4/21 Calc:

$$f = \frac{1}{5}x^5 - 2x^4 + 5x^3 + 75$$

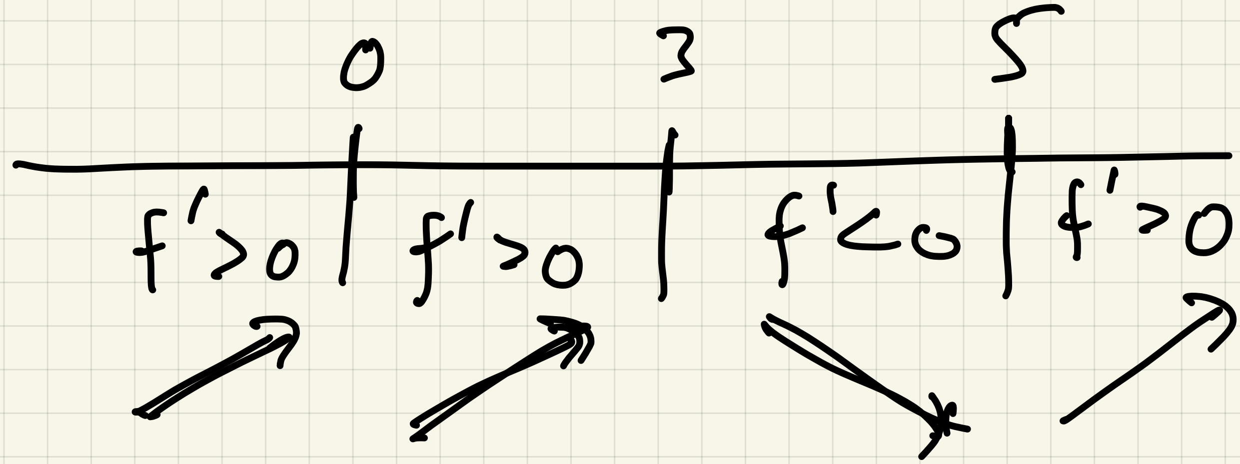
$$f' = x^4 - 8x^3 + 15x^2$$

$$= x^2(x^2 - 8x + 15)$$

$$x^2(x-3)(x-5) = 0$$

$$x = 0, 3, 5$$

2.



f incr on $(-\infty, 3) \cup (5, \infty)$

f decr on $(3, 5)$

rel max at $x=3$

rel min at $x=5$

§ 4.5

L'Hospital's rule

Last time:

If $\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0$

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)} \quad \left(\text{if exists}\right)$$

Ex 0: $\lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} \overset{LH\ 0/0}{=} \lim_{x \to 2} \frac{3x^2}{2x} = \frac{12}{4} = 3$

$=$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$$

Ex 1 (a) $\lim_{x \to 0} \frac{\sin x}{x} \overset{LH\ 0/0}{=} \lim_{x \to 0} \frac{\cos x}{1} = 1$ ✓

(b) $\lim_{x \to 0} \frac{\cos x - 1}{x} \overset{0/0}{=} \lim_{x \to 0} \frac{-\sin x}{1} = 0$

(c) $\lim_{x\to 0} \frac{\cos x - 1}{x^2} \overset{0/0}{=} \lim_{x\to 0} \frac{-\sin x}{2x} \overset{0/0}{=}$

$\lim_{x\to 0} \frac{-\cos x}{2} = -\frac{1}{2}$

(d) $\lim_{x\to 0} \frac{x}{e^{5x}-1} \overset{0/0}{=} \lim_{x\to 0} \frac{1}{5e^{5x}} = \frac{1}{5}$

(e) $\lim_{x\to 1} \frac{x}{e^{5x}-1} = \frac{1}{e^5-1}$

<u>Note</u>: L'Hospital's rule still works

if (a) for Left/right limits

(b) $c = \pm\infty$ $\quad 0/0$

(c) $\left.\begin{array}{l}\lim f = \pm\infty \\ \lim g = \pm\infty\end{array}\right\}$ $\quad \infty/\infty$

<u>Ex?</u>

4t

(a) $\displaystyle\lim_{x\to\infty} \frac{2x}{3x^2+5} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{2}{6x} \overset{2/\infty}{=} 0$

(b) $\displaystyle\lim_{x\to-\infty} \frac{2x^2+5x}{3x^2+5} \overset{\infty/\infty}{=}$

$\displaystyle\lim_{x\to-\infty} \frac{4x+5}{6x} \overset{-\infty/-\infty}{=} \lim_{x\to-\infty} \frac{4}{6} = \frac{2}{3}$

(c) $\displaystyle\lim_{x\to\infty} \frac{x^{10}}{e^x} = \lim_{x\to\infty} \frac{10x^9}{e^x} =$

