4/18 Calc: Exam 3

avg 114 ~ 76%

med 110

150 8

135 0

120

105 4

90 4

4

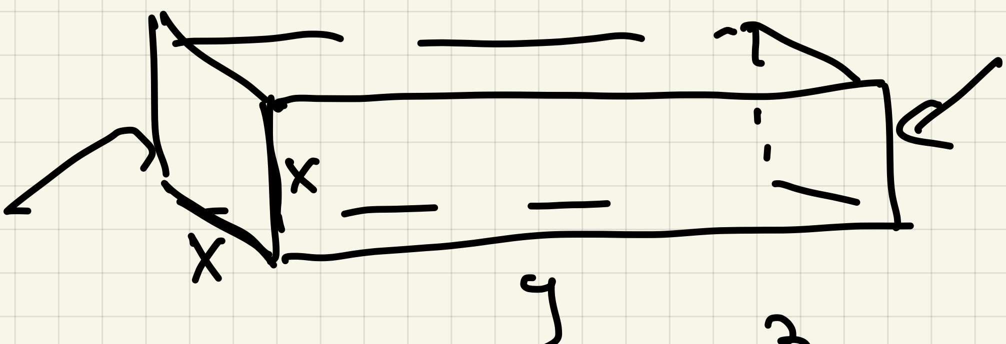
6.(c)

$$\lim_{x\to 0}(3e^x+9x)^{1/x}$$

0^0 1^∞

$$\ln L = \lim_{x\to 0}\frac{\ln(3e^x+9x)}{x}$$

7.



$$\left.\begin{array}{l}\text{Volume} = 36000 \text{ in}^3\\ \overset{4}{x^2y}\end{array}\right\} \Rightarrow y = \frac{36000}{x^2}$$

$$A = 2x^2 + 3xy$$

$$A = 2x^2 + 3x\left(\frac{36000}{x^2}\right), \quad x > 0$$

$$= 2x^2 + \frac{108000}{x}$$

$$A' = 4x - \frac{108000}{x^2} = 0$$

$$4x^3 = 108000$$

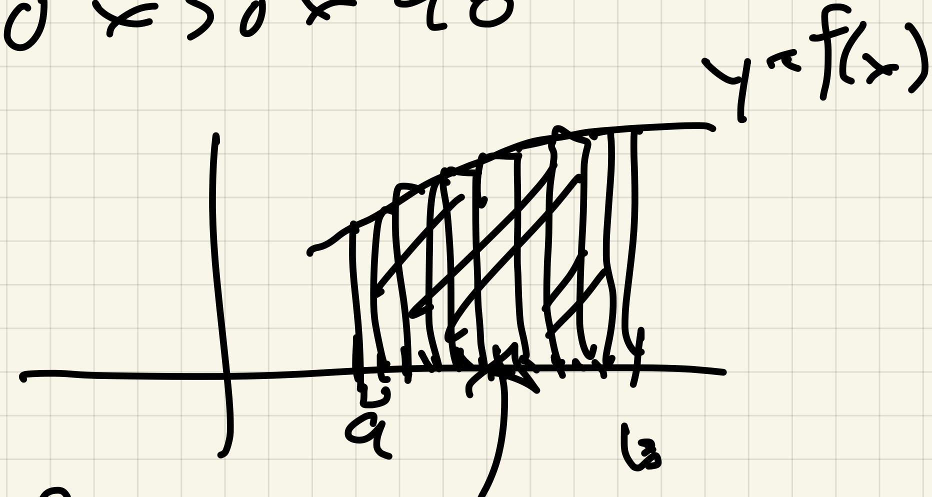
$$x^3 = 27000$$

$x = 30$ in, $y = 40$

<u>dims</u>:

$30 \times 30 \times 40$

<u>§5.1–5.2</u>



<u>Method</u>

to find to find A = area

(1) Break $[a,b]$ into n equal ~~pieces~~ intervals of width $\Delta x = \frac{b-a}{n}$

(2) Choose c_k in each interval

(easy choice: $c_k = $ right end pt)
$$= a + k \cdot \Delta x$$

③ Esimate: $\sum_{k=1}^{n} f(c_k) \cdot \Delta x$

④ Take limit $n \to \infty$

§5.2

Defn: If $f(x)$ is any function on $[a,b]$, the definite integral of $f(x)$ on $[a,b]$

$$= \int_a^b f(x)\,dx$$

$$= \lim_{n \to \infty} \sum_{k=1}^{n} f(c_k) \cdot \Delta x$$

$\|$ book

$$\lim_{n\to\infty} \sum_{k=1}^{n} f(a+k\cdot\Delta x)\cdot\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

