4/4/ Calc: Quiz 16

avg 90%

$$f' = \frac{1}{5}x^5 - 3x^4 + \frac{20}{3}x^3$$

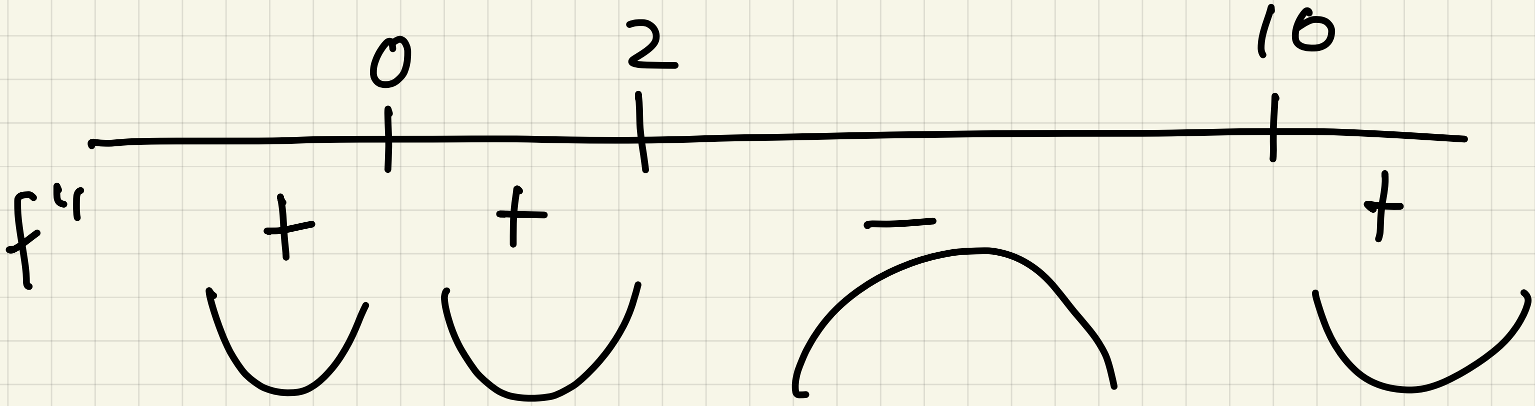
$$f'' = x^4 - 12x^3 + 20x^2$$

$$= x^2(x^2 - 12x + 20)$$

$$x^2(x-2)(x-10) = 0$$

$x = 0, 2, 10$

$$\frac{12 \pm \sqrt{144 - 4(1)(20)}}{2(1)} = 2, 10$$




conc up $(-\infty, 2) \cup (10, \infty)$

down $(2, 10)$

3. $x = 2, 10$

Last time L'Hospital's rule

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

if $\frac{f(c)}{g(c)}$ $\boxed{\frac{0}{0}, \frac{\pm\infty}{\pm\infty}}$

or $c = \pm\infty$

Other forms:

$$\lim_{x \to c} f(x) \cdot g(x)$$

$f(x) \to 0$, $g(x) \to \infty$

Idea: $\boxed{\text{rewrite as fraction}}$

$$\lim_{x \to c} f(x)^{g(x)}$$

$1^{\infty}$ $0^{0}$

Idea: $\boxed{\text{take logarithm}}$

**Ex1** $$\lim_{x \to 0^+} \left(e^x + 7x\right)^{1/x} = L$$

$$\ln L = \lim_{x \to 0^+} \ln\left(e^x + 7x\right)^{1/x}$$

$$= \lim_{x \to 0^+} \underbrace{\frac{1}{x}}_{+\infty} \underbrace{\ln\left(e^x + 7x\right)}_{0}$$

$$= \lim_{x \to 0^+} \frac{\ln\left(e^x + 7x\right)}{x}$$

$$= \lim_{x \to 0^+} \frac{\left(\dfrac{e^x + 7}{e^x + 7x}\right)}{1} = \frac{8}{1}$$