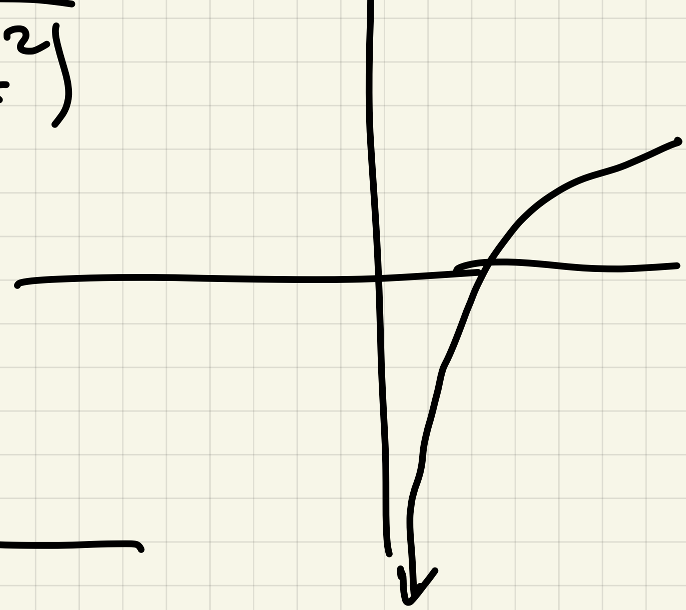
$\displaystyle\lim_{x\to\infty} \frac{90x^8}{e^x} = \lim_{x\to\infty} \frac{720x^7}{e^x} =$

$\displaystyle\ldots = \lim_{x\to\infty} \frac{10\cdot 9\cdot 8\cdot 7\cdots\cdot 2\cdot 1}{e^x} = 0$

(d) $\displaystyle\lim_{x\to 0^+} \frac{e^x}{x} = +\infty$

LH does not apply

(e) $\lim_{x \to 0^+} \frac{\ln(5x^2+x)}{\ln(3x^2)} = \frac{-\infty}{-\infty}$



$\overset{LH}{=}$

$$\lim_{x \to 0^+} \frac{\frac{10x+1}{5x^2+x}}{\frac{6x}{3x^2}}$$

$$\lim_{x \to 0^+} \frac{3x^2(10x+1)}{6x(5x^2+x)} =$$

$$\lim_{x \to 0^+} \frac{30x^3+3x^2}{30x^3+6x^2} =$$

$$\lim_{x \to 0^+} \frac{90x^2+6x}{90x^2+12x}$$

$$\lim_{x \to 0^+} \frac{180x+6}{180x+12} = \lim_{x \to 0^+} \frac{180}{180} = 1$$

(f) $\lim_{x \to 0} \frac{7^x - 1}{e^x - 1} = \lim_{x \to 0} \frac{7^x \cdot \ln 7}{e^x}$

$= \frac{\ln 7}{1} = \ln 7.$

(g) $\lim_{x \to \infty} \frac{x^3}{\sqrt{x^6+2}} =$ ∞/∞

$\lim_{x \to \infty} \frac{3x^2}{\frac{1}{2}(x^6+2)^{-1/2} \cdot 6x^5}$

$= \lim_{x \to \infty} \frac{x^2}{\frac{x^5}{\sqrt{x^6+2}}} =$

$\lim_{x \to \infty} \frac{\sqrt{x^6+2}}{x^3}$ - - keeps going, but no result!!

Fails!

$$\lim_{x \to \infty} \frac{x^3}{\sqrt{x^6+2}} = \lim_{x \to \infty} \sqrt{\frac{x^6}{x^6+2}}$$

$$= \sqrt{\lim_{x \to \infty} \frac{x^6}{x^6+2}} = \sqrt{1} = 1,$$

We've looked at "indeterminate forms"

$$0/0 \quad \infty/\infty \quad \infty/-\infty \quad -\infty/\infty$$

$$-\infty/-\infty$$

Other possibilities,

"$\infty - \infty$"

Ex 3

$$\lim_{x \to 0^+} \left(\frac{9x+2}{x} - \frac{2}{\sin x} \right)$$

$+\infty - \infty$

Unclear

$$\lim_{x \to 0^+} \frac{(9x+2)\sin x - 2x}{x \sin x} \quad \to \frac{0}{0}$$

$$\lim_{x \to 0^+} \frac{9\sin x + \boxed{(9x+2)\cos x} - 2}{\sin x + x\cos x} \quad \to \frac{0}{0}$$

$$\lim_{x \to 0^+} \frac{9\cos x + 9\cos x - (9x+2)\sin x}{\cos x + \cos x - x\sin x}$$

$$\frac{18}{2} = 9$$

Others

(A) $0 \cdot \infty$

(B) 1^∞

(C) 0^0

(A) $0 \cdot \infty$ Idea: write as fraction

Ex 4 (a) $\lim_{x \to 0^+} x \ln x$

$$= \lim_{x \to 0^+} \frac{\ln x}{\left(\frac{1}{x}\right)} =$$

$$\lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+} \frac{-x^2}{x}$$

$$= \lim_{x \to 0^+} -x = 0$$

(B) 0^0 1^∞

Idea: take logarithm

(a) $L = \lim_{x \to 0^+} x^x$

$$\ln L = \ln\left(\lim_{x \to 0^+} x^x\right) =$$

$$= \lim_{x\to 0^+} (\ln x^x)$$

$$= \lim_{x\to 0^+} x \ln x = 0$$

use previous example

$$\therefore \ln L = 0$$

$$L = e^{\ln L} = e^0 = 1$$

(b) $L = \lim_{x\to 0^+} (1-2x)^{1/x}$ (Qu.25) $\approx .135335$

$$\ln L = \lim_{x\to 0^+} \ln(1-2x)^{1/x}$$

$$= \lim_{x\to 0^+} \frac{1}{x} \ln(1-2x)$$

$$= \lim_{x\to 0^+} \frac{\ln(1-2x)}{x} \quad 0/0 \quad =$$

$$\lim_{x \to 0^+} \frac{\frac{1}{1-2x} \cdot (-2)}{1}$$

$$= -2$$

so $\ln L = -2$

$$L = e^{\ln L} = e^{-2} = \frac{1}{e^2}$$