Thm If f continuous the limit exists

$\boxed{\begin{array}{ll} \text{Extended definition:} & \text{If } a=b,\ \int_a^b f(x)\,dx \\ & \text{(natural)} \\ & \text{If } a>b, \\ & \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx \end{array}}$

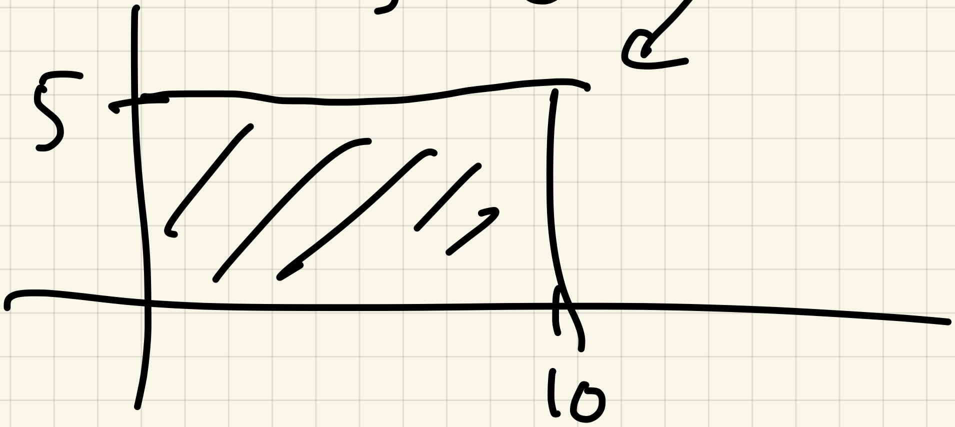
Rmks: If $f(x) \geq 0$, ~~then~~

$\int_a^b f(x)\,dx =$ Area under curve

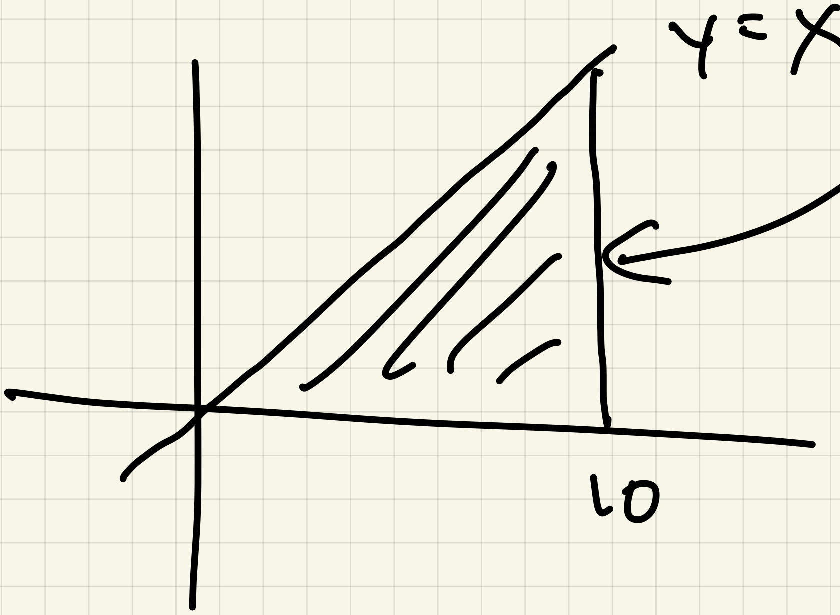
Ex1:

(a) $\int_0^{10} 5\,dx =$ Area under curve $y=5$

$5 \cdot 10 = 50$



(b) $\int_0^{10} x\,dx = \text{Area} = \frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 10 \cdot 10 = \frac{100}{2} = 50$

