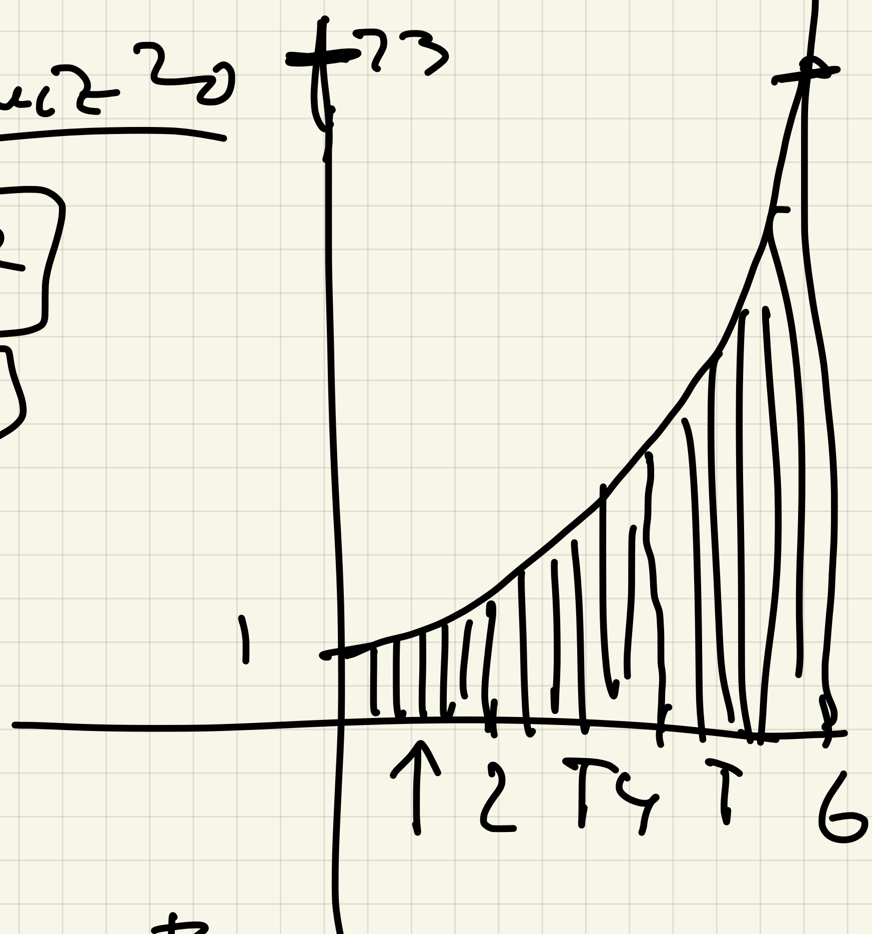
4/23/ Calc I

Quiz 20

$f(x) = \boxed{1+2x^2}$

on $[0,6]$



$n = 3$

(a) right endpoints

$$\sum_{k=1}^{3} f(c_k)\,\Delta x = f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2$$

$$9\cdot 2 + 33\cdot 2 + 73\cdot 3$$

$$= 230$$

(b) left endpoints

$$f(0)\cdot 2 + f(2)\cdot 2 + f(4)\cdot 2$$

$$1\cdot 2 + 9\cdot 2 + 33\cdot 2$$

$$2 + 18 + 66 = 86$$

#2. $\sum_{k=1}^{n} f(c_k)\Delta x$

$$\sum_{k=1}^{n}\left(1+\frac{72k^2}{n^2}\right)\frac{6}{n}$$

$$\underbrace{\sum_{k=1}^{n}\frac{6}{n}}_{6} + \underbrace{\sum_{k=1}^{n}\frac{72\cdot k^2}{n^2}\cdot\frac{6}{n}}$$

$$\frac{72\cdot 6}{n^3}\sum k^2$$

$$\frac{72\cdot\cancel{6}}{n^{\cancel{3}2}}\cdot\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}}$$

$$\lim_{n\to\infty}\underbrace{\frac{72}{\cancel{n^3}}}_{1}\underbrace{\frac{(2n^2+3n+1)}{n^2}}_{2}$$

