{h} = \lim_{h\to 0} \frac{h}{h} = 1 \\ \lim_{h\to 0^-} \frac{|h|}{h} = \lim_{h\to 0} \frac{-h}{h} = -1 \end{cases}$$

$f'(0) = \text{DNE}$

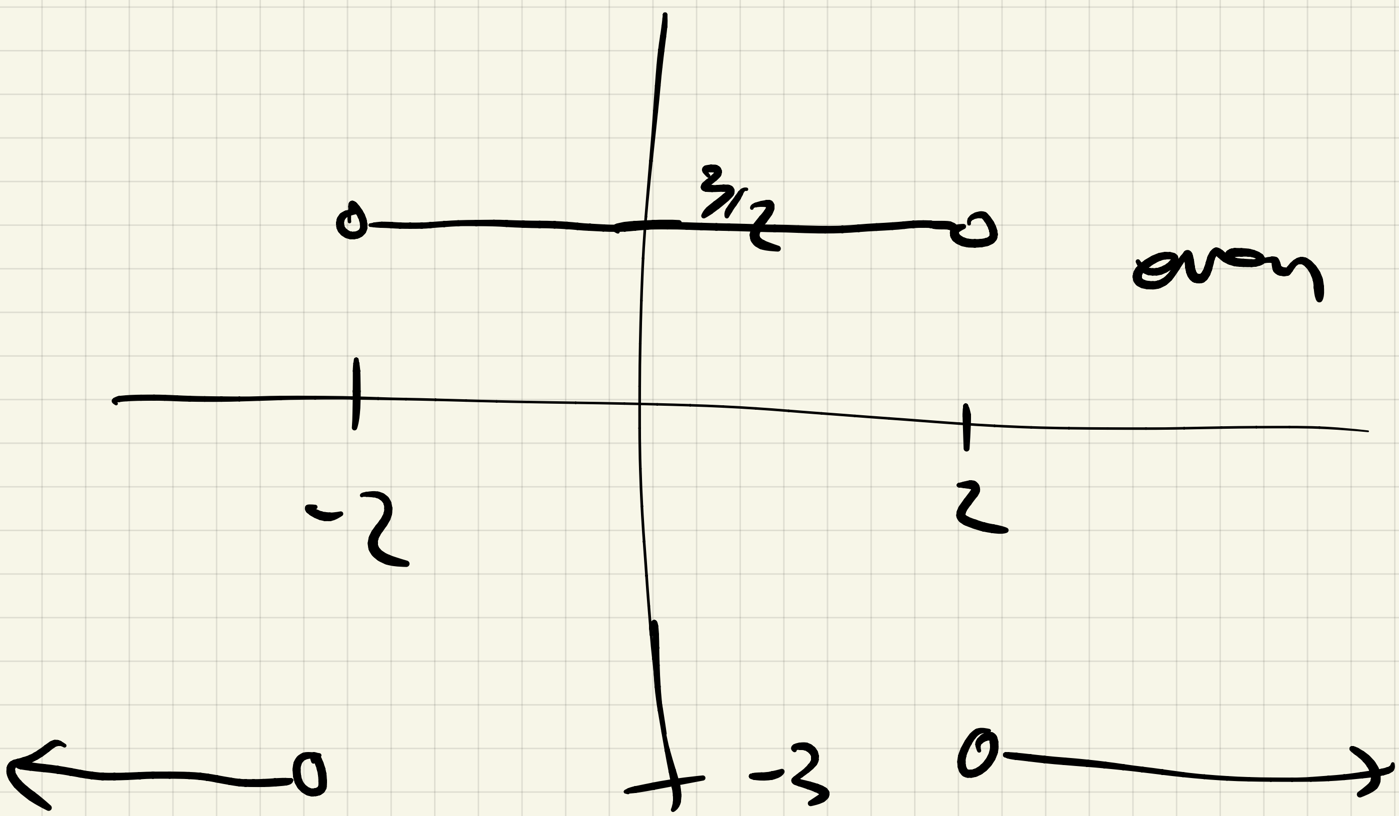
(c)



