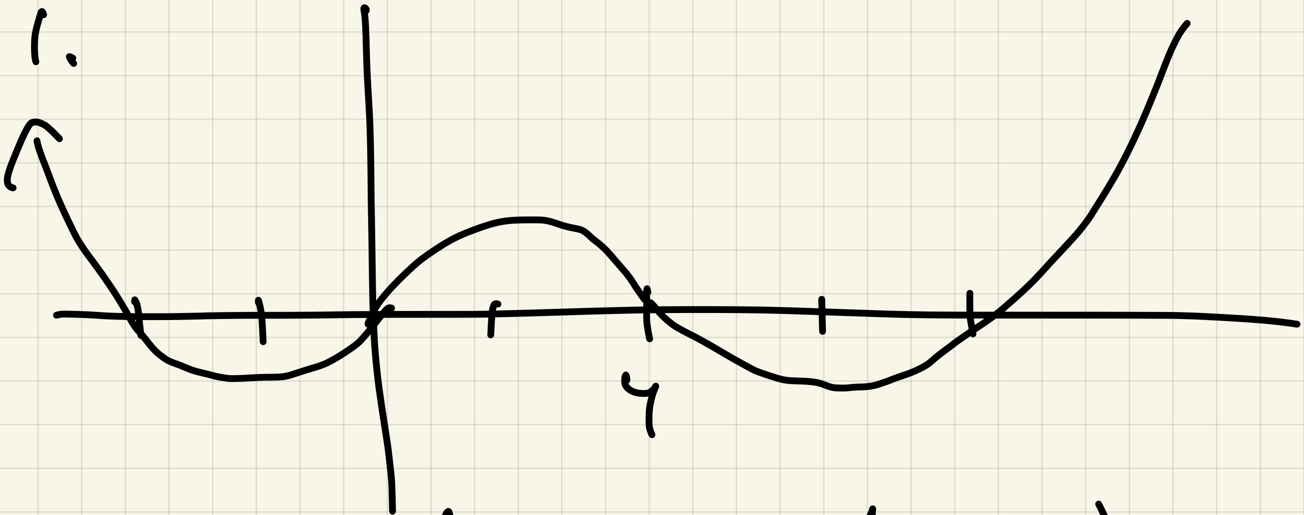
4/5 | Calc I   Quiz 17

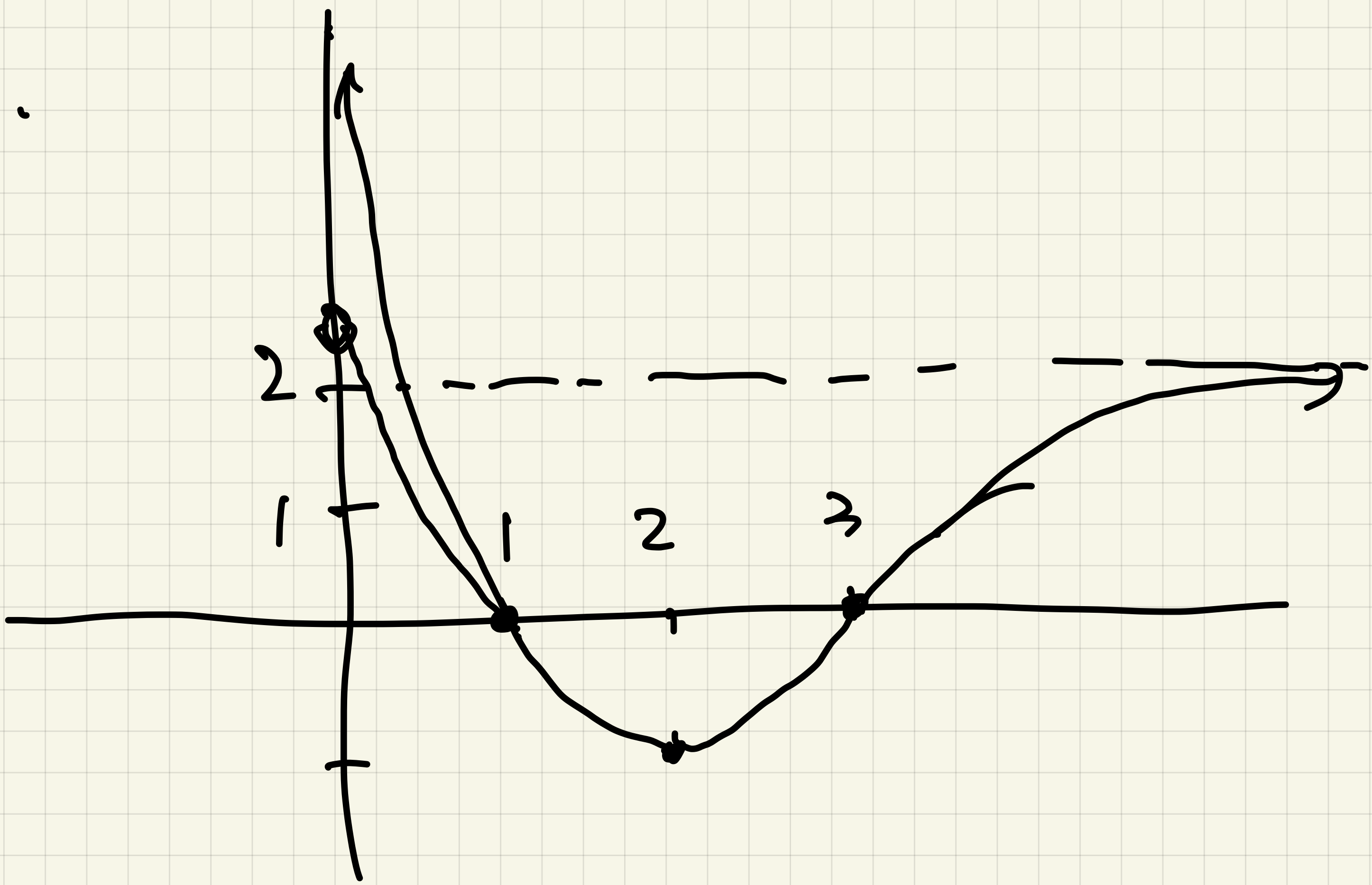