$$72\cdot 2$$

$$6+144=150.$$

§5.4

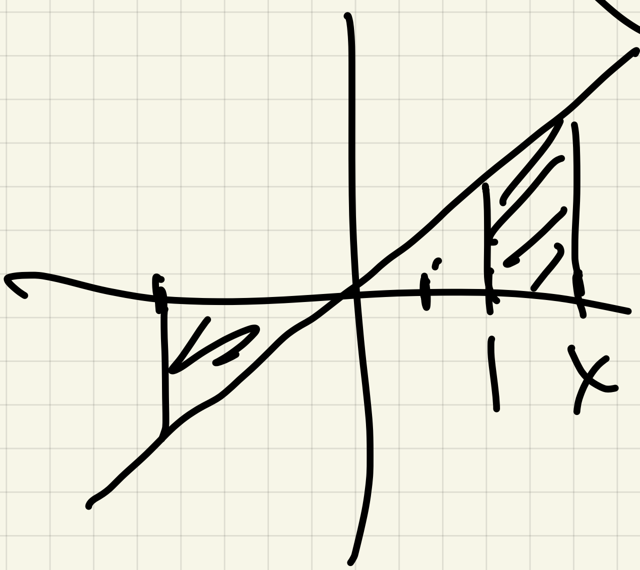
Definite Integral

Last time

$$\lim_{n\to\infty}\sum_{k=1}^{n} f(c_k)\Delta x_k = \int_a^b f(x)\,dx$$

$$\boxed{\text{complicated}} \qquad \int_a^b f(x)\,dx \;\, \| \;\, \text{Signed area}$$

Ex 0 If $A(x) = \int_1^x t\,dt = \dfrac{x^2-1}{2}$



$$\Downarrow$$

$$A'(x) = x$$

Theorem (FTC, Part 1).

If $f(x)$ is continuous on $[a,b]$,

$$\boxed{A(x) = \int_a^x f(t)\,dt}$$

is differentiable and

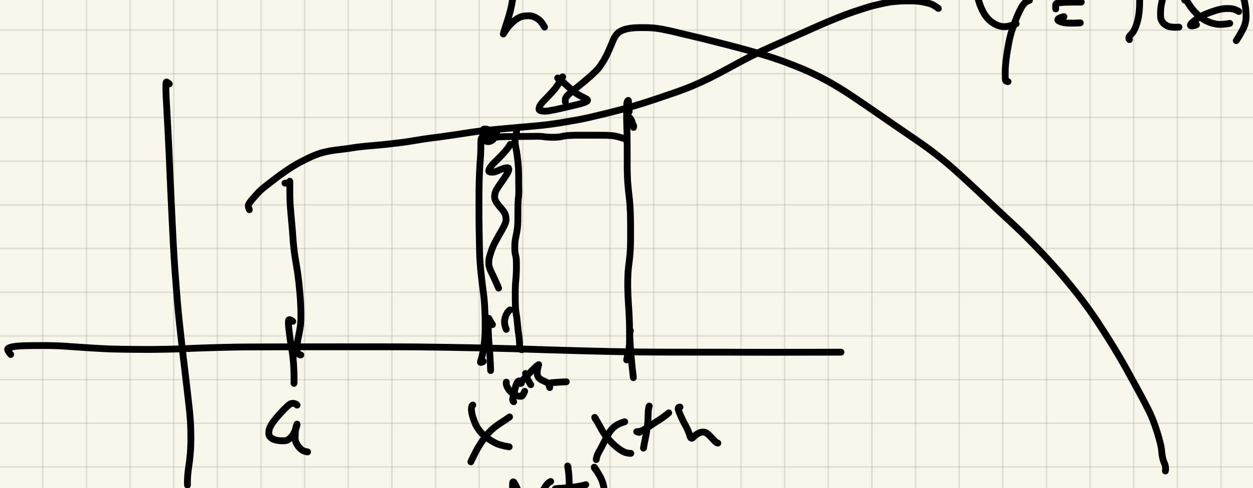
$$\boxed{A'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)}$$

Why?