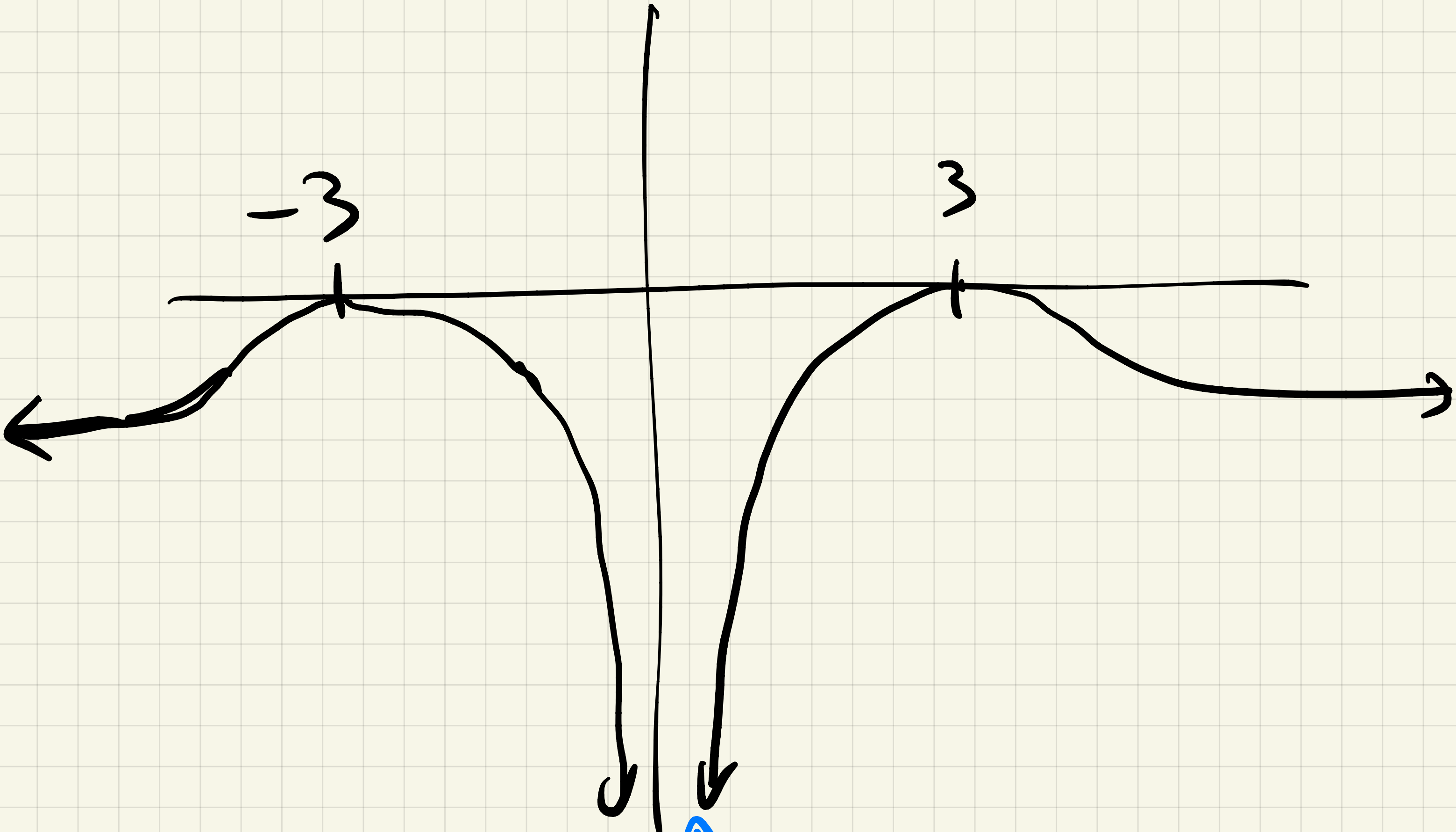
odd

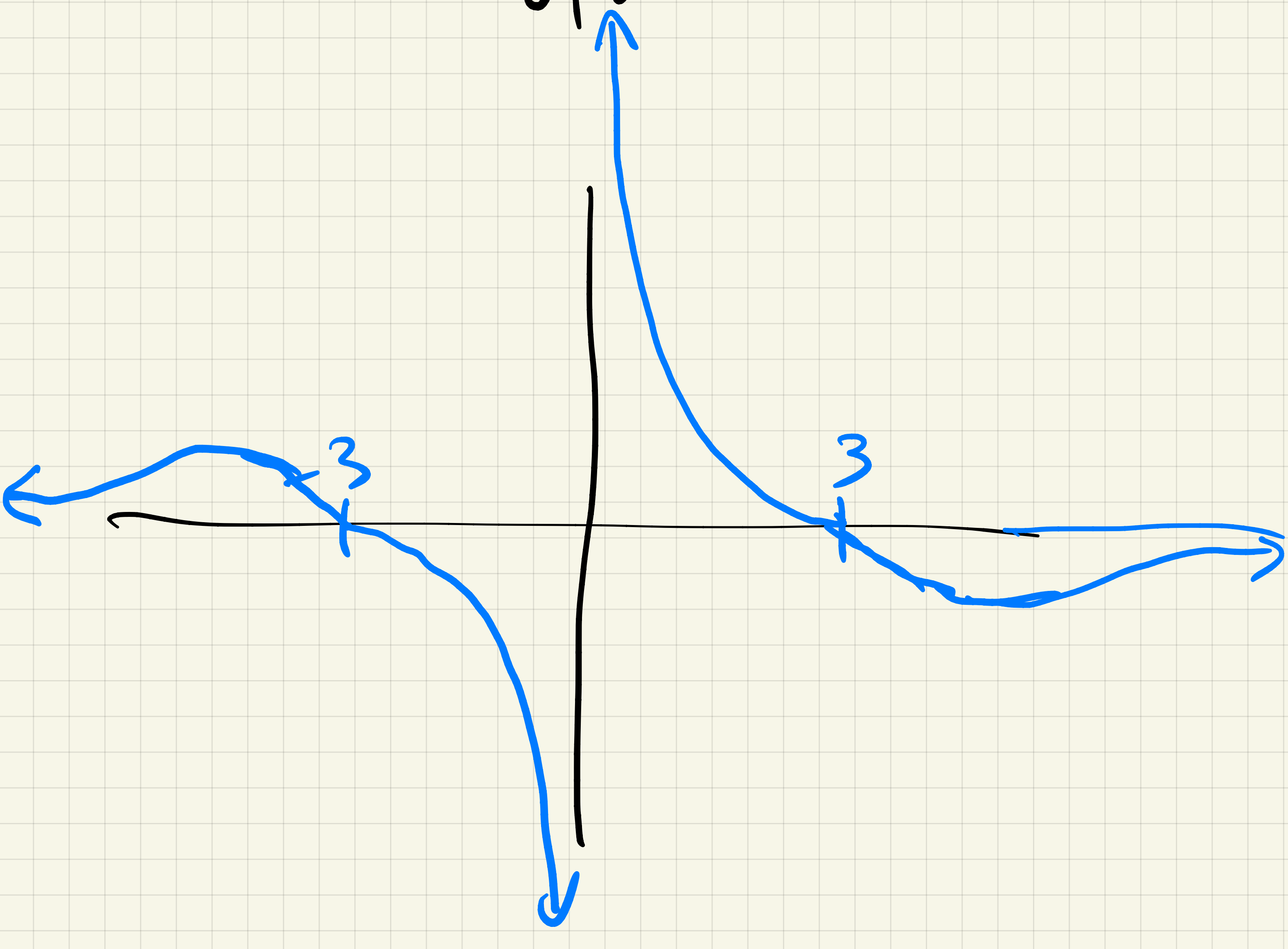
$f'(x) = 3/2 \qquad -2 < x < 2$

$f'(x) = -3 \qquad x > 2, \text{ or } x < -2$

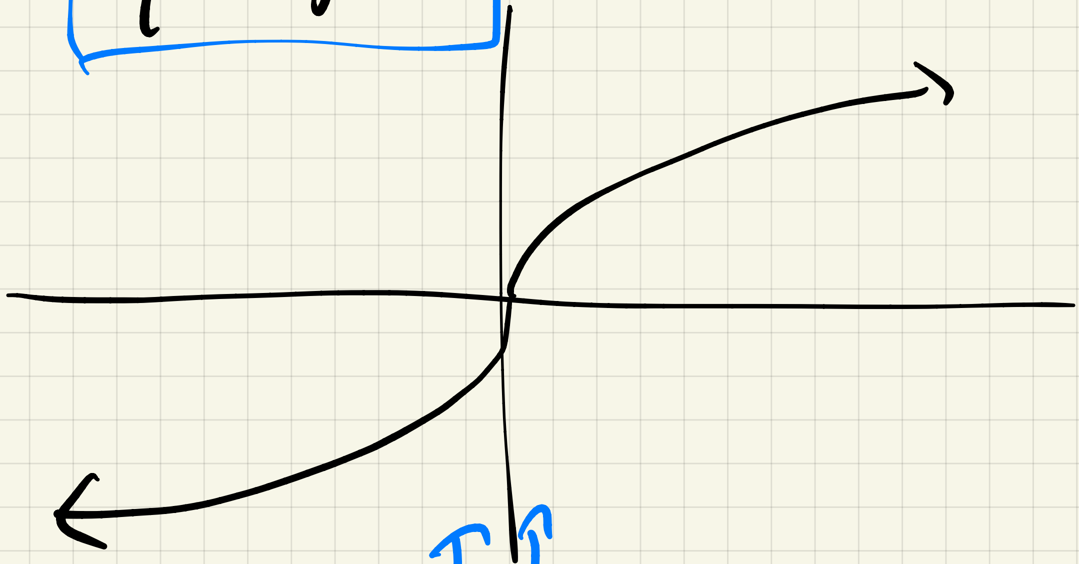


