4/30/ Calc

Quiz 22

1. Differentiate

$$F(x) = \int_0^x t^3 - 7\tan t \, dt$$

FTC → $F'(x) = x^3 - 7\tan x$

~~$\frac{1}{4}t^4 - \Big|_2^x$~~

LONG way:

$$F(x) = \int_0^x t^3 - 7\tan t \, dt =$$

$$\frac{1}{4}t^4 + 7\ln|\cos t| \Big|_0^x =$$

$$\frac{1}{4}x^3 + 7\ln|\cos x|$$

$x^3 - 7\tan x$

(b) $F(x) = \int_0^{\sin x} \sqrt{1+t^2}\, dt$

$y = F(x) = \int_0^{u} \sqrt{1+t^2}\, dt$ $\quad u = \sin x$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \sqrt{1+u^2} \cdot \cos x$

FTC

$= \sqrt{1+\sin^2 x} \cdot \cos x$

2. $\int_1^3 4x - x^2\, dx =$

$2x^2 - \frac{1}{3}x^3 \Big|_1^3 =$

$$(18-9] - (2 - \tfrac{1}{3})$$

$$9$$

$$7 + \tfrac{1}{3} - \tfrac{22}{3}$$

Last time: u-substitution

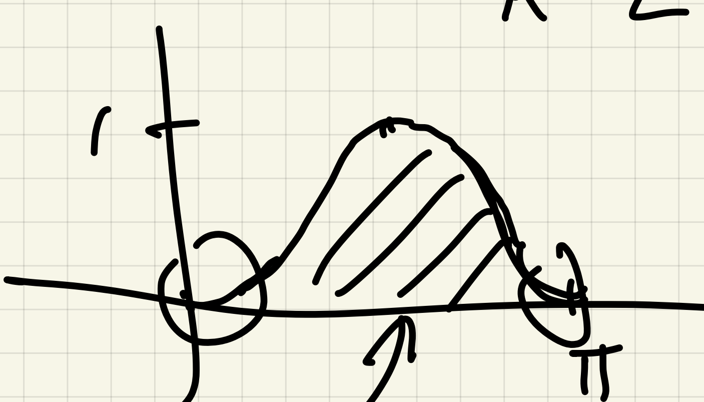
trig integrals

u-sub with endpoints

Ex|

$y = \sin x$

$A = 2$

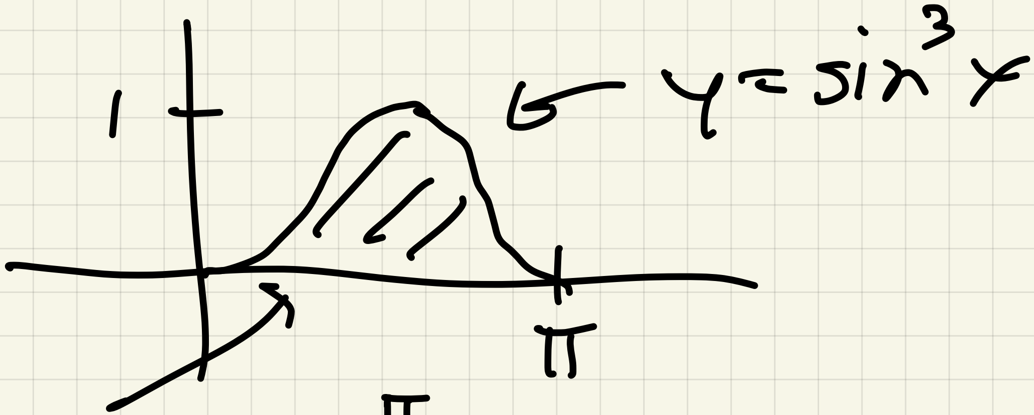


$y = \sin^2 x$

$A = \frac{\pi}{2}$

$$\int \sin^2 x \, dx$$

$$\int_0^{\pi} \frac{1 - \cos 2x}{2} \, dx$$



$$A = \int_0^{\pi} \sin^3 x \, dx =$$

$$\int_0^{\pi} \underbrace{\sin^2 x} \sin x \, dx =$$

$$\int_{x=0}^{x=\pi} (1-\cos^2 x) \underbrace{\sin x \, dx}$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$x = \pi$