$$A'(x) = \lim_{h\to 0} \frac{A(x+h) - A(x)}{h} =$$

$$\lim_{h\to 0} \frac{\int_a^{x+h} f(t)\,dt - \int_a^x f(t)\,dt}{h}$$

$$\lim_{h\to 0} \frac{\int_x^{x+h} f(t)\,dt}{h}$$



but

$$\frac{\int_x^{x+h} f(t)\,dt}{h} \approx \frac{f(x)\cdot h}{h} = f(x)$$

so

Ex1 Differentiate

(a) $y = \int_0^x \cos(5t)\,dt$

$\frac{dy}{dx} = \cos(5x)$

(b) $y = \int_8^x e^{7t^3}\,dt \Rightarrow$

$\frac{dy}{dx} = e^{7x^3}$

(c) $y = \int_{-10}^x \tan t + t^3\,dt$

$y' = \tan x + x^3$

(d) $y = \int_x^0 (t^3+1)^8\,dt$

$= -\int_0^x (t^3+1)^8\,dt$

so $y' = \quad (\cancel{x}^3+1)^8$

(e) $y = \int_{-3}^{x^2} e^{7t^3}\,dt$

$$y = \int_{-3}^{u} e^{7t^3}\,dt, \quad u = x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Thm

$$e^{7u^3} \cdot 2x = e^{7x^6} \cdot 2x$$

(f) $y = \int_{-50}^{\sin x} (t^3+2)\,dt$

$$= \int_{-50}^{u} (t^3+2)\,dt, \quad u = \sin x$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (u^3+2) \cdot \cos x$$

$$= (\sin^3 x + 2) \cdot \cos x$$

(9)

$$y = \int_{x^2}^{x^3} \arctan t \, dt =$$

$$\int_0^{x^3} \arctan t \, dt - \int_0^{x^2} \arctan t \, dt$$

LHS RHS

$u = x^3$ $u = x^2$

$$y' \quad \arctan x^3 \cdot 3x^2 - \arctan x^2 \cdot 2x$$

<u>Ex2</u> Solve IVP for $F(x)$

$$\begin{cases} F'(x) = \boxed{\cos(x^2)} \\ F(2) = 3 \end{cases}$$

Thm $$F(x) = \int_0^x \cos(t^2)\,dt + C$$

$$\underbrace{3 = F(2) = \int_0^2 \cos(t^2)\,dt + C}$$

$$C = \underline{3 - \int_0^2 \cos(t^2)\,dt}$$

so $$F(x) = \int_0^x \cos(t^2)\,dt + 3 - \int_0^2 \cos(t^2)\,dt$$

$$= \int_2^x \cos(t^2)\,dt + 3$$

Theorem (FTC, Part 2)

If f is continuous on $[a,b]$

and $F'(x) = f(x)$

(F is antiderivative for $f(x)$)