1.



conc up $(-\infty, 0) \cup (4, \infty)$

down $(0, 4)$

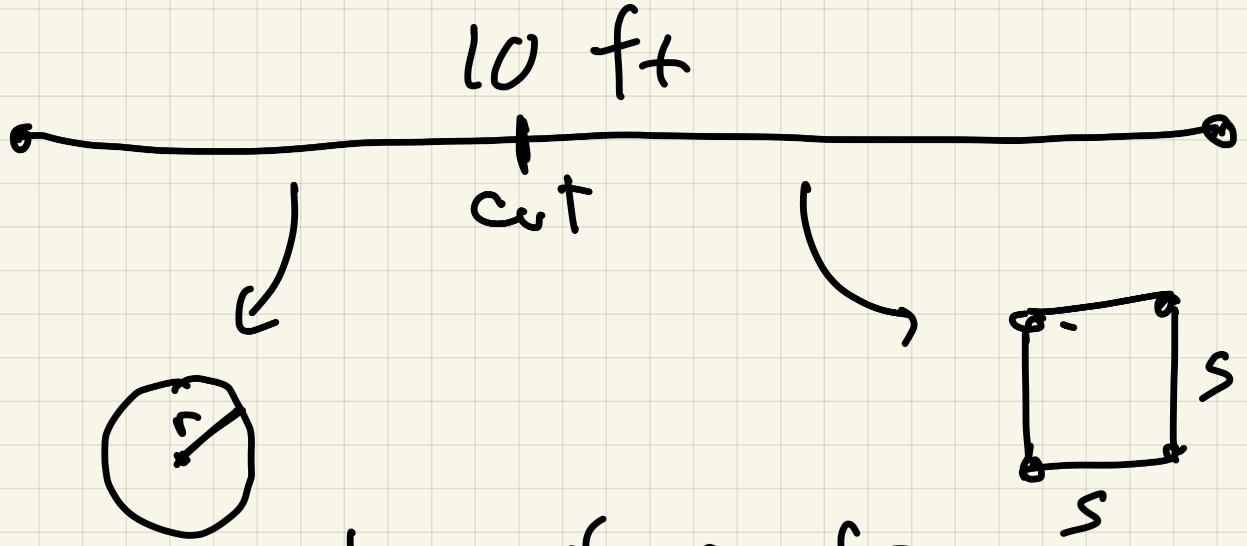
2.



