2/1/Calc 1

avg 84%
med 93%

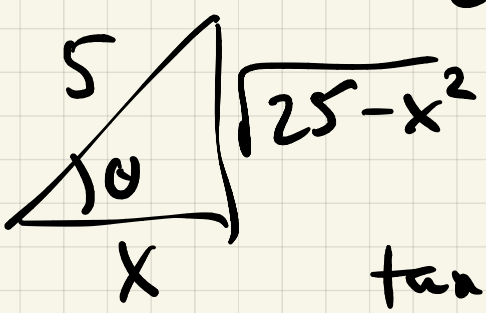
1. $f(x) = \sqrt[3]{5x+7}$

$y = \sqrt[3]{5x+7}$ $\quad f$

$x = \sqrt[3]{5y+7}$ $\quad f^{-1}$

$x^3 = 5y+7$ $\qquad \dfrac{x^3-7}{5} = y$

2. $\tan(\underbrace{\arccos \tfrac{x}{5}}_{\theta})$



$\cos\theta = \dfrac{x}{5}$

$\tan\theta = \dfrac{\sqrt{25-x^2}}{x}$

3. (a) $\ln(4x-3) = 10$

$4x-3 = \underline{e^{\ln(4x-3)}} = e^{10}$

$x = \dfrac{e^{10}+3}{4}$

(b) $e^{3x-9} = 20$

take ln

$$3x-9 = \ln\left(e^{3x-9}\right) = \ln 20$$

$$3x = 9 + \ln 20$$

$\leftarrow$ $(\ln 20)+9$

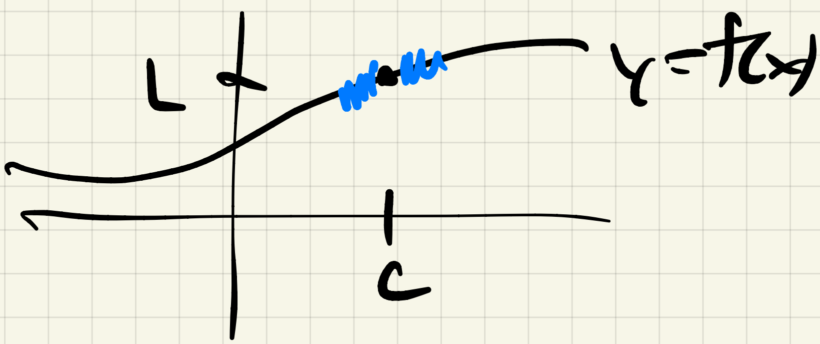
$\ln(20+9)$

$$x = \frac{9+\ln 20}{3}$$

Quiz 5 Friday — 2.2 earlier

6 Thu 2.2

Last time limits $\lim_{x \to c} f(x) = L$



Pictures

Calculator estimates

Algebraic limit rules

p.9 polynomials, →

$$\lim_{x \to c} p(x) = p(c)$$

$$\lim_{x \to c} \frac{p(x)}{q(x)} = \frac{p(c)}{q(c)}, \quad c \neq 0$$

Ex/ (a) $\displaystyle \lim_{x \to 3} x^2 + 7x = 30$

(b) $\displaystyle \lim_{x \to 3} \frac{x^3 + 3x}{2x+4} = \frac{27+9}{6+4} = \frac{36}{10}$

(c) $\displaystyle \lim_{x \to 3} \sqrt[5]{\frac{x^3+3x}{2x+4}} = \sqrt[5]{\frac{36}{16}}$

(d) $\displaystyle \lim_{x \to 0} (1+x)^{3/x} \approx 20.0856\ldots$ ✓

Note : $\displaystyle e = \lim_{x \to 0} (1+x)^{1/x}$

$$\therefore \lim_{x \to 0} \left((1+x)^{1/x} \right)^3 = e^3$$

$$\left((1+x)^{1/x} \right)^3 = (1+x)^{3/x}$$

Less obvious limits

(a) $\lim_{x\to 10} \frac{x^2-100}{x-10} = \lim_{x\to 10} \frac{(x-10)(x+10)}{(x-10)} =$