Then $\int_a^b f(t)\,dt = F(b) - F(a)$

<u>Why?</u> $F(x)$ and $\int_a^x f(t)\,dt$

both antiderivatives for $f(x)$

$\Downarrow$ §4.8

$$\int_a^x f(t)\,dt = F(x) + C$$

can solve for C by taking $x = a$

$$\underbrace{\int_a^a f(t)\,dt}_{= 0} = F(a) + C$$

so $C = -F(a)$

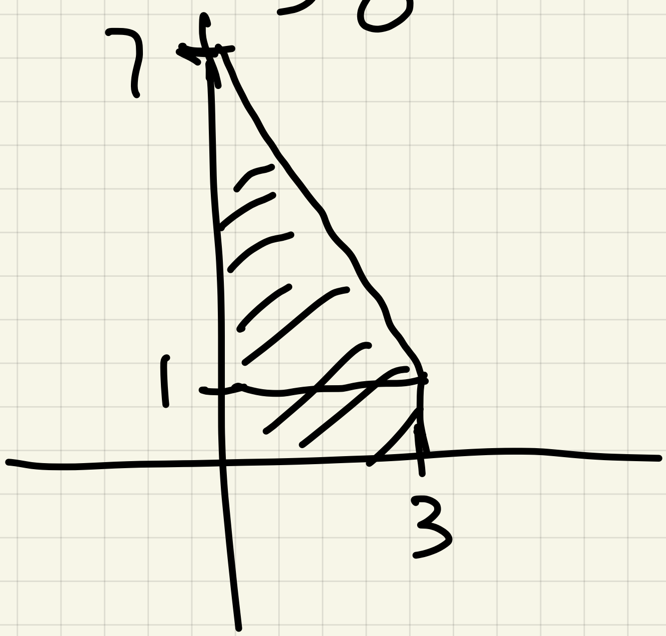
so $\int_a^x f(t)\,dt = F(x) - F(a)$

$\underline{\underline{x=b}}$ $\int_a^b f(t)\,dt = F(b) - F(a)$ ✓

Notation $\quad F(b) - F(a) = F(x)\Big|_a^b$

Ex3 $\quad$ Find definite integrals

(a) $\displaystyle\int_0^3 7-2x\,dx \overset{FTC}{=} 7x - x^2\Big|_0^3$



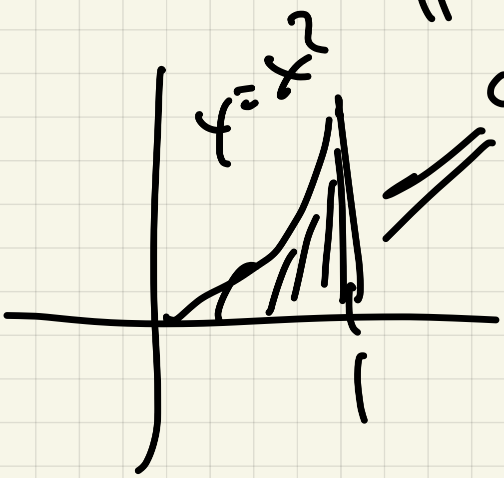
$= (21-9) - (0-0)$

$12 - 0 = 12$ ✓

(b) $\displaystyle\int_0^1 x^2\,dx \overset{FTC}{=} \frac{1}{3}x^3\Big|_0^1$

$= \frac{1}{3} - 0 = \frac{1}{3}$

$=$ area



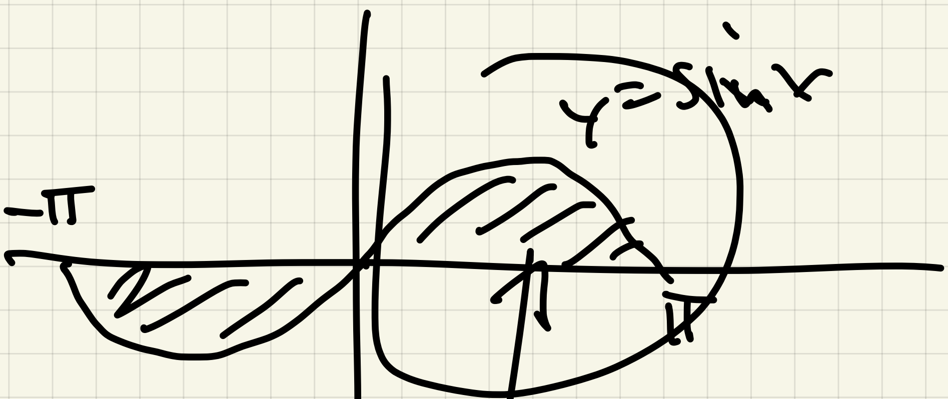
(c) $\int_{-\pi}^{\pi} \sin x \, dx = -\cos x \Big|_{-\pi}^{\pi} =$

$\Vert$ signed area

$-(-1) - (-(-1))$

$\Vert$

0



But <u>now</u> we know more:

Area $= \int_0^{\pi} \sin x \, dx =$

$-\cos x \Big|_0^{\pi} = -(-1) - (-1)$

$= 2$

So signed area $= 0$

but <u>total area</u> $= 2 + 2 = 4$

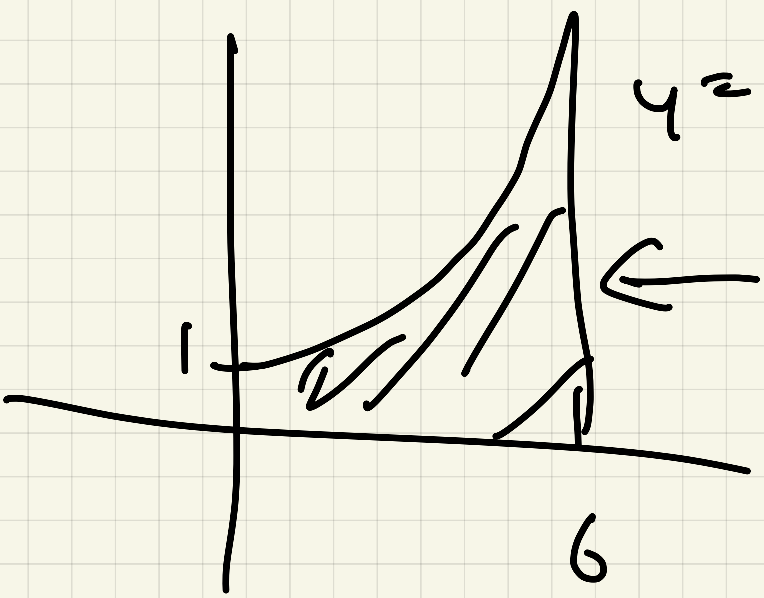
(d) $\int_a^b x^3\,dx \overset{FTC}{=} \frac{1}{4}x^4\Big|_a^b =$

$a<b$

$\frac{1}{4}b^4 - \frac{1}{4}a^4 =$

$\frac{b^4-a^4}{4}$

(e) Quiz 20



$y = 1+2x^2$

Area $= \int_0^6 1+2x^2\,dx$

$\overset{FTC}{=}$

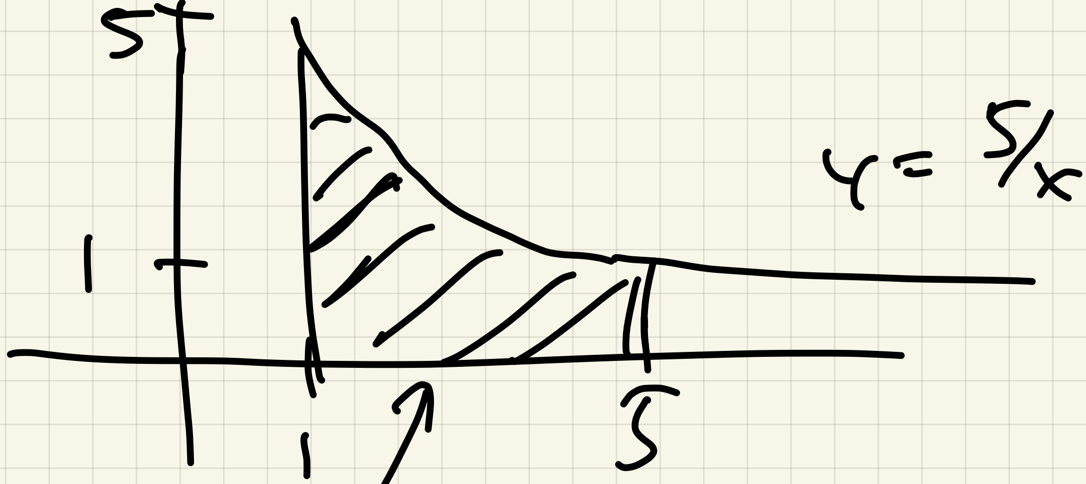
$x+\frac{2}{3}x^3\Big|_0^6 =$

$6+\frac{2}{3}(6^3) - 0$

$6+144 = 150$

Ex 4 find shaded area!

(a)