Last time: Optimization

Ex1: A 10 ft. wire is cut into 2 pieces, one bent into a circle, other into a square:

where should wire be cut to minimize total area (maximize total area)



r = radius of circle

s = side of square

Area $= A = \pi r^2 + s^2$

relation $\quad 2\pi r + 4s = 10$ ft

circum $\quad$ perim

$2\pi r + 4s = 10$

$2\pi r = 10 - 4s$

$r = \frac{10-4s}{2\pi} = \frac{5-2s}{\pi}$

$$A = \pi \left(\frac{5-2s}{\pi}\right)^2 + s^2$$

$$= \frac{(5-2s)^2}{\pi} + s^2$$

$0 \le s \le 5/2 = 2.5$

find max/min of

$$A = \frac{(5-2s)^2}{\pi} + s^2 \quad \text{on } [0, 5/2]$$

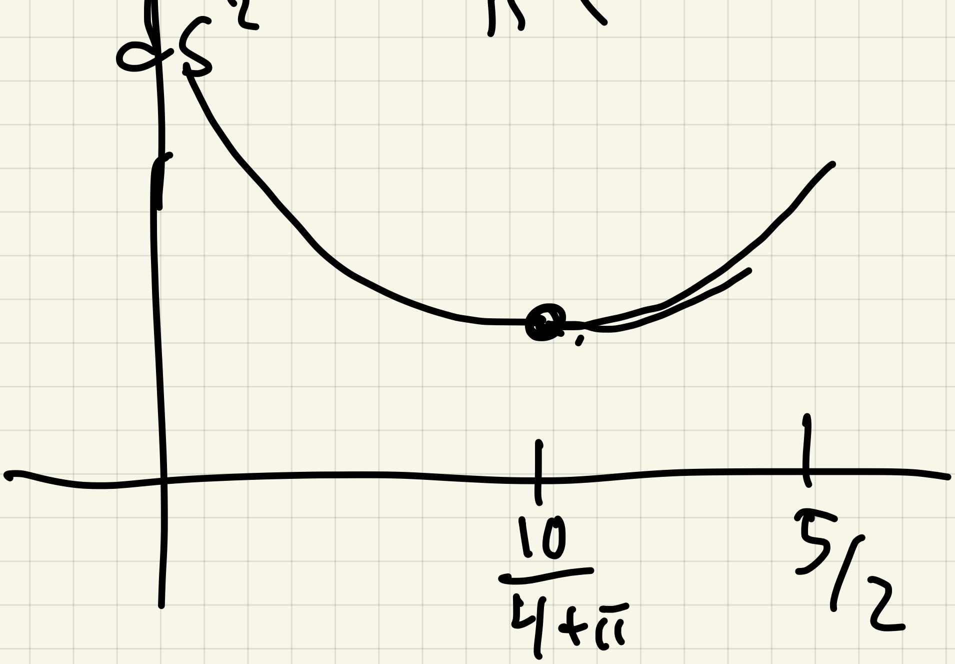
$$\frac{dA}{ds} = \frac{1}{\pi} 2(5-2s)^1(-2) + 2s$$

