4/25/ Calc Quiz 21

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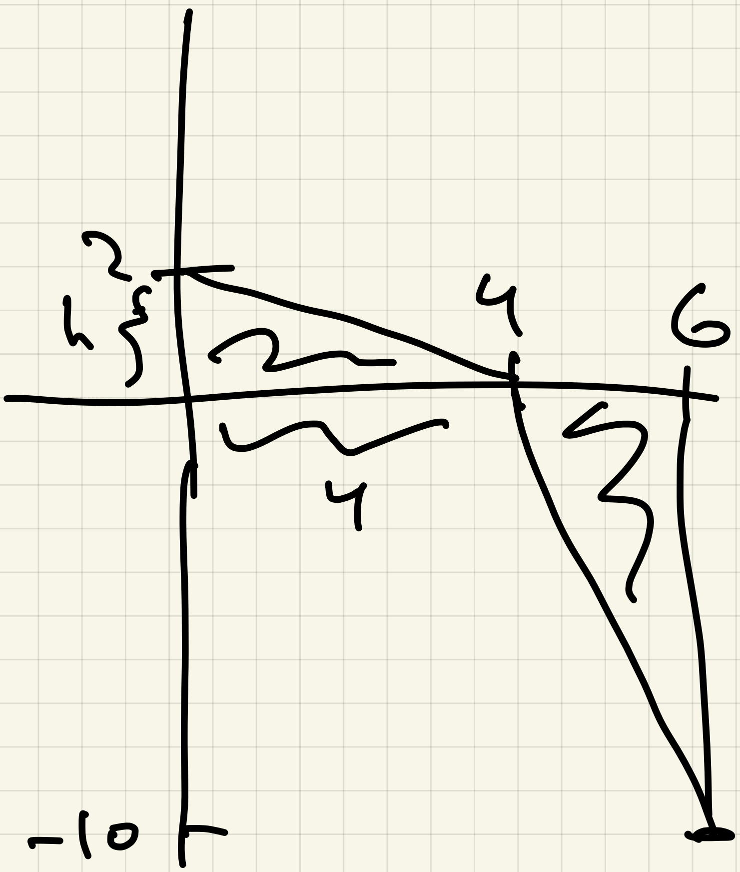
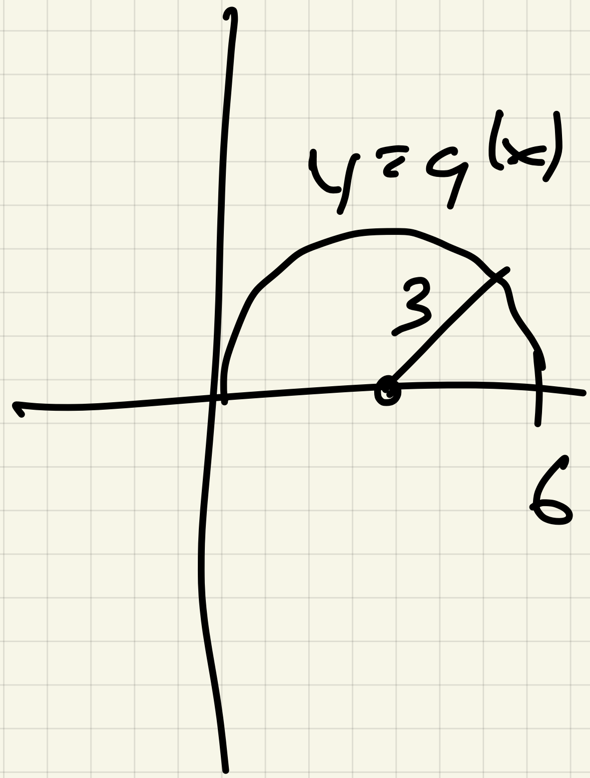




