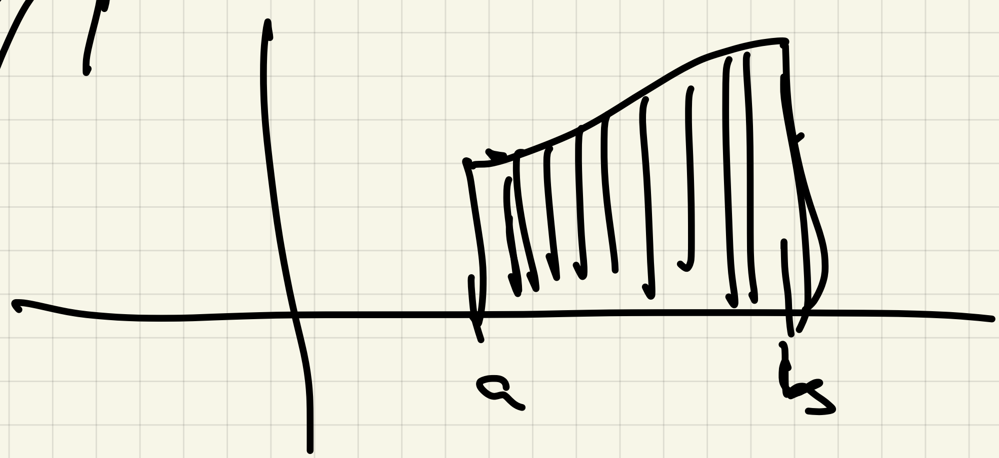
4/19| Calc I

Definite Integral

Last time

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(c_k)\,\Delta x$$

$\Delta x = \frac{b-a}{n}$

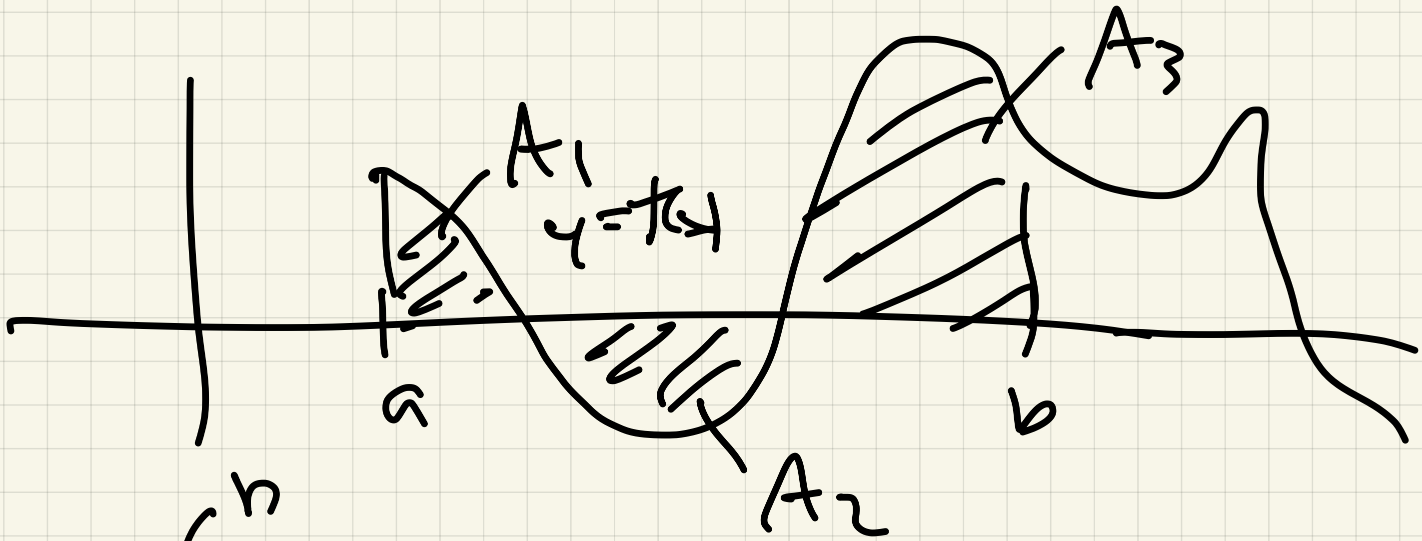


$$\lim_{n\to\infty}\sum_{k=1}^{n} f\left(a+k\left(\frac{b-a}{n}\right)\right)\cdot\left(\frac{b-a}{n}\right)$$