$$\ln L = 8$$

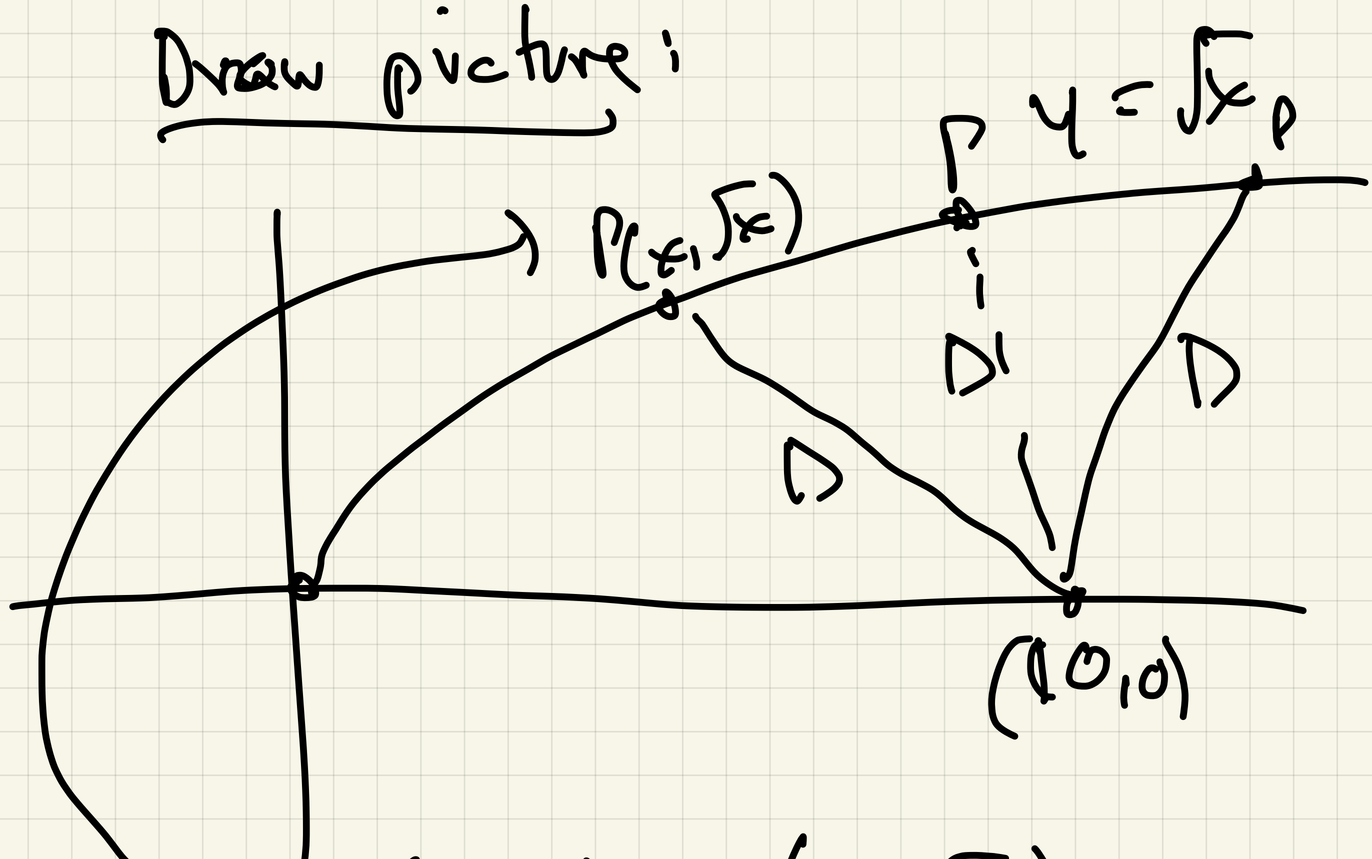
$$L = e^{\ln L} = e^8 \approx 2980.95\ldots$$

§4.6 Optimization:

[ex] Which point $P$ on curve $y = \sqrt{x}$ is closest to $(10, 0)$?

How to start?

Draw picture:




$P = (x, y) = (x, \sqrt{x})$, $\underline{x \geq 0}$

$D$ = distance from $P = (x, \sqrt{x})$

to $(10,0)$ =

$$\sqrt{(x-10)^2 + (\sqrt{x}-0)^2}$$

$$= \sqrt{x^2 - 20x + 100 + x}$$

$$= \sqrt{x^2 - 19x + 100} = (\quad)^{1/2}$$

Want to make as small as possible, i.e., find $\underline{\underline{\text{minimum}}}$.