cancel for $x \neq 10$

$\lim_{x\to 10} \frac{x+10}{1} = 20$

(b) $\lim_{x\to 1} \frac{x^3-12x+11}{x-1} =$

$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x-11)}{\cancel{(x-1)}} =$

$\lim_{x\to 1} \frac{x^2+x-11}{1} = -9$

(c) $\lim_{x\to 1} \frac{x^3-12x+11}{(x-1)^2} =$

$\lim_{x\to 1} \frac{(x-1)(x^2+x-11)}{(x-1)(x-1)} =$

$\lim_{x\to 1} \frac{x^2+x-11}{x-1} = \frac{\to -9}{\to 0} \to -\infty$

DNE

$A^2-B^2=(A-B)(A+B)$

(d) $\lim_{x\to 0} \left(\frac{\sqrt{x+4}-2}{x}\right) \frac{(\sqrt{x+4}+2)}{(\sqrt{x+4}+2)} =$

$\lim_{x\to 0} \frac{x+\cancel{4}-\cancel{4}}{x(\sqrt{x+4}+2)}$

$\lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+4}+2)} = \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2}$

$= \frac{1}{4}$

(e) $\lim_{x\to 9} \frac{x-9}{(3-\sqrt{x})} \frac{(3+\sqrt{x})}{(3+\sqrt{x})} =$

$\lim_{x\to 9} \frac{(x-9)(3+\sqrt{x})}{(9-x)}$

$= \lim_{x\to 9} \frac{-1(3+\sqrt{x})}{1} = -6$

(f) $\lim_{x\to 1} \frac{\frac{1}{5}-\frac{1}{x+4}}{x-1} =$

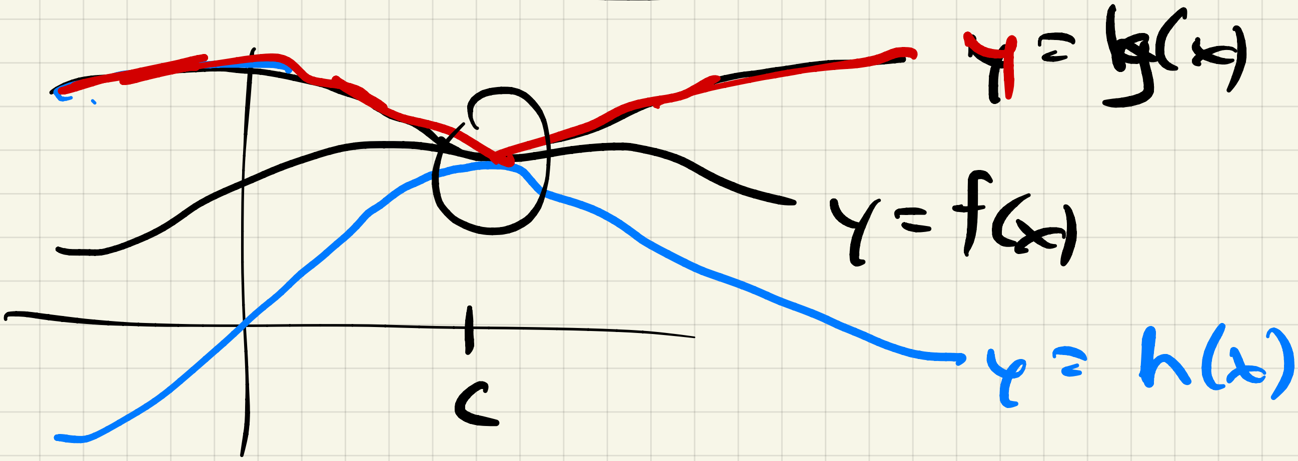
$\lim_{x\to 1} \frac{\frac{(x+4)-5}{5(x+4)}}{x-1} =$