Geometric Interpretation: $\int_a^b f(x)\,dx =$

signed area

Area above x-axis − Area below



$$\int_a^b f(x)\,dx = A_1 - A_2 + A_3$$

Properties:

def

① $\int_a^a f(x)\,dx = 0$

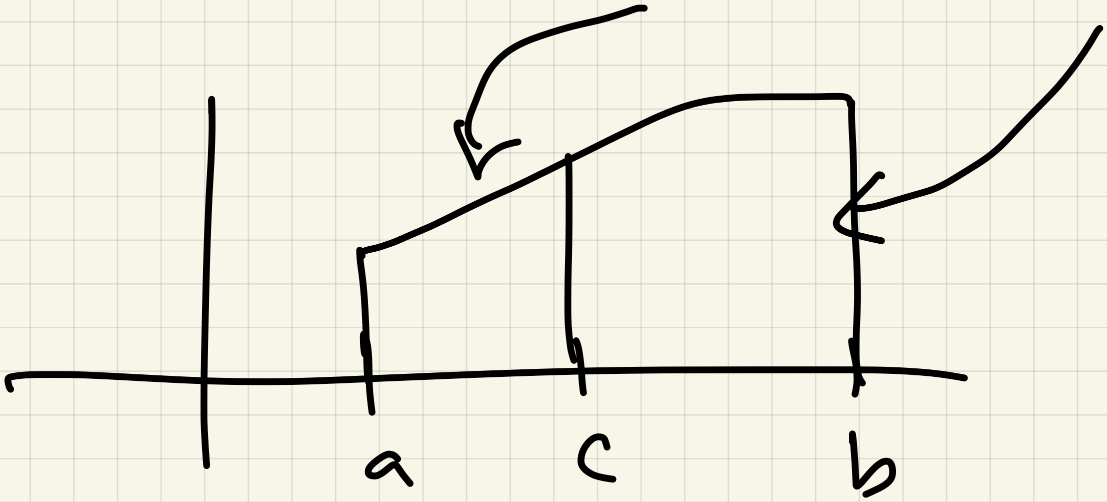
② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ $\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$

c const

④ $\int_a^b (f+g)\,dx = \int_a^b f\,dx + \int_a^b g\,dx$

⑤ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

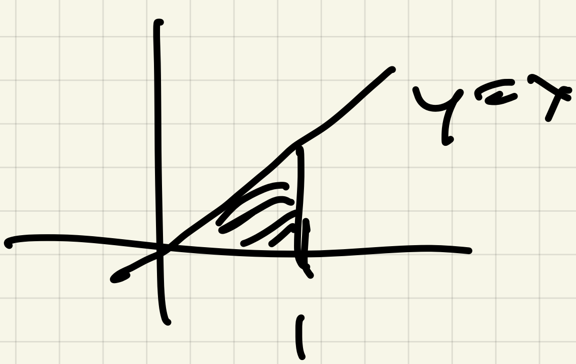


6) $f \le g \Rightarrow \int_a^b f\,dx \le \int_a^b g\,dx$

Ex] $$\int_0^1 7x + 60x^2\,dx$$

$$= \int_0^1 7x\,dx + \int_0^1 60x^2\,dx$$

$$7\underbrace{\int_0^1 x\,dx} + 60\underbrace{\int_0^1 x^2\,dx}_{1/3}$$



$$7\left(\frac{1}{2}\right) + 60\left(\frac{1}{3}\right)$$

$$7/2 + 20 = \frac{47}{2}$$

Ex2 $\int_a^b x^3\,dx \quad , \quad a < b$

Yesterday:

$c_k = a + k\Delta x$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \underbrace{(a + k\Delta x)^3}_{} \cdot \Delta x \quad \boxed{\Delta x = \frac{b-a}{n}}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(a^3 + 3a^2 k\Delta x + 3ak^2\Delta x^2 + k^3\Delta x^3\right)\Delta x$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} a^3\left(\frac{b-a}{n}\right) + 3a^2k\left(\frac{b-a}{n}\right)^2 + 3ak^2\left(\frac{b-a}{n}\right)^3 + k^3\left(\frac{b-a}{n}\right)^4$$

$$\lim_{n\to\infty} a^3\left(\frac{b-a}{n}\right)\sum_{k=1}^{n} 1 + 3a^2\left(\frac{b-a}{n}\right)^2 \sum_{k=1}^{n} k +$$

$$3a\left(\frac{b-a}{n}\right)^3 \sum k^2 + \left(\frac{b-a}{n}\right)^4 \sum k^3$$

n

$$\frac{n(n+1)(2n+1)}{6}$$

$$\frac{n(n+1)}{2}$$

$$\frac{n^2(n+1)^2}{4}$$

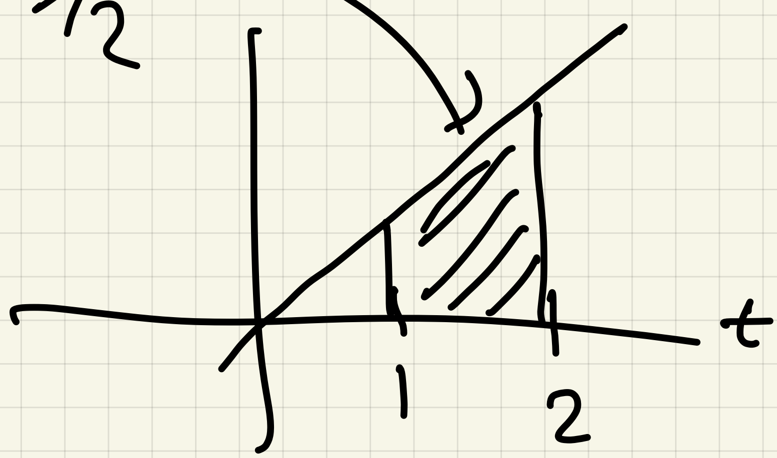
take $\lim_{n\to\infty}$

answer $\dfrac{b^4-a^4}{4}$

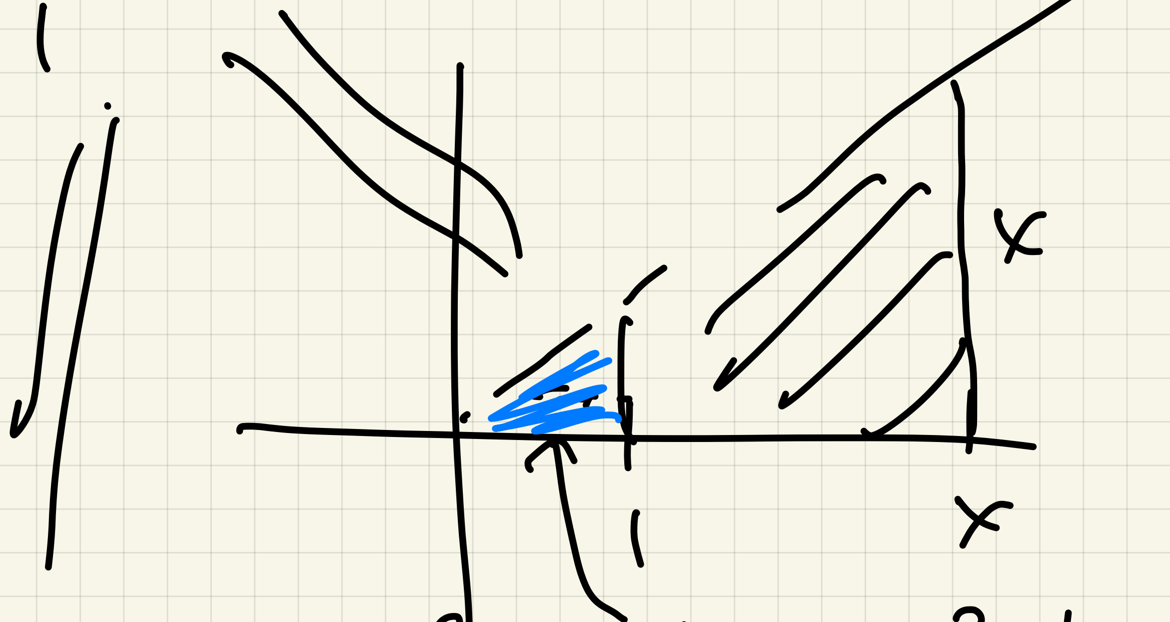
<u>Ex3</u> For $f(t) = t$, compute

(a) $\displaystyle\int_1^1 t\,dt = 0$

(b) $\displaystyle\int_1^2 t\,dt = 3/2$

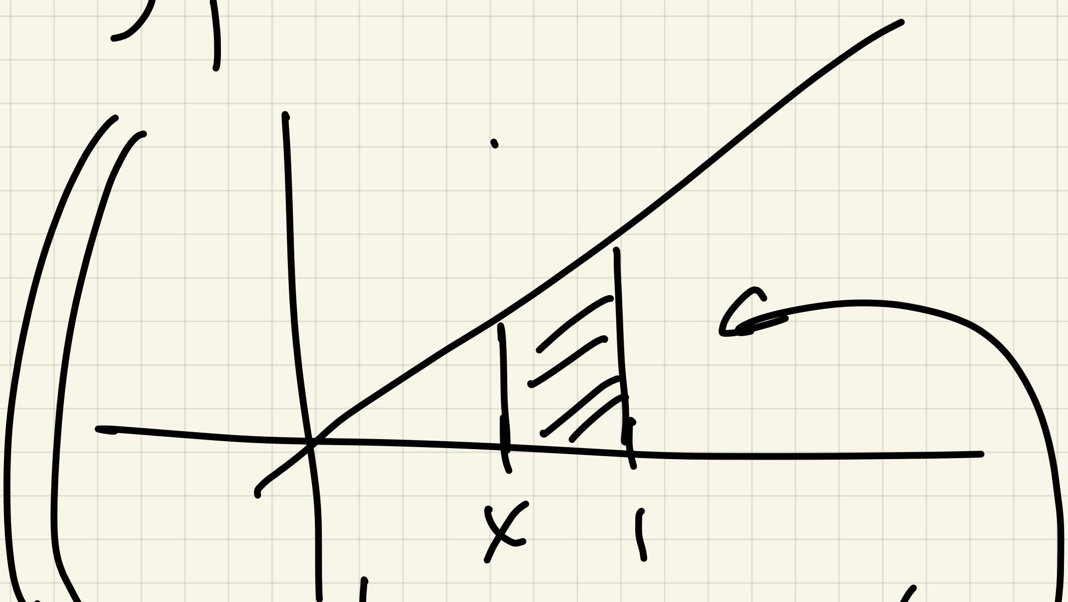


(c) $\int_1^x t\,dt, \quad x > 1$



$$\text{Area} = \frac{x^2}{2} - \frac{1}{2} = \frac{x^2-1}{2}$$