1. $\int_0^4 f(x)\,dx$ = Area = $\frac{1}{2}bh = 4$

2. $\int_4^6 f(x)\,dx = -\frac{1}{2}bh = -\frac{1}{2}\cdot 2\cdot 10 = -10$

3. $\int_0^6 f\,dx = -6$

4. $\int_6^0 f\,dx = 6$

5. $\int_0^6 g(x)\,dx = \frac{1}{2}\pi \cdot 3^2 = \frac{9\pi}{2}$

6. $\int_0^6 3g = 3\int_0^6 = 3\left(\frac{9\pi}{2}\right) = \frac{27\pi}{2}$

7. $\int_0^6 2f - 3g =$

$$2\boxed{\int_0^6 f} - 3\boxed{\int_0^6 g}$$

$= 2(-6) - 3\left(\frac{9\pi}{2}\right) =$

$-12 - \frac{27\pi}{2}$

<u>Last time</u> FTC

<u>Part 1</u> $\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$

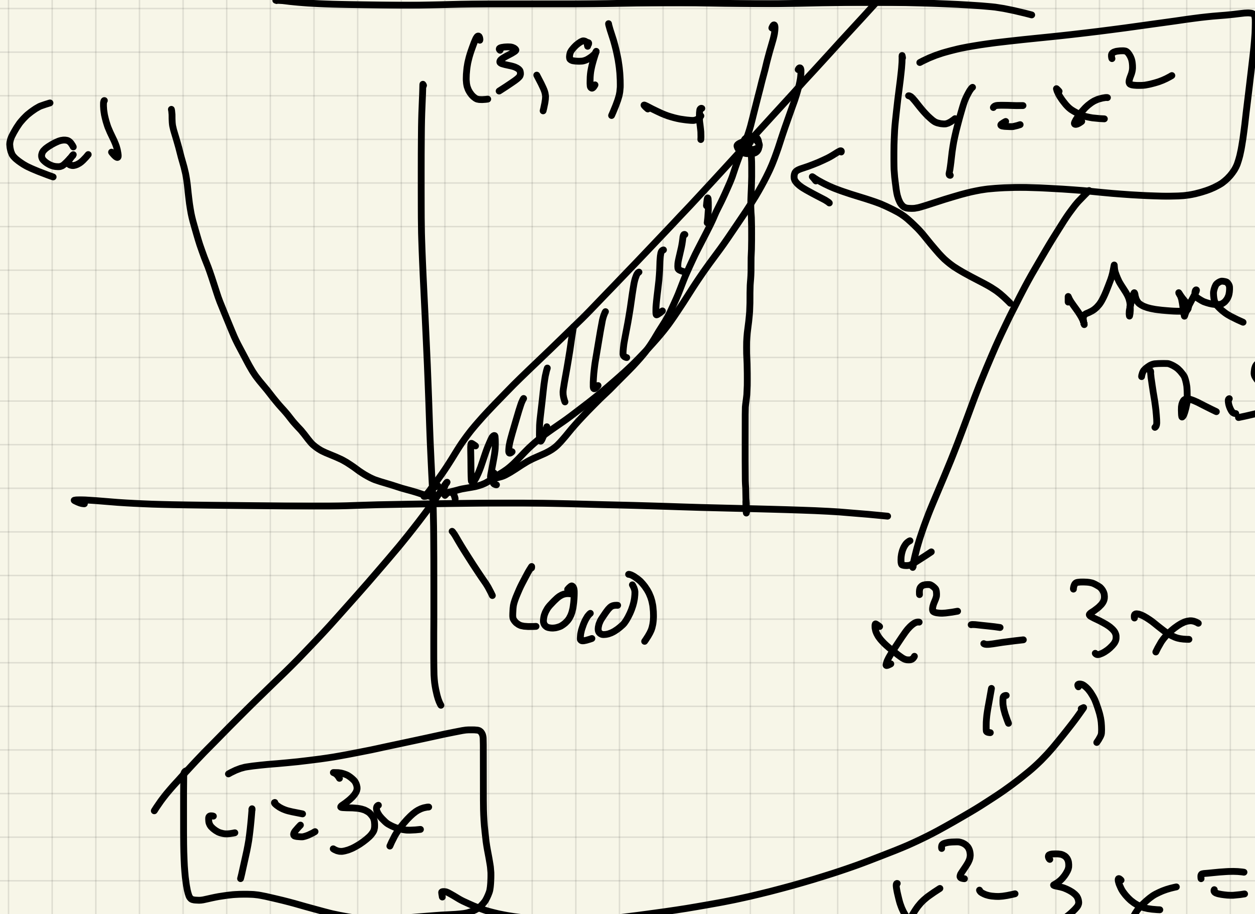
const

Part 2 $F'(x) = f(x) \Rightarrow$

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

Ex Find shaded area

(a)



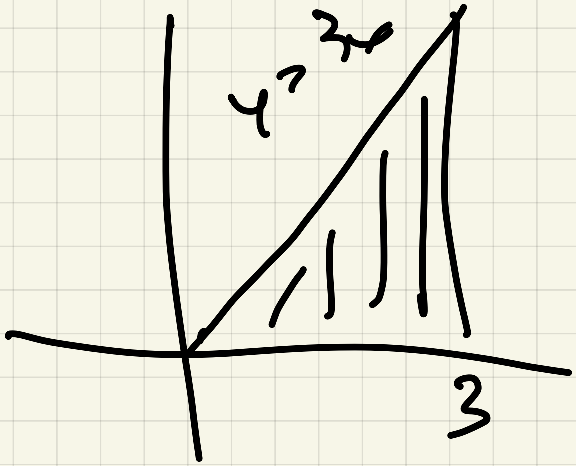
$$x^2 = 3x$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x = 0, \quad x = 3$$