$\Downarrow$

$\cos x = -1$

$x = 0$

$\Downarrow$

$\cos 0 = 1$

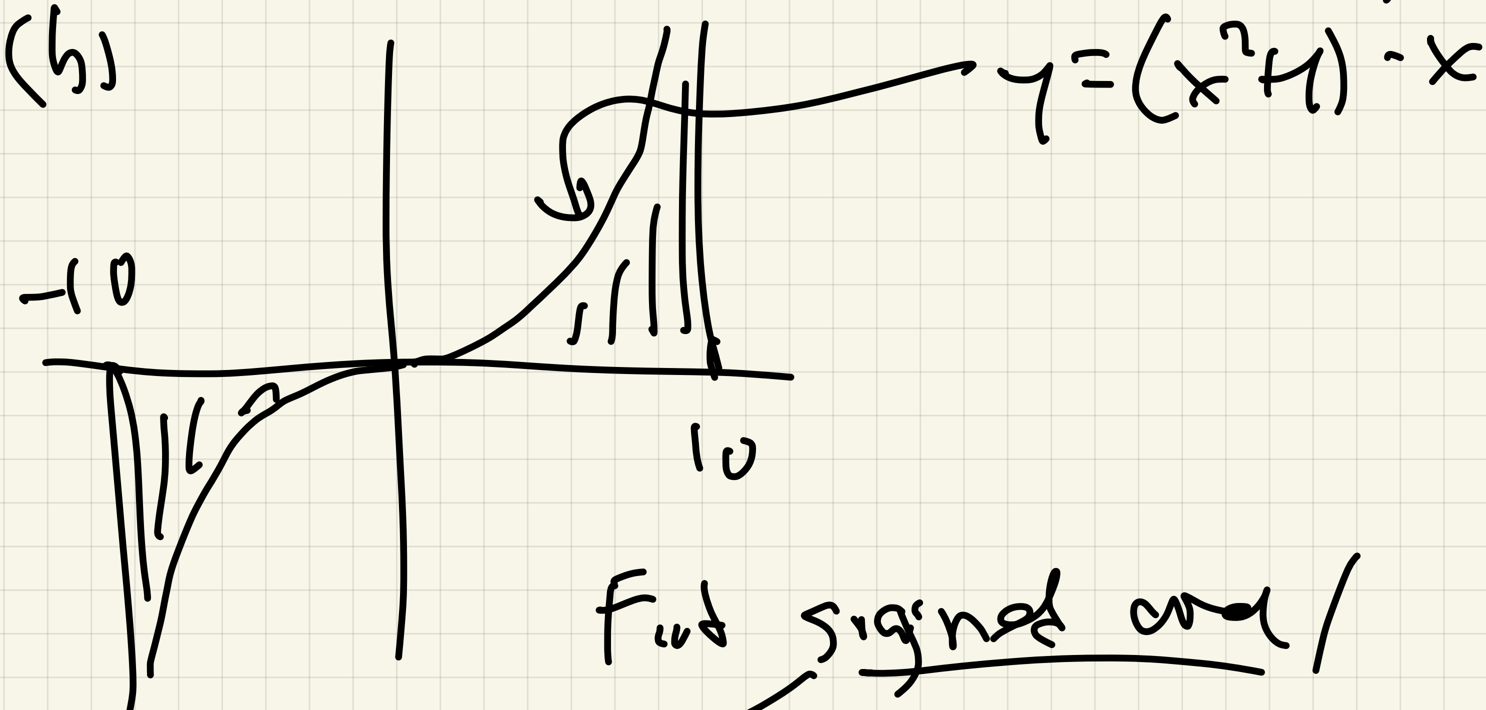
$$-\int_{u=1}^{u=-1} (1-u^2) \, du$$

$$\int_{-1}^{1} (1-u^2)\, du = u - \frac{1}{3}u^3 \Big|_{-1}^{1} =$$

$$\left(1-\frac{1}{3}\right) - \left(-1+\frac{1}{3}\right) =$$

$$\frac{2}{3} - \left(-2/3\right) = 4/3.$$

(b)