$$\frac{dA}{ds} = \frac{-20 + 8s + 2\pi s}{\pi} = 0$$

$$s(8+2\pi) = 20$$

$$s = \frac{20}{8+2\pi} = \frac{10}{4+\pi}$$

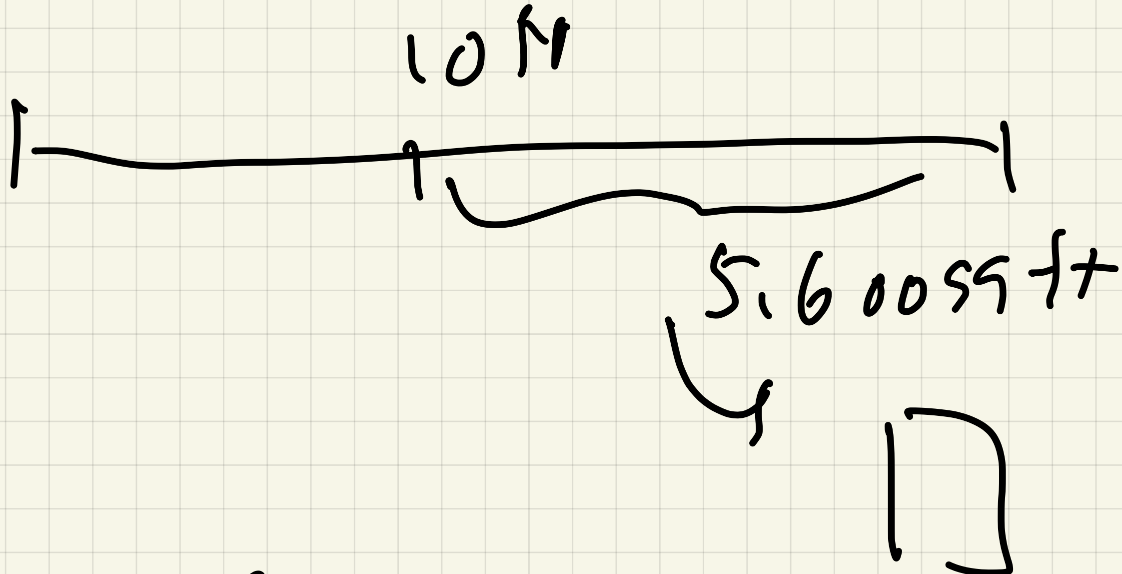
$$\frac{d^2A}{ds^2} = \frac{1}{\pi}(8+2\pi) > 0$$



<u>min area</u>: $s = \frac{10}{4+\pi}$

cut at $4s = \frac{40}{4+\pi} = 5.60099$ ft

rest to circle



<u>max area</u> $s = 0$

↳ put all wire into circle.

§4.8 <u>Antiderivatives</u>

<u>Defn</u>: A function $F(x)$ is an <u>antiderivative</u> of $f(x)$

on an interval I if

$$F'(x) = f(x) \quad \text{all } x \text{ in } I$$

Ex1 $F(x) = x^2$ ✓ is antiderivative for $f(x) = 2x$

also $F(x) = x^2 + 5$

$x^2 + 100$

$x^2 + C \qquad C$ constant

Thm: If $F(x)$ is an antiderivative for $f(x)$ on I, then all antiderivatives for $f(x)$ are

$F(x) + C, \quad C$ constant

Ex 2 Find all antiderivatives of $f(x) = x^4$ on $I = (-\infty, \infty)$:

$$F(x) = \frac{1}{5}x^5 + C$$

C constant

Notation: If $F'(x) = f(x)$, on I we write

$$\int f(x)\, dx = F(x) + C$$

integral sign

integrand

any antiderivative

integration constant

Indefinite integral:

Ex 3

$$\int x^4\,dx = \frac{1}{5}x^5 + C$$

(a)

$$\int 7e^x\,dx = 7e^x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C$$

$$\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$$