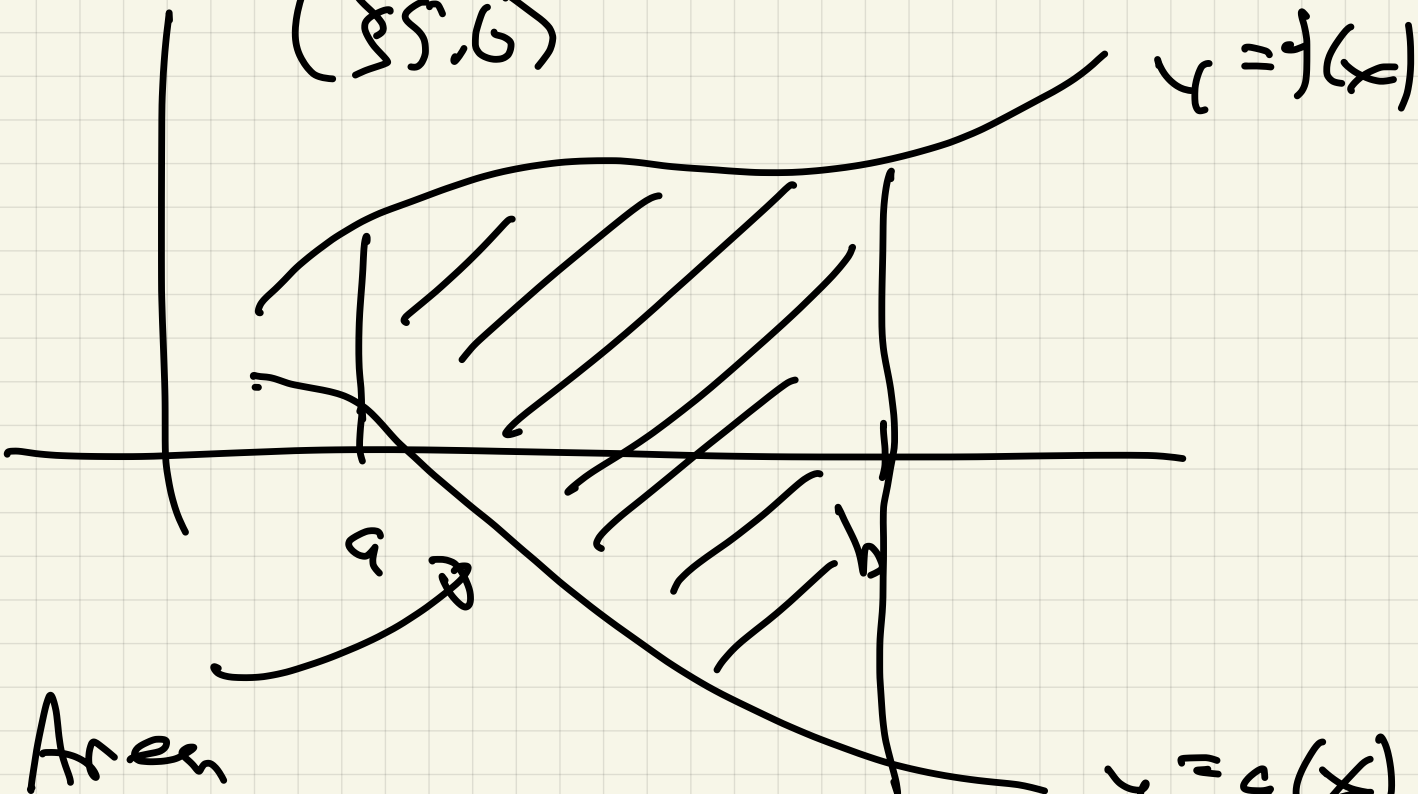
4/26 Calc 1 Final
Thursday, May 9,
2 - 430

Last time: Area between curves
(§5.6)



Area
=
$$\int_a^b f(x) - g(x)\,dx$$

Integration by u-substitution (§5.5)

Change variable

Ex! (a) $\int \tan x \, dx =$

$\int \frac{\sin x}{\cos x} \, dx = \int \frac{1}{\cos x} \sin x \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow du = -\sin x \, dx$