$$D' = \frac{dD}{dx} = \frac{1}{2}\left(\underline{\underline{x^2 - 19x + 100}}\right)^{-1/2} \cdot (2x - 19)$$

$$= \frac{2x-19}{2\sqrt{x^2-19x+100}} = 0 \rightarrow$$

$$2x - 19 \Rightarrow x = \frac{19}{2} = 9.5$$

$$\frac{19}{2}$$

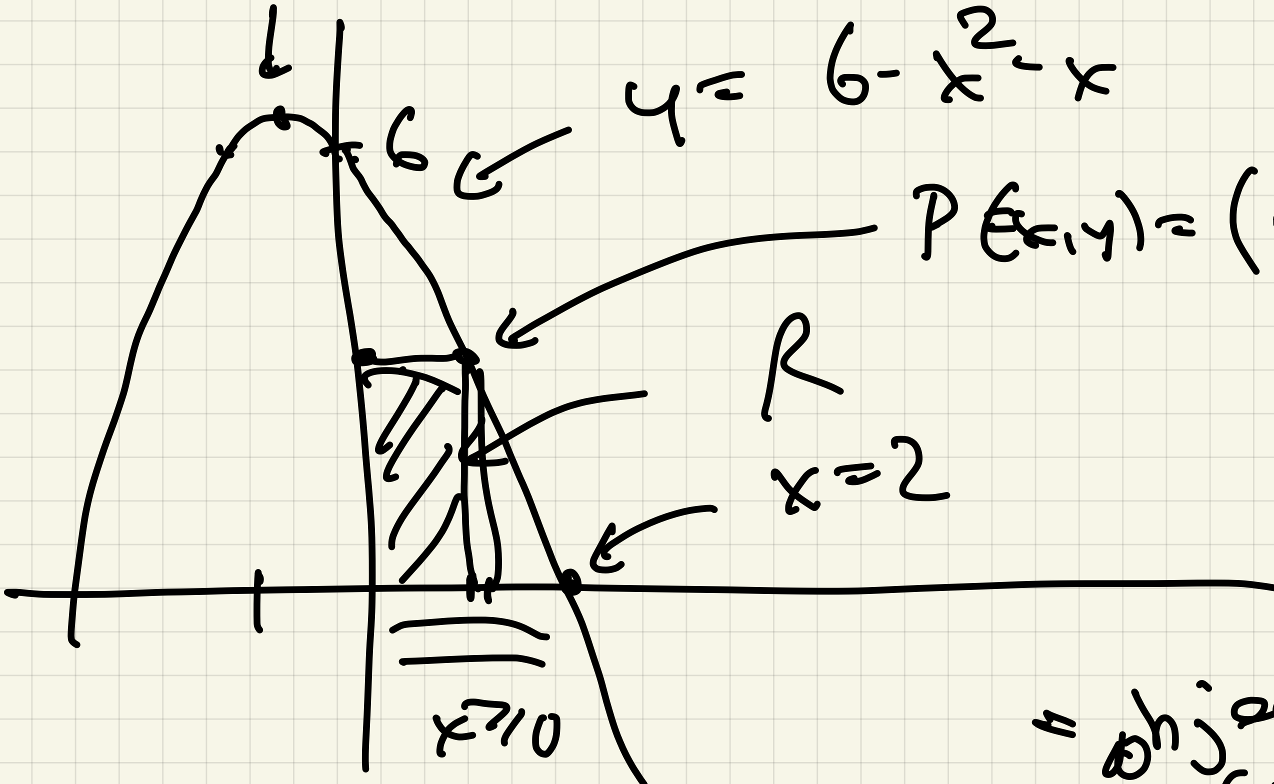
$D' < 0$ | $D' > 0$

$\frac{19}{2}$ is min

$>0$ $\quad P = \left(\frac{19}{2}, \sqrt{\frac{19}{2}}\right)$

**Ex2**: Find the largest area of a rectangle R in quad I with 2 sides along x/y axis, and the upper right corner on curve

$$y = 6 - x^2 - x$$




"objective function"

Area of R =

$$A = w \cdot h$$

$$= \boxed{x \cdot (6 - x^2 - x)}$$

$$0 \leq x \leq 2$$

$\leftarrow$ x-int

$$A = 6x - x^3 - x^2$$

<u>crit pts</u>:

$$\frac{dA}{dx} = 6 - 3x^2 - 2x = 0$$

$$0 = -3x^2 - 2x + 6$$

$ax^2 + bx + c$

$$0 = 3x^2 + 2x - 6$$

$$x = \frac{-2 \pm \sqrt{4 - 4(3)(-6)}}{2(3)}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{76}}{6}$$

$$\cancel{\frac{-2 - \sqrt{76}}{6}}$$

$x < 0$

$$\boxed{\frac{-2 + \sqrt{76}}{6}}$$

max / min ?