$$\lim_{x \to 1} \frac{\frac{x-1}{5(x+4)}}{x-1} =$$

$$\lim_{x \to 1} \frac{\cancel{x-1}}{5(x+4)} \cdot \frac{1}{\cancel{x-1}} =$$

$$\lim_{x \to 1} \frac{1}{5(x+4)} = \frac{1}{25}$$

Sandwich Theorem



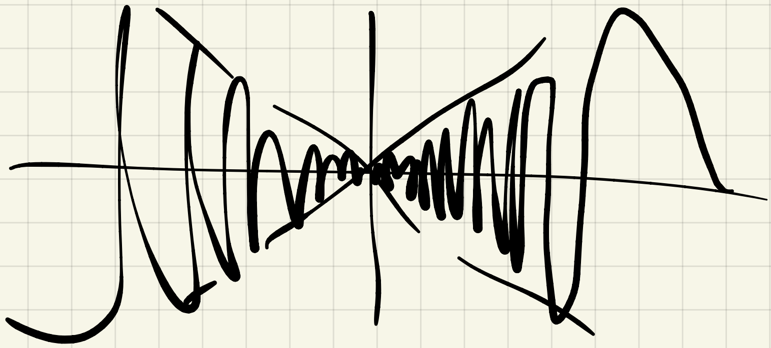
If $h(x) \le f(x) \le g(x)$

and $\lim_{x \to c} h(x) = \lim_{x \to c} g(x) = L$

Then $\lim_{x \to c} f(x) = L$

Ex 9 On Tuesday, we "saw"

$$\lim_{x \to 0} x\left(\sin \frac{1}{x}\right) = 0$$



Use Sandwich to prove it:

$$-1 \le \sin \frac{1}{x} \le 1$$

$$-|x| \le x \sin \frac{1}{x} \le |x| \qquad x \geq 0$$

(<u>recall</u> $\lim_{x \to c} |x| = |c|$)

$$\boxed{\lim_{x \to 0} -|x| = \lim_{x \to 0} |x| = 0}$$

So Sandwich Thm $\Rightarrow$

$$\lim_{x \to 0} x\left(\sin \frac{1}{x}\right) = 0$$

<u>Ex 6</u> From §1.3

$\longrightarrow$ $-|x| \le \sin x \le |x|$

$$\boxed{\lim_{x\to 0} \sin x = 0} \ ??$$

b/c $\lim -|x| = 0$

$\lim |x| = 0$

$+ \quad -|x| \le 1-\cos x \le |x|$

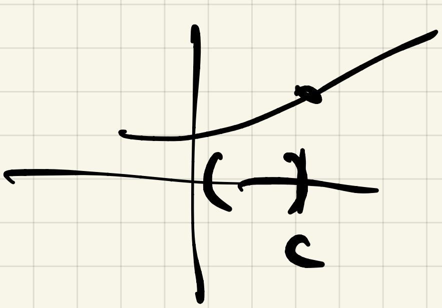
so $$\boxed{\lim_{x\to 0} 1-\cos x = 0}$$