Shaded area:



Area − Area

$$\int_0^3 3x\,dx - \int_0^3 x^2\,dx$$

$$\|$$

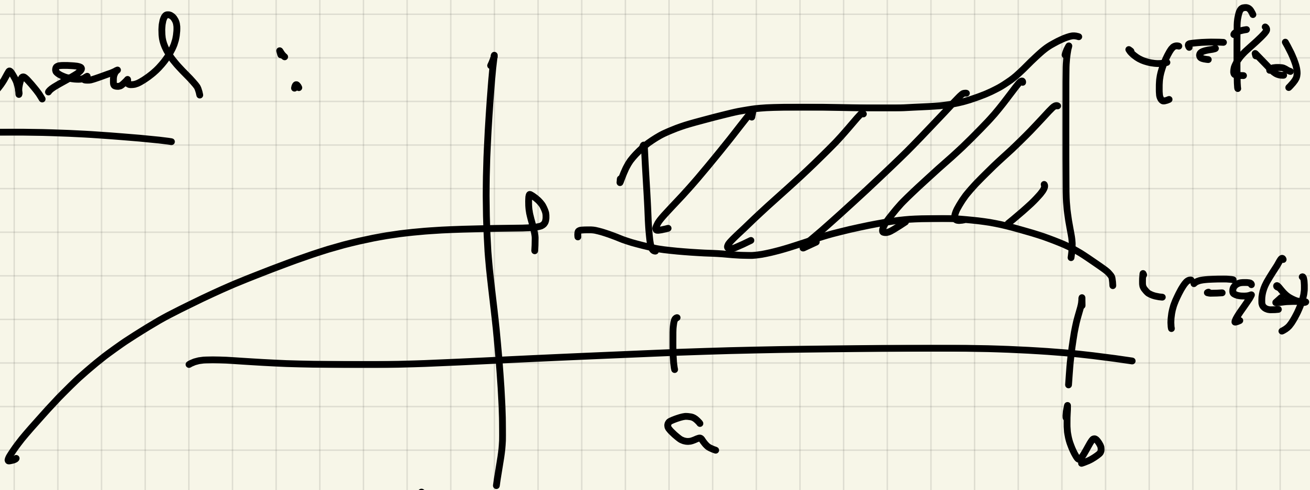
$$\int_0^3 (3x^1 - x^2)\,dx =$$

$$\frac{3}{2}x^2 - \frac{1}{3}x^3 \Big|_0^3 =$$

$$\left(\frac{3}{2}\cdot 3^2 - \frac{1}{3}\cdot 3^3\right) - (0-0)$$

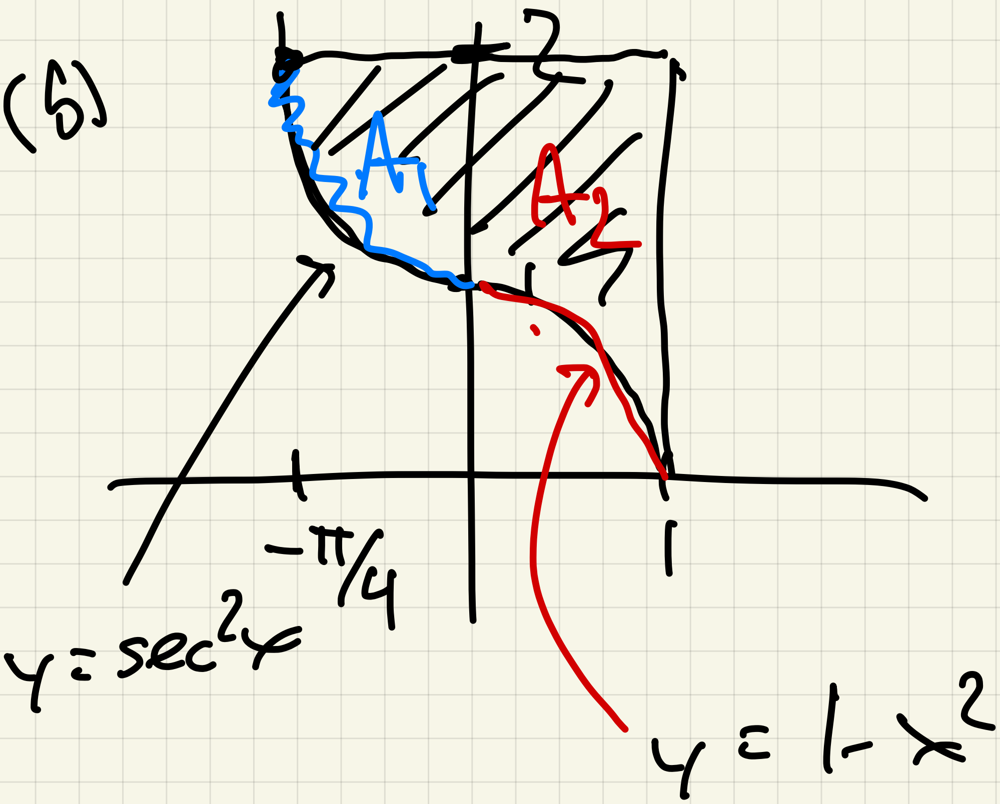
$$\frac{27}{2} - 9 = \frac{27}{2} - \frac{18}{2} = \frac{9}{2} = 4{,}5$$