Find signed area / total area

$$\int_{-10}^{10} (x^2+1)^{25} \cdot x\, dx =$$

$u = x^2+1$

$du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

$$\frac{1}{2}\int_{u=101}^{u=101} u^{25}\,du = 0 \checkmark$$

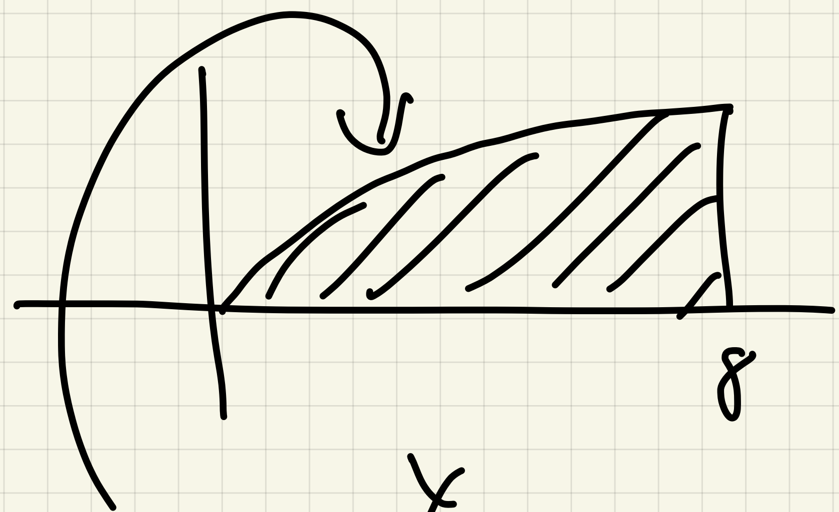
<u>Total</u> <u>Area</u> $$2\int_0^{\pi}(x^2+1)^{25}\cdot dx$$

$$=$$

$$2\cdot\frac{1}{2}\int_1^{101} u^{25}\,du = \sout{\frac{1}{3}(u^3)\Big|^{101}}$$

$$\frac{u^{26}}{26}\Bigg|_1^{101} = \frac{101^{26}-1}{26}$$

<u>Ex</u>



$$y = \frac{x}{\sqrt{x+1}}$$

$$A = \int_0^8 \frac{x}{\sqrt{x+1}}\,dx$$

$u = x+1 \Rightarrow x = u-1$

$du = dx$

$$\int_1^9 \frac{u-1}{\sqrt{u}}\,du = \int_1^9 \sqrt{u} - \frac{1}{\sqrt{u}}\,du =$$