(d) $\int_1^x t\,dt, \quad 0 < x < 1$

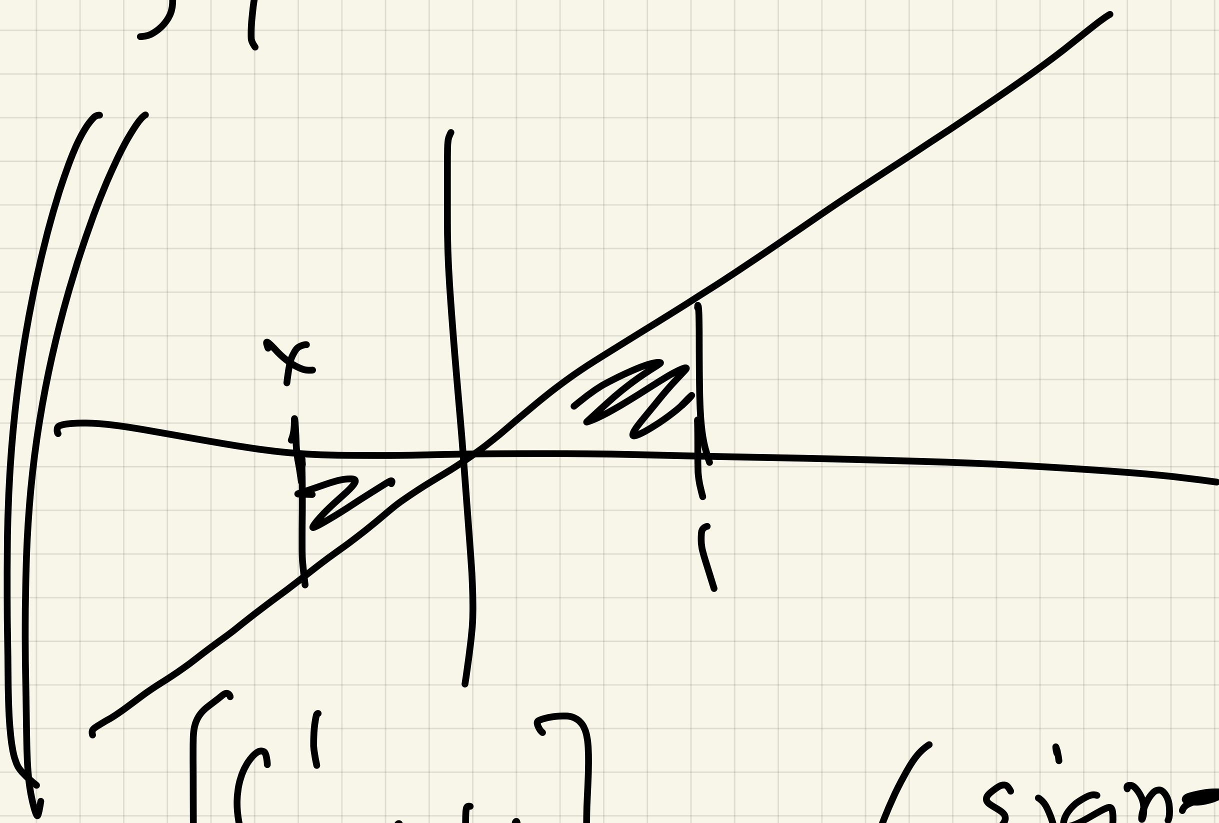


$$-\int_x^1 t\,dt = -(\text{area})$$

$$= -\left(\frac{1^2}{2} - \frac{x^2}{2}\right)$$

$$= \frac{x^2-1}{2}$$

(d) $\int_1^x t\,dt \qquad x < 0$



$$-\left[\int_x^1 t\,dt\right] = -\left(\text{signed area}\right)$$

$$= -\left(\frac{1^2}{2} - \frac{(-x)^2}{2}\right)$$

$$= \frac{x^2-1}{2}$$

So $\int_1^x t\,dt = \frac{x^2-1}{2}$

$$\frac{d}{dx}\int_1^x t\,dt = \frac{d}{dx}\left(\frac{x^2-1}{2}\right) = x$$

same function

Thm If $A(x) = \int_a^x f(\cancel{x}t)\,dt$

a constant

then $A'(x) = f(x)$