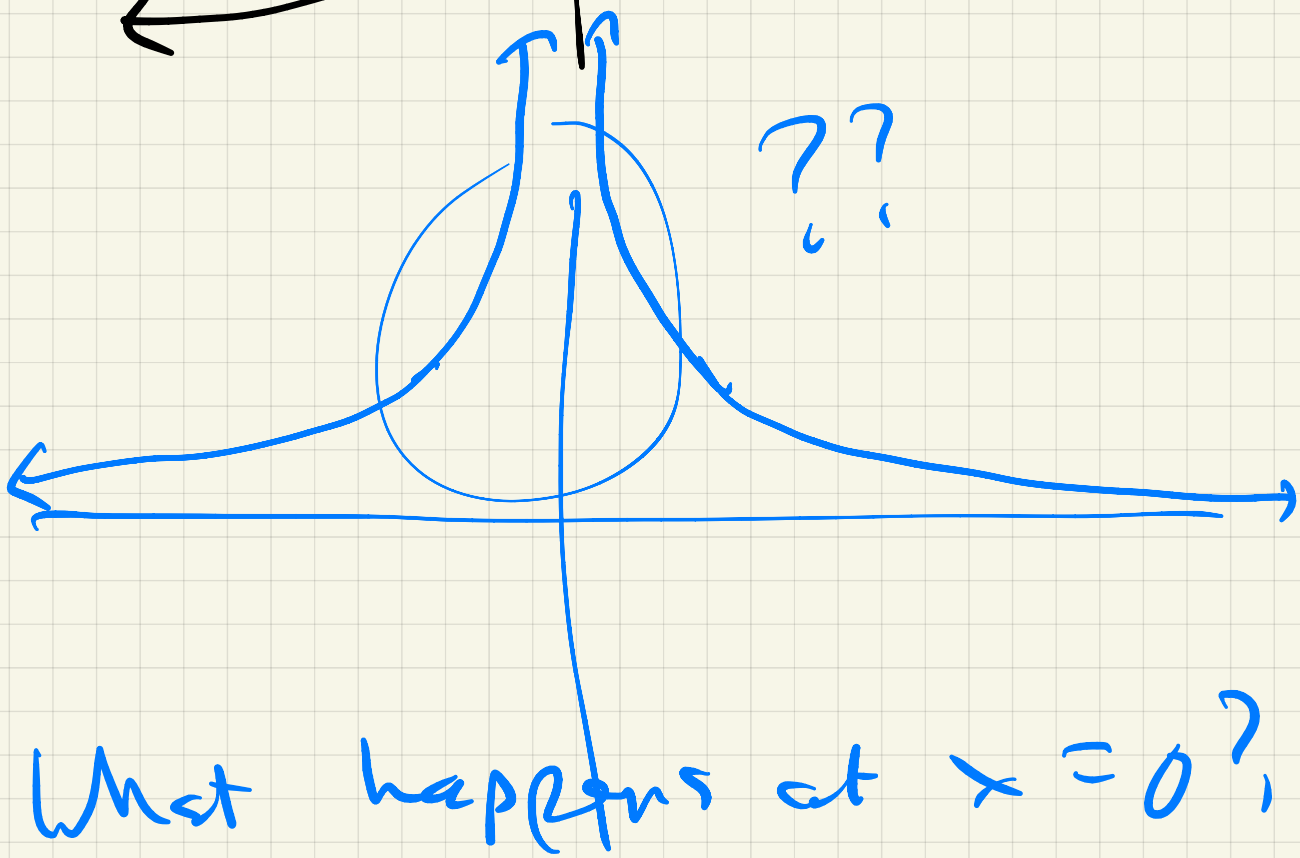
even


-3
3


-3
3

(d) $\boxed{y = \sqrt[3]{x}}$





What happens at $x = 0$?

$$\frac{dy}{dx}(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt[3]{h}}{h} = \lim_{h \to 0} \frac{1}{h^{2/3}} =$$

$$\frac{h^{1/3}}{h^1} \qquad \frac{\sqrt[3]{h}}{h^1} = h^{?}$$

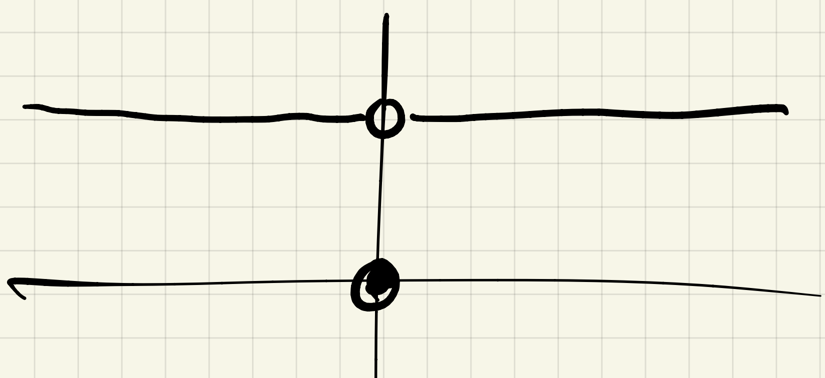
$$= h^{1/3-1} = h^{-2/3}$$

$$\lim_{h \to 0} \frac{1}{h^{2/3}}$$

DNE

Take away

$\boxed{\text{If } f(x) \text{ is not continuous at } x=c, \text{ then } f'(c) \text{ DNE}}$



Thm If $f'(c)$ exists, then f is continuous at $x=c$