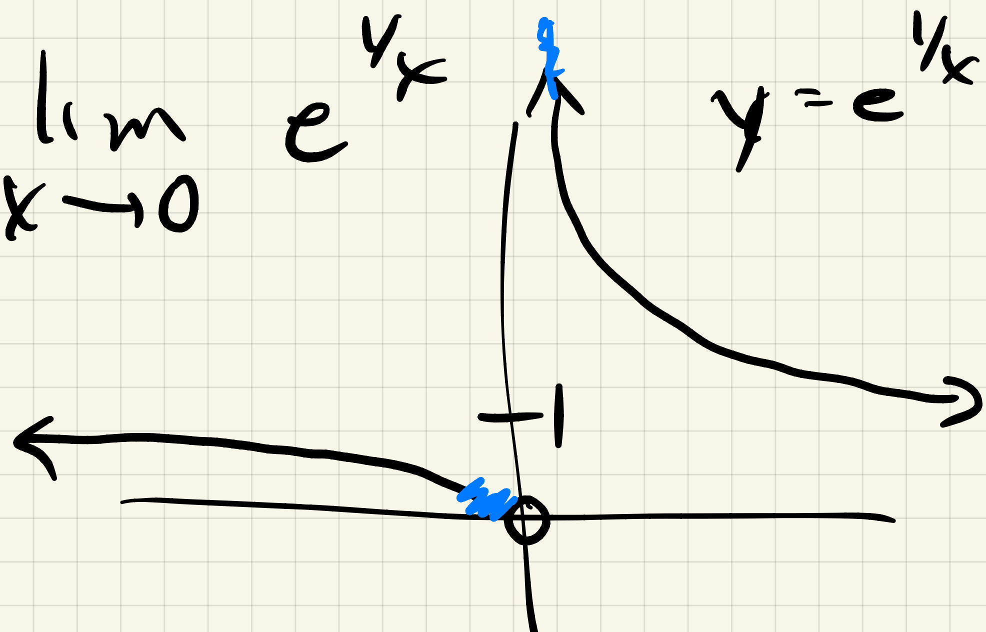
$$\boxed{\lim_{x\to 0} \cos x = 1}$$

§2.3 skip

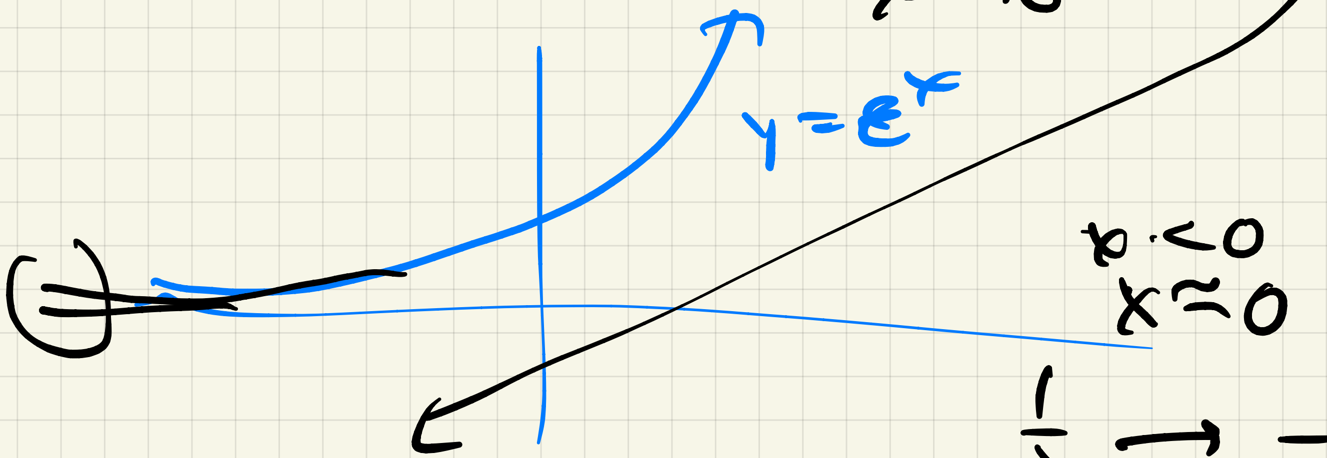
§2.4

Ex1 $$\lim_{x\to 0} e^{1/x}$$





Picture $\Rightarrow$ $\lim_{x\to 0} e^{1/x}$ DNE



$x<0$
$x \approx 0$
$\frac{1}{x} \to -\infty$

But the limit from left side is 0

Defn If domain f contains a interval (d,c) (Resp. (c,e)), then $f(x)$ has a left-hand limit L at c ($f(x)$ has a right hand limit of L at c) if we can make $f(x)$ as close to L as we like by choosing $x<c$ (resp. $x>c$)