Easy : $d^2A/dx^2 = -6x - 2 < 0$

conc down $\Rightarrow$ crit pt is max

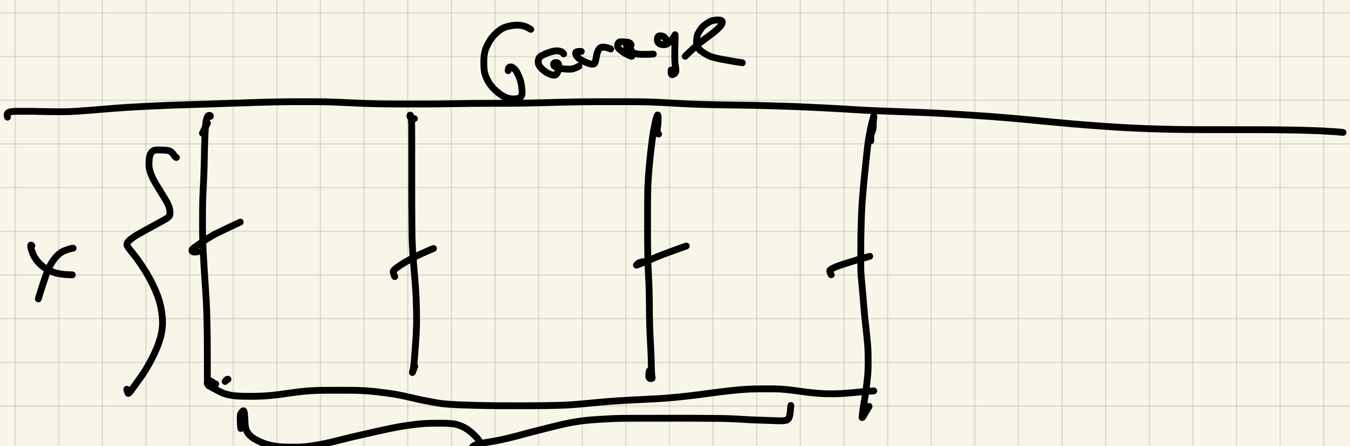
so $x = \frac{-2+\sqrt{26}}{6} \approx 1.1196$

$$\left(\frac{-2+\sqrt{26}}{6}\right)\left(6-\left(\frac{-2+\sqrt{26}}{6}\right)^2-\left(\frac{-2+\sqrt{26}}{6}\right)\right)$$

$$\shortparallel$$

$$4.061$$

<u>Ex 3</u> Beth has 100 of fencing and wants to enclose a garden next to her garage:




Garden y has 3 sub plots:

What $\boxed{\text{dimensions}}$ give the largest area for garden??

$y =$ width of garden

$x =$ depth

$$\left.\begin{aligned} \text{Area} = A &= xy \\ \text{Fence:} \quad 100 &= y + 4x \end{aligned}\right\} \Rightarrow$$

$$\boxed{y = 100 - 4x}$$

$$A = xy = x(100 - 4x)$$

$$(0 \leq x \leq 25)$$

$$A = 100x - 4x^2$$

$$\frac{dA}{dx} = 100 - 8x = 0$$

$$x = \frac{100}{8} = 12.5$$

$d^2A/dx^2 = -8 < 0$

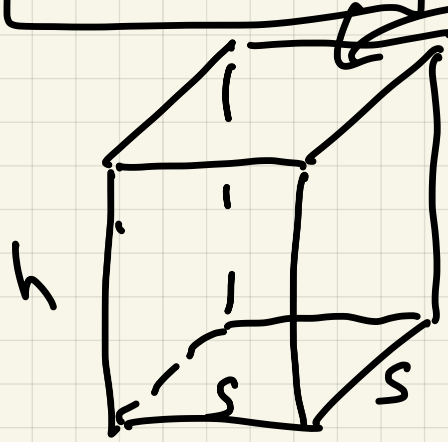
$x = 12.5$ (rel max)

<u>Dimensions</u> garden $y = 100 - 4x$

$$\boxed{12.5 \text{ ft} \times 50 \text{ ft}}$$

<u>Ex 4</u> Find <u>dimensions</u> of a box with a square base and no top and volume is $500 \text{ in}^3$ and smallest surface area.

no top



s = side of square base

h = height

$$\boxed{\begin{aligned} SA = A &= s^2 + 4sh \\ V = s^2 h &= 500 \end{aligned}}$$

combine

$$h = \frac{500}{s^2}$$

$$A = s^2 + 4s\left(\frac{500}{s^2}\right)$$

$$\cancel{\frac{dA}{ds}} = \quad s^2 + \frac{2000}{s}$$

$$\frac{dA}{ds} = 2s - \frac{2000}{s^2} = 0$$

$$2s = \frac{2000}{s^2}$$

$s^3 = 1000 \qquad s = 10$ in

$$d^2A/ds^2 = 2 + \frac{4000}{s^3} > 0$$

$\cup \leftarrow$ min ✓

$s = 10$

dimensions : $10 \times 10 \times 5$