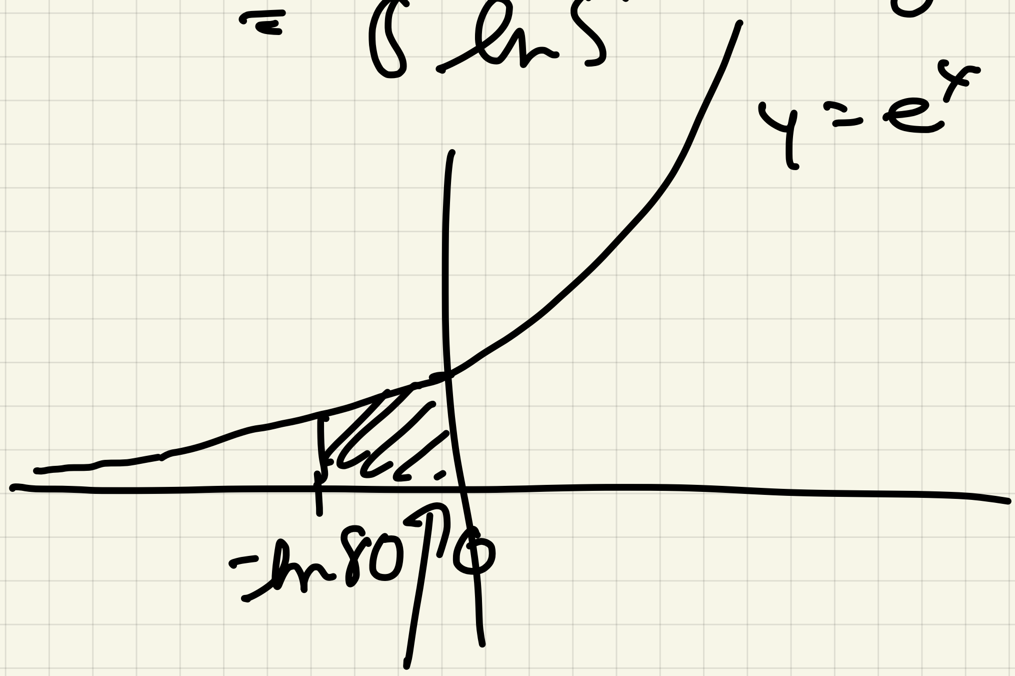
$$\text{Area} = \int_1^5 \boxed{\frac{5}{x}}\,dx \overset{FTC}{=}$$

$$= 5\ln|x| \Big|_1^5$$

$$= 5\ln 5 - 5\underbrace{\cancel{\ln 1}}_{=0} \to 0$$

$$= 5\ln 5$$

(b)



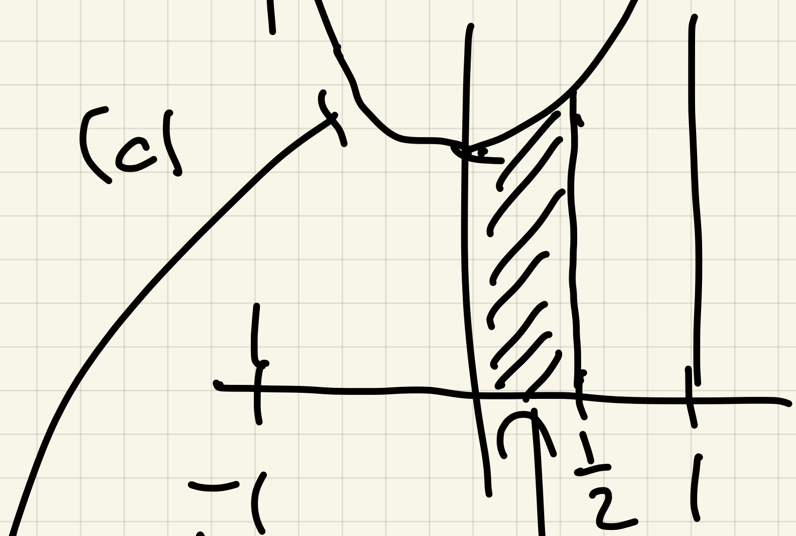
$$\text{Area} = \int_{-\ln 80}^{0} e^x \, dx =$$

$$e^x \Big|_{-\ln 80}^{0} = e^0 - \boxed{e^{-\ln 80}}$$

$$= 1 - \frac{1}{\boxed{e^{\ln 80}}} = 1 - \frac{1}{80}$$

$$\frac{79}{80}$$

(c)



$$y = \frac{1}{\sqrt{1-x^2}}$$

$$\text{Area} = \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} \, dx =$$

$$\arcsin x \Big|_0^{\frac{1}{2}} =$$

$$\arcsin\frac{1}{2} - \arcsin 0$$

$$\Downarrow$$

$$\frac{\pi}{6} - 0 = \frac{\pi}{6}$$