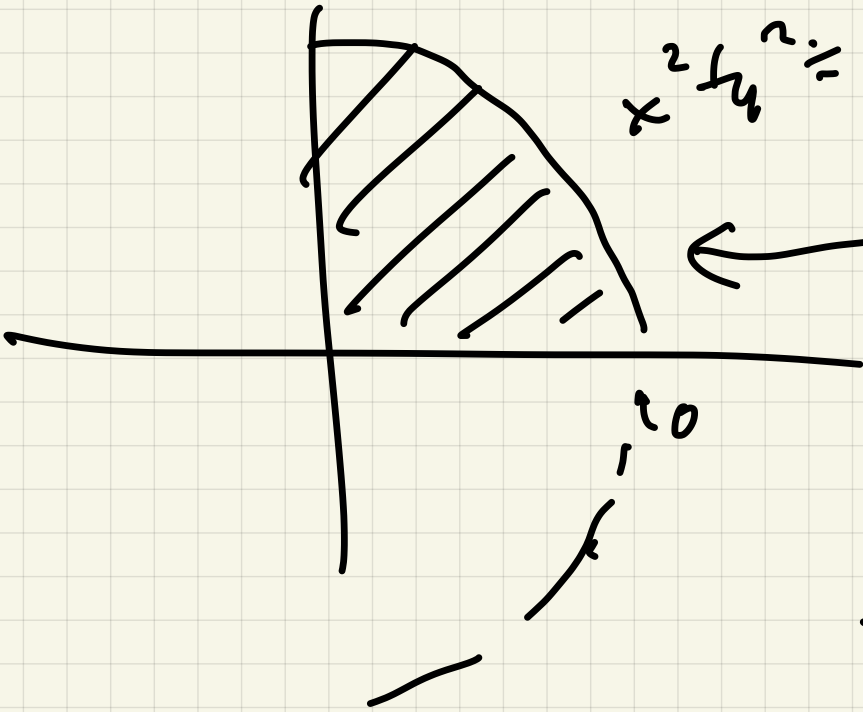


(c) $\int_0^{10} \sqrt{100 - x^2}\,dx$ = area

$y = \sqrt{100 - x^2}$

$y^2 = 100 - x^2$

$x^2+y^2=100=10^2$



$\frac{1}{4}$ (Area circle rad = 10)

$\frac{1}{4}(100\pi)=25\pi$

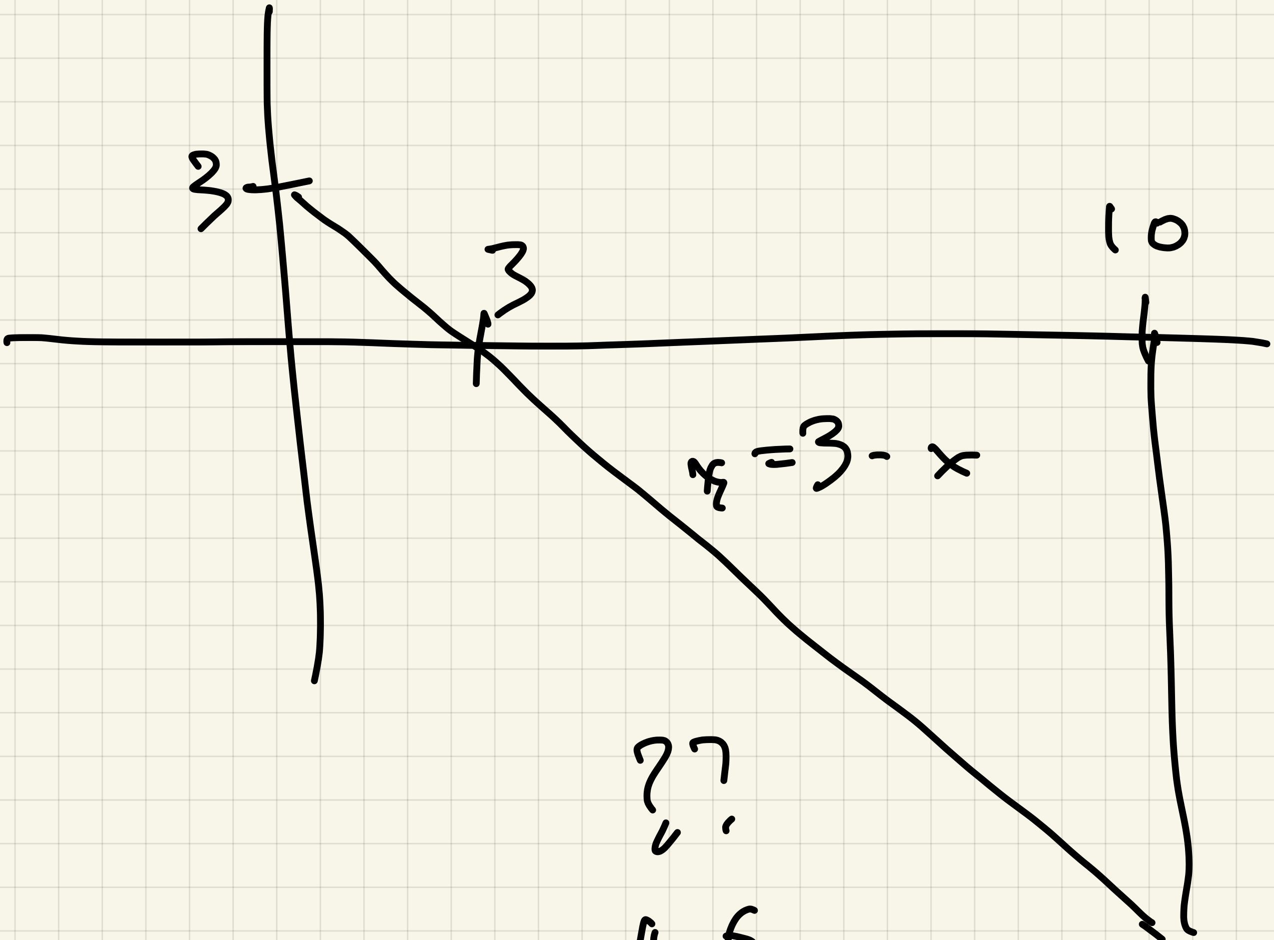
(d) $\displaystyle\int_0^0 \sqrt{100-x^2}\,dx = 0$