In general:



$$\text{Area} = \int_a^b (f(x) - g(x))\,dx$$

(5)



$$\text{Area} = A = A_1 + A_2$$

$$A_1 = \int_{-\pi/4}^{0} 2 - \sec^2 x\,dx$$

$$= 2x - \tan x \Big|_{-\pi/4}^{0} =$$

$$= (0 - 0) - \left(2\left(-\frac{\pi}{4}\right) - \tan -\frac{\pi}{4}\right)$$

$$= -\left(-\frac{\pi}{2} - (-1)\right)$$

$$= \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$$

$$A_2 = \int_0^1 2 - (1 - x^2)\, dx$$

$$= \int_0^1 1 + x^2\, dx =$$

$$x + \frac{1}{3}x^3 \Big|_0^1 = 1 + \frac{1}{3} = \frac{4}{3}$$

so $A = A_1 + A_2 = \frac{\pi - 2}{2} + \frac{4}{3}$

$$= \frac{3\pi + 2}{6}$$

§5.5 <u>Idea</u>: undo the chain rule / change of variable / u-substitution

<u>Observe</u>: $F'(x) = f(x)$ and $\boxed{u = g(x)}$

$$\frac{d}{dx}\left(F(g(x))\right) = F'(g(x)) \cdot g'(x)$$
$$= f(g(x)) \cdot g'(x)$$
$$= f(u) \cdot \frac{du}{dx}$$

So $\displaystyle\int f(u) \cdot \frac{du}{dx} \cdot dx = F(u) + C$

$F(g(x)) + C$

<u>Key</u>: recognize $u = g(x)$

<u>Ex 1</u> Find the indefinite integral

(a) $\int \underbrace{e^{x^2}}\underbrace{2x\,dx}$

$\int e^u \quad du$

$u = x^2$

$\boxed{\frac{du}{dx} = 2x}$

$du = 2x\,dx$

$e^u + C = e^{x^2} + C$

(b) $\int 2\cos(\underset{u}{2x})\,dx$

$\int \cos(2x) \cdot \underbrace{2\,dx}$

$u = 2x, \quad du = 2\,dx$

$\int \cos u \; du$

$\sin u + C = \sin(2x) + C$

(c) $\int \sec^2(\underbrace{e^x}) \cdot \underbrace{e^x\,dx}$

$u = e^x$

$du = e^x\,dx$

$\int \sec^2 u \, du = \tan u + C$

$\tan(e^x) + C$

(d) $\int \frac{\cos x}{\sin x}\, dx = \int \frac{1}{\sin x} \cdot \underbrace{\cos x\, dx}_{du}$

$u = \sin x$
$du = \cos x\, dx$

$\int \frac{1}{u}\, du =$

$\ln|u| + C = \ln|\sin x| + C$

Ex2 More of same

$\int \frac{2x}{(x^2+1)^2}\, dx$

$u = x^2 \leftarrow$
$u = x^2+1 \leftarrow$

$u = x^2+1$ best:
$du = 2x\, dx$

$\int \frac{2x\, dx}{(x^2+1)^2} = \int \frac{du}{u^2} = -\frac{1}{u} + C$

$= \frac{-1}{(x^2+1)} + C$ ✓

Also possible:

$$u = x^2$$
$$du = 2x\,dx$$

$$\int \frac{2x\,dx}{(x^2+1)^2} = \int \frac{du}{(u+1)^2}$$

$$\boxed{v = u+1}$$

$$\frac{dv}{du} = 1 \Rightarrow dv = du$$

$$\int \frac{dv}{v^2} = -\frac{1}{v} + C$$

$$= -\frac{1}{u+1} + C = \frac{-1}{x^2+1} + C$$

(5) $\int (\sin x + 2)^4 \cos x\,dx$

$$u = \sin x + 2$$
$$du = \cos x\,dx$$

$$\int u^4\,du = \frac{1}{5}u^5 + C$$

$$= \frac{1}{5}(\sin x + 2)^5 + C$$

(c) $\int \underbrace{\sin(5x)}\,\underbrace{dx}$ $\quad u = 5x$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$$\int \sin u \,\frac{1}{5}du = \int \frac{1}{5}\sin u\,du =$$

$$-\frac{1}{5}\cos u + C = -\frac{1}{5}\cos 5x + C$$

(d) $\int (7x+10)^{12}\,dx = \frac{1}{7}\int u^{12}\,du =$

$u = 7x + 10$

$du = 7\,dx$

$\frac{1}{7}du = dx$

$$\frac{1}{7}\,\frac{1}{13}u^{13} + C = \frac{1}{91}(7x+10)^{13} + C$$

(e) $\int \underbrace{(e^x)}^{39} \cdot \underbrace{e^x dx}$

$u = e^x \quad || \quad du = e^x dx$

$$\int u^{39} du = \frac{1}{40} u^{40} + C$$

$$= \frac{1}{40}(e^x)^{40} + C$$

ALSO

$$\int (e^x)^{39} e^x = \int e^{39x} \cdot e^x dx =$$

$$\int e^{40x} dx$$

$u = 40x$

$du = 40 dx$

$\frac{1}{40} du = dx$

$$= \frac{1}{40} \int e^u du$$

$$= \frac{1}{40} e^u + C = \frac{1}{40} e^{40x} + C$$

(a) $\int \sqrt{8x+5}\,dx$

$u = 8x+5$

$du = 8\,dx$

$\frac{1}{8}du = dx$

$\int \frac{1}{8}\sqrt{u}\,du = \frac{1}{8}\,\frac{2}{3}\,u^{3/2} + C$

$u^{1/2}$

$= \frac{1}{12}u^{3/2} + C$

$= \frac{1}{12}(8x+5)^{3/2} + C$

(b) $\int x\sqrt{8x+5}\,dx$

$u = 8x+5$

$du = 8\,dx \rightarrow \frac{1}{8}du = dx$

$\frac{1}{8}\int x \sqrt{u}\,du$

$u = 8x+5 \Rightarrow u-5 = 8x \rightarrow$

$x = \frac{u-5}{8}$

$$\frac{1}{8}\int \frac{u-5}{8}\sqrt{u}\,du = \frac{1}{64}\int (u-5)\sqrt{u}\,dx$$

$$= \frac{1}{64}\int u^{3/2} - 5u^{1/2}\,du$$

$$= \frac{1}{64}\left(\frac{2}{5}u^{5/2} - 5\,\frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{1}{160}u^{5/2} - \frac{5}{96}u^{3/2} + C$$

$$= \frac{1}{160}(8x+5)^{5/2} - \frac{5}{96}(8x+5)^{3/2} + C$$

(c) $\displaystyle\int \frac{e^{2x}}{1+e^{2x}}\,dx$

$u = 1+e^{2x}$

$du = 2e^{2x}\,dx$

$\frac{1}{2}du = e^{2x}\,dx$

$$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C =$$

$$\frac{1}{2}\ln(1+e^{2x}) + C$$

(c!) $\int \frac{e^{2x}}{1+e^{2x}} dx = \int \frac{1+e^{2x}-1}{1+e^{2x}} dx$

$= \int 1 dx - \frac{1}{1+e^{2x}} dx$

$x - \int \frac{dx}{1+e^{-2x}}$ e^{-2x} e^{-2x}

$x - \left(\int \frac{e^{-2x} dx}{e^{-2x}+1} \right)$

$u = e^{-2x}+1$

$du = -2e^{-2x} dx$

$-\frac{1}{2} du = e^{-2x} dx$

$-\frac{1}{2} \int \frac{du}{u}$

$= -\frac{1}{2} \ln|u| + C$

$x + \frac{1}{2} \ln(e^{-2x}+1) + C$

(d) $\int \frac{e^x}{1+e^{2x}}\,dx$ $\qquad u = e^x$, $du = e^x\,dx$

$e^{2x} = (e^x)^2$

$$\int \frac{du}{1+u^2} = \arctan u + C$$

$= \arctan(e^x) +$