close enough to c

$$\lim_{x\to c^-} f(x) = L$$

left

$$\lim_{x\to c^+} f(x) = L$$

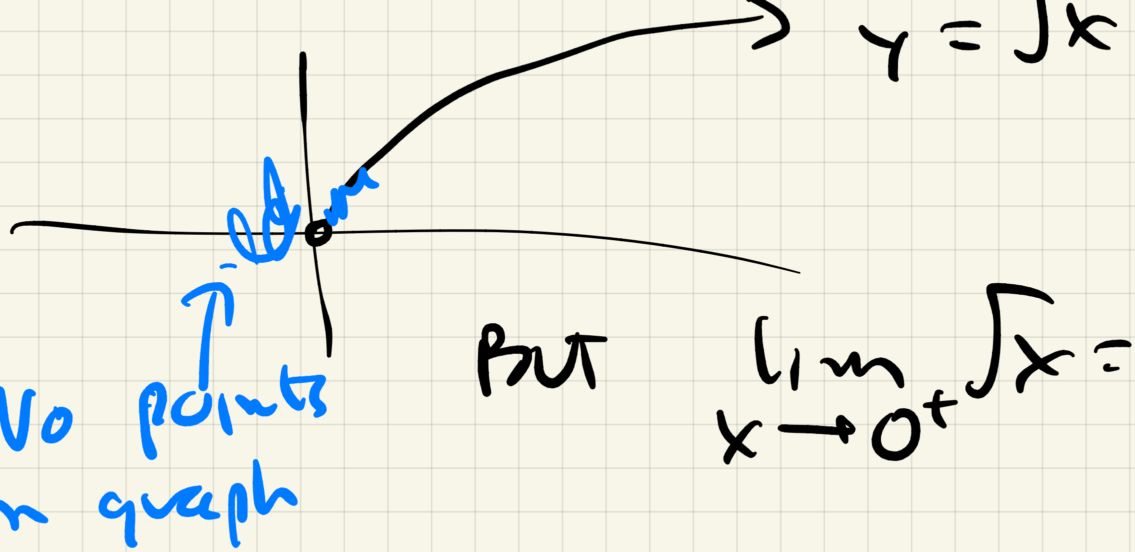
right

$$\lim_{x\to 0} e^{1/x} \text{ DNE}$$

BUT

$$\lim_{x\to 0^-} e^{1/x} = 0$$

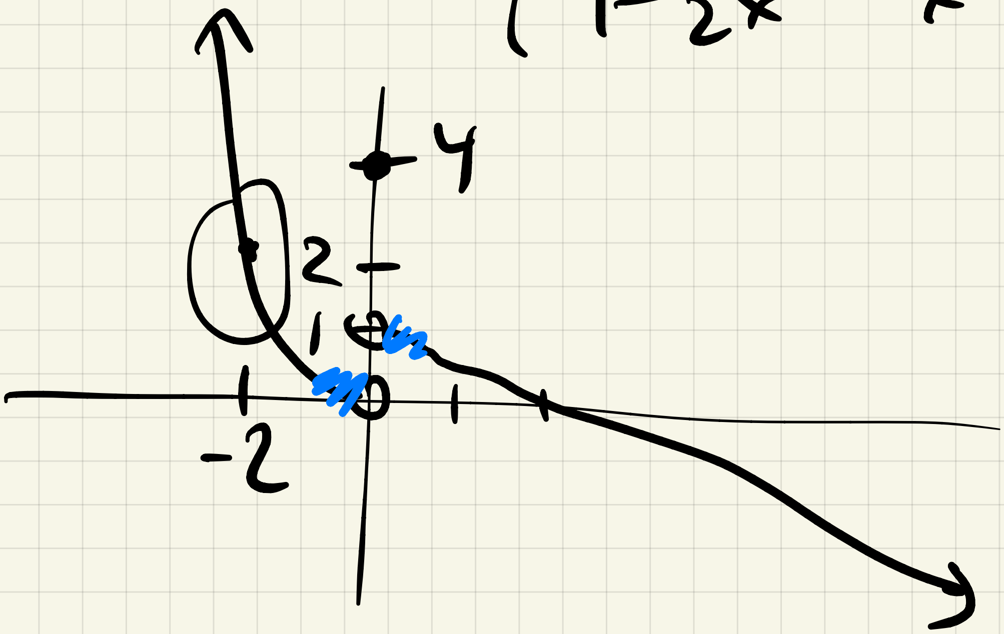
<u>Ex2</u> $\lim_{x\to 0} \sqrt{x}$ DNE



BUT $\lim_{x\to 0^+} \sqrt{x} = 0$

<u>Ex3</u>

$$f(x) = \begin{cases} x^2 & x < 0 \\ 4 & x = 0 \\ 1 - \frac{1}{2}x & x > 0 \end{cases}$$