$$\int_1^9 u^{1/2} - u^{-1/2}\,du =$$

$$\frac{2}{3}u^{3/2} - 2u^{1/2}\Big|_1^9$$

$$\left(\frac{2}{3}27 - 2\cdot 3\right) - \left(2/3 - 2\right)$$

$$= 40/3$$

OR

$w = \sqrt{x+1} = (x+1)^{1/2}$

$$dw = \frac{1}{2}(x+1)^{-1/2} = \frac{dx}{2\sqrt{x+1}}$$

$$2\,dw = \frac{dx}{\sqrt{x+1}}$$

$$w = \sqrt{x+1} \Rightarrow w^2 = x+1 \Rightarrow$$

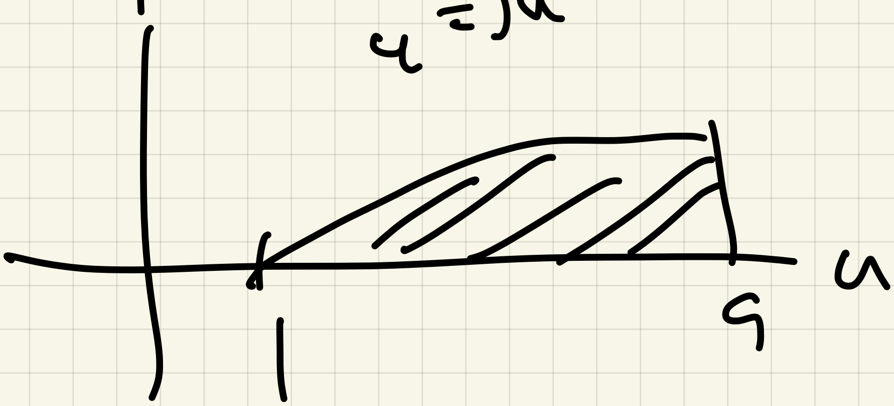
$$x = 1 - w^2$$

so $$\int_0^8 \frac{x\,dx}{\sqrt{x+1}} = \int_{w=1}^{w=3} 2(1-w^2)\,dw =$$

$$\int_1^3 2 - 2w^2\,dw = 2w - \frac{2}{3}w^3\Big|_1^3 =$$

$$40/3$$

$$y = \sqrt{u} - \frac{1}{\sqrt{u}}$$



(a) $\int_{e}^{e^e} \frac{3\,dx}{x(\ln x)}$ ← $u = \ln x$, $du = \frac{1}{x}dx$

$\int_{u=1}^{u=e} \frac{3\,du}{u} = 3\ln u \Big|_1^e =$

$3\ln e - 3\ln 1$

$3 - 0 = 3$

(b) $\int_1^2 \frac{5\,dx}{\sqrt{2x - x^2}}$ ← complete square

$\sqrt{2x - x^2}$

$-(x^2 - 2x) = -(x^2 - 2x + 1) + 1$

$= 1 - (x^2 - 2x + 1) =$

$1 - (x-1)^2$

$$\int_{x=1}^{x=2} \frac{5\,dx}{\sqrt{1-(x-1)^2}}$$

$u = x-1$

$du = dx$

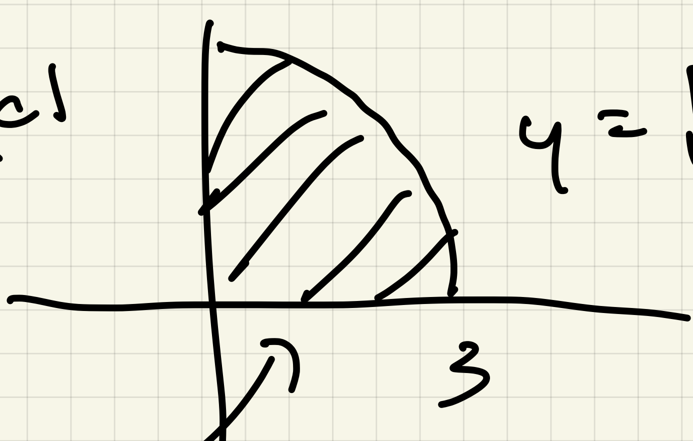
$$\int_{u=0}^{u=1} \frac{5\,du}{\sqrt{1-u^2}} = 5\arcsin u\Big|_0^1 =$$

$$5\arcsin 1 - 5\arcsin 0$$

$$= 5\cdot\frac{\pi}{2} - 5\cdot 0$$

$$= 5\pi/2$$

(c)



$y = \sqrt{9-x^2}$

$$A = \int_0^3 \sqrt{9-x^2}\,dx$$

like: $x = 3\sin u$

$\frac{x}{3} = \sin u$

$u = \arcsin \frac{x}{3}$

$$du = \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3}\,dx$$

$$= \frac{dx}{3\sqrt{1-\frac{x^2}{9}}} = \frac{dx}{\sqrt{9-x^2}}$$

$$\int_0^3 \frac{9-x^2}{\sqrt{9-x^2}}\,dx$$

$$\int (9-9\sin^2 u)\,du$$

$$9\int_0^{\pi/2} \cos^2 u\,dt$$

$$\int_0^{\pi/2} 9\left(\frac{1+\cos 2u}{2}\right) =$$

$$9\left(\frac{u}{2} + \frac{\sin 2u}{4}\right)\Bigg|_0^{\pi/2} =$$

$$9\left(\frac{\pi/2}{2}\right) = \frac{9\pi}{4}$$

<u>Exam</u>: May 9, Thursday

2 - 430

Ch 2 Limits	20%
3 Derivatives	20%
4 Applications	20%
5 Integration	40%

Graphing

max/min ← (Applications)

related rates

Integration:

A. $\int_a^b f(x)\,dx$ — defn $\lim_{n\to\infty} \sum_{k=1}^{n} f(c_k)\,\Delta x$; estimate; signed area

Definite integral

B. Indefinite Integral

$\int f(x)\,dx$ = Set of antiderivatives

fUPs

u-substitution

C. Relationship

FTC I: $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$

FTC II If $F' = f$ then

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

Area between curves