(e) $\displaystyle\int_{10}^0 \sqrt{100-x^2}\,dx =$

$\displaystyle -\int_0^{10} \sqrt{100-x^2}\,dx =$

-25π

(f) $\displaystyle\int_0^{10} 3-x\,dx$



??

$$\int_0^{10} (3-x)\,dx \stackrel{\text{def}}{=} \qquad f(x) = 3 - x$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} f\left(0 + k\cdot\frac{10}{n}\right)\cdot\frac{10}{n}$$

$$\Delta x = \frac{10}{n}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(3 - k\frac{10}{n}\right)\left(\frac{10}{n}\right) =$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{30}{n} - k\frac{10^2}{n^2}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{30}{n} - \sum_{k=1}^{n} k\frac{100}{n^2}$$

$$\lim_{n\to\infty} \frac{30}{n}\underbrace{\sum_{k=1}^{n} 1} - \frac{100}{n^2}\sum_{k=1}^{n} k$$

$$\lim_{n\to\infty} \frac{30}{\cancel{n}}\cdot\cancel{n} - \frac{100}{n^{\cancel{2}}}\,\frac{\cancel{n}(n+1)}{2}$$

$$\lim_{n\to\infty} 30 - \frac{100}{2}\left(\frac{n+1}{n}\right)$$

$$30 - 50 = \cancel{20}$$

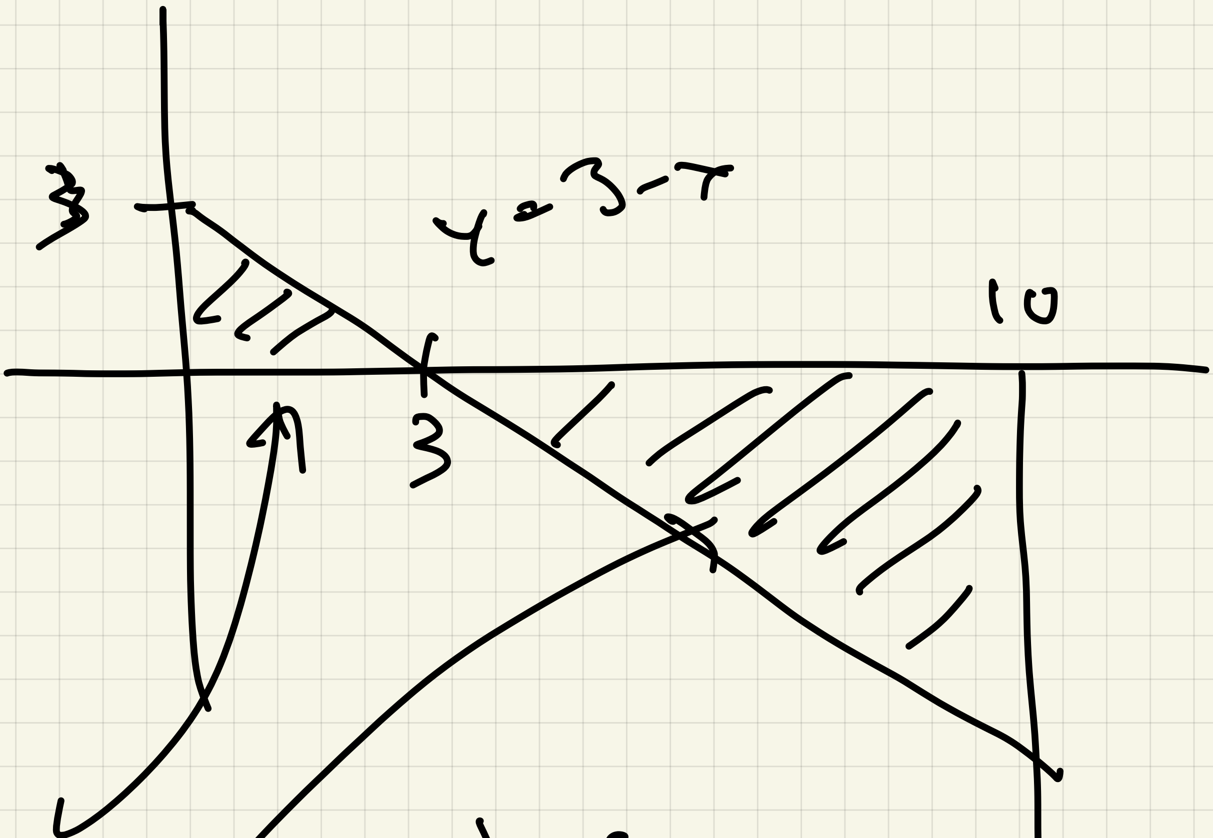
$$\to -20$$

But can interpret $\int_a^b f(x)$

as signed area

//

Area above x-axis − Area below x-axis



A_1 = area above

$= \frac{1}{2}bh = \frac{1}{2}3\cdot 3 = \frac{9}{2}$

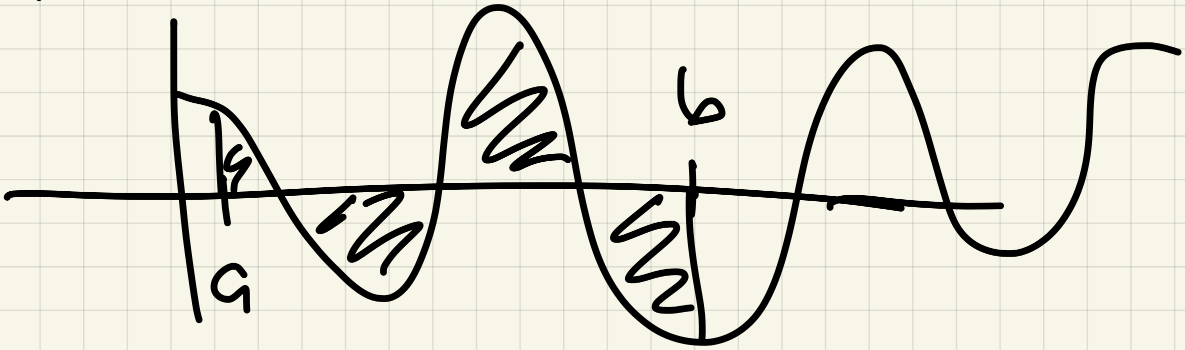
A_2 = area below $= \frac{1}{2}bh =$

$\frac{1}{2}\cdot 7\cdot 7 = \frac{49}{2}$

$\int_0^{10} 3-x\,dx = A_1 - A_2$

$$9/2 - 49/2 = -\frac{40}{2} = -20 \checkmark$$

In general

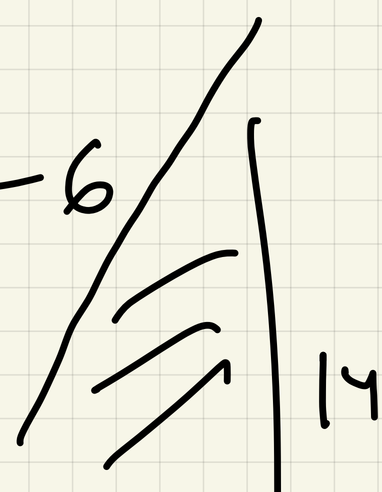