(a) $\lim\limits_{x \to -2} f(x) = \lim\limits_{x \to -2} x^2 = 4$

(b) $\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^-} x^2 = 4$

(c) $\lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2^+} x^2 = 4$

(d) $f(0) = 4$

(e) $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} x^2 = 0$

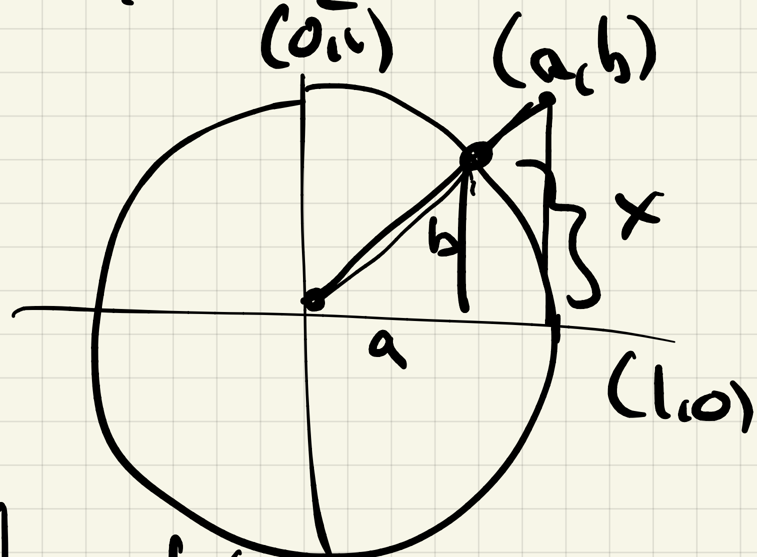
(f) $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} 1 - \frac{1}{2}x = 1$

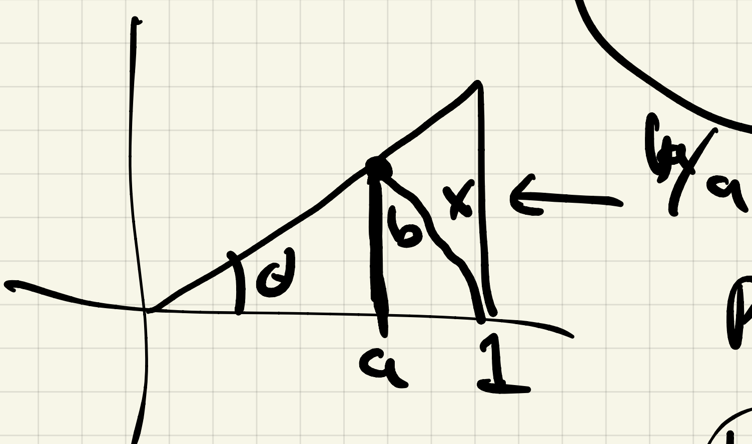
(9) $\lim_{x \to 0} f(x) = \text{DNE}$

<u>Ex1</u> $\lim_{x \to 0^+} \frac{\sin x}{x} = 1$

For $0 < x < \pi/2$

Unit circle





picture tells us

$$b \le x \le b/a$$

$$\sin x \le x \le \tan x = \frac{\sin x}{\cos x}$$

$$\div \sin x > 0$$

$$1 \le \frac{x}{\sin x} \le \frac{1}{\cos x}$$

$$\lim_{x\to 0^+} 1 = 1 \qquad \lim_{x\to 0} \frac{1}{\cos x} = 1$$

sandwich ~~Thm~~

$$\lim_{x\to 0^+} \frac{x}{\sin x} = 1$$

$$\therefore \lim_{x\to 0^+} \frac{\sin x}{x} = 1$$

[<u>Notice</u> also $\lim_{x\to 0^-} \frac{\sin x}{x} = 1$
b/c ~~…~~ symmetry]

<u>Example</u>

(a) $\lim_{x\to 0} \frac{2\sin x}{7x} = \lim_{x\to 0} \frac{2}{7}\left(\frac{\sin x}{x}\right)$

$= 2/7 \lim_{x\to 0} \frac{\sin x}{x} = 2/7$

(b) $\lim_{x\to 0} \frac{\sin 3x}{6x} =$ $\qquad \frac{\sin x}{x}$

$\underline{\text{Idea}}$: $u = 3x$

as $x \to 0$, $3x = u \to 0$

$$\lim_{x \to 0} \frac{\sin 3x}{6x} = \lim_{u \to 0} \frac{\sin u}{6x = 2 \cdot 3x}$$

$$ = 2(u)$$

$$= \lim_{u \to 0} \frac{1}{2}\left(\frac{\sin u}{u}\right) = \frac{1}{2} \cdot 1 = \frac{1}{2}$$