$-1 \, du = \sin x \, dx$

$-\int \frac{1}{u} \, du = -\ln|u| + C$

$= -\ln|\cos x| + C$

$= \ln\left|\frac{1}{\cos x}\right| + C$

$= \ln|\sec x| + C$

(b) $\int \sec x \, dx = \int \frac{dx}{\cos x}$

$\shortparallel$

$\int \sec x (1) \, dx =$

$$\int \sec x \, \frac{\sec x + \tan x}{\sec x + \tan x}\, dx$$

$$\parallel$$

$$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$$

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x \, dx$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\ln|\sec x + \tan x| + C$$

Summary:

$$\int \sin x \, dx = -\cos x + C$$

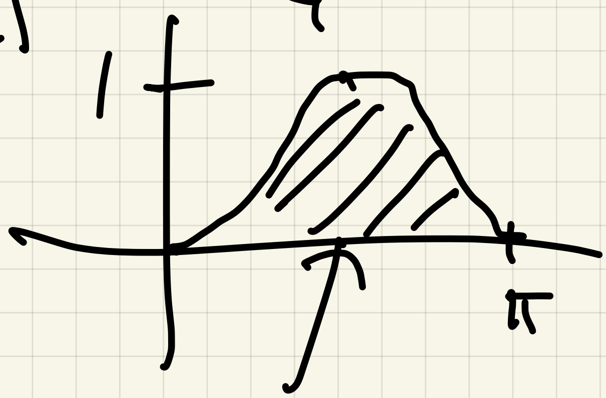
$$\int \cos x \, dx = \sin x + C$$

$$\int \cot x \, dx = \ln|\sin x| + C$$

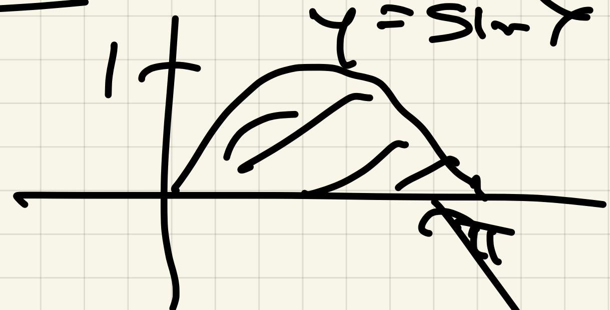
$$\int \tan x \, dx = \ln|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$
$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

(c) $y = \sin^2 x$



$y = \sin x$



$$\text{Area} = ? = \int_0^{\pi} \sin^2 x\,dx =$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\frac{1}{2}\int_0^{\pi} 1 - \boxed{\cos 2x}\,dx =$$

$$\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)\Bigg|_0^{\pi} = \frac{1}{2}\pi = \frac{\pi}{2} < 2$$

(d) $$\int \frac{\ln\sqrt{t}}{t}\,dt = \int \frac{\frac{1}{2}\ln t}{t}\,dt =$$

$\ln t^{1/2}$

$$\int \frac{1}{2}\ln t \left(\frac{1}{t}\,dt\right)$$

$$u = \ln t, \quad du = \frac{1}{t}\,dt$$

$$\int \frac{1}{2} u \, du = \frac{1}{4}u^2 + C =$$

$$= \frac{1}{4}(\ln t)^2 + C$$

(e) $$\int \frac{dx}{x \ln x}$$ $$u = \ln x$$ $$du = \frac{1}{x}\,dx$$

$$\int \frac{1}{\ln x}\left(\frac{1}{x}\,dx\right) = \int \frac{1}{u}\,du =$$

$$\ln|u| + C = \ln(\ln x) + C$$

(f) $$\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx$$ $$u = \arcsin x$$ $$du = \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\int e^u\,du = e^u + C =$$

$e^{\arcsin x} + C$

(9) $\int \sqrt{\frac{x^3-3}{x^{11}}}\,dx = \int \sqrt{\frac{1-3/x^3}{x^8}}\,dx$

$= \int \frac{\sqrt{1-3/x^3}}{x^4}\,dx$ $\quad u = 1-3x^{-3}$

$du = 9x^{-4}$

$= \frac{9}{x^4}\,dx$

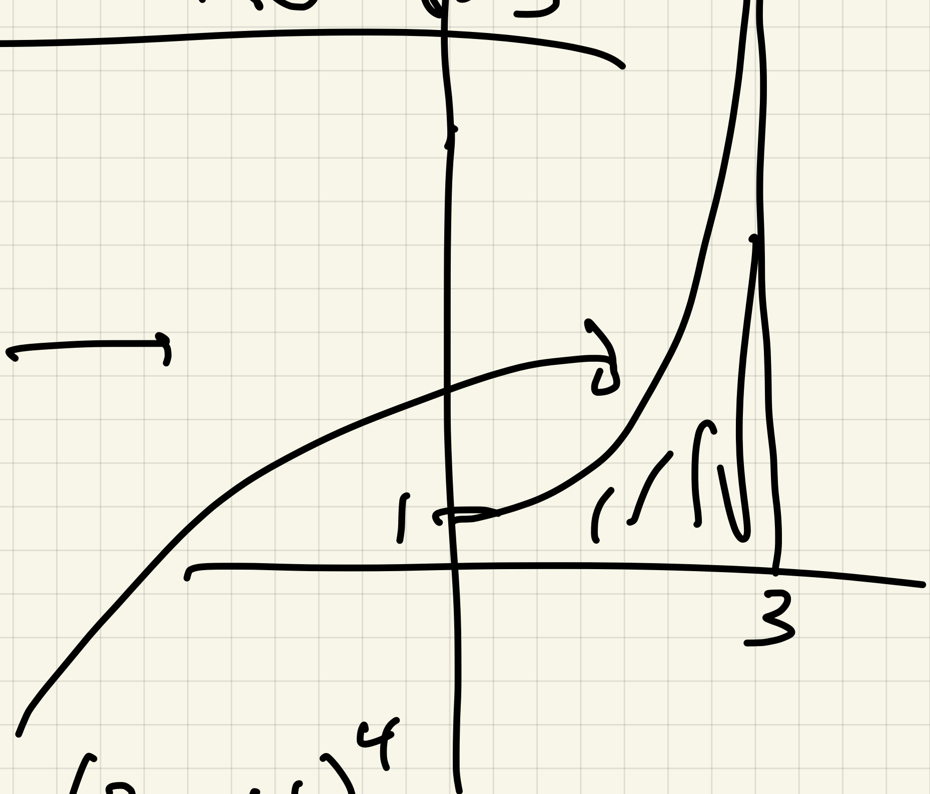
$\frac{1}{9}\int \sqrt{1-3/x^3}\,\frac{9}{x^4}\,dx =$

$\frac{1}{9}\int \sqrt{u}\,du = \frac{1}{9}\frac{2}{3}u^{3/2} + C$

$= \frac{2}{27}\left(1-3/x^3\right)^{3/2} + C$

2 Important points about

definite integrals !



Ex)

Find area

$$y = (3x+1)^4$$

$$\text{Area} = \int_0^3 (3x+1)^4\, dx$$

$u = 3x+1$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$\frac{1}{3}\int u^4\, du = \frac{1}{15}u^{5} + C$$

$$\frac{1}{15}(3x+1)^5 + C$$

$$A = \int_0^3 (3x+1)^4\,dx =$$

$$\frac{1}{15}(3x+1)^5 + C \Big|_0^3 =$$

$$\left(\frac{1}{15} 10^5 + \cancel{C}\right) - \left(\frac{1}{15}\cdot 1^5 + \cancel{C}\right) =$$

$$= \frac{1}{15}(10^5 - 1)$$

Shortcuts ..

(1) Don't need "+C"

(2) Could have changed endpoints on definite

$$\int_{x=0}^{x=3} (3x+1)^4\,dx = \frac{1}{3}\int_{u=1}^{u=10} u^4\,du$$

$$u = 3x+1$$

$$\frac{1}{15}u^5 \Big|_1^{10} = \frac{1}{15}(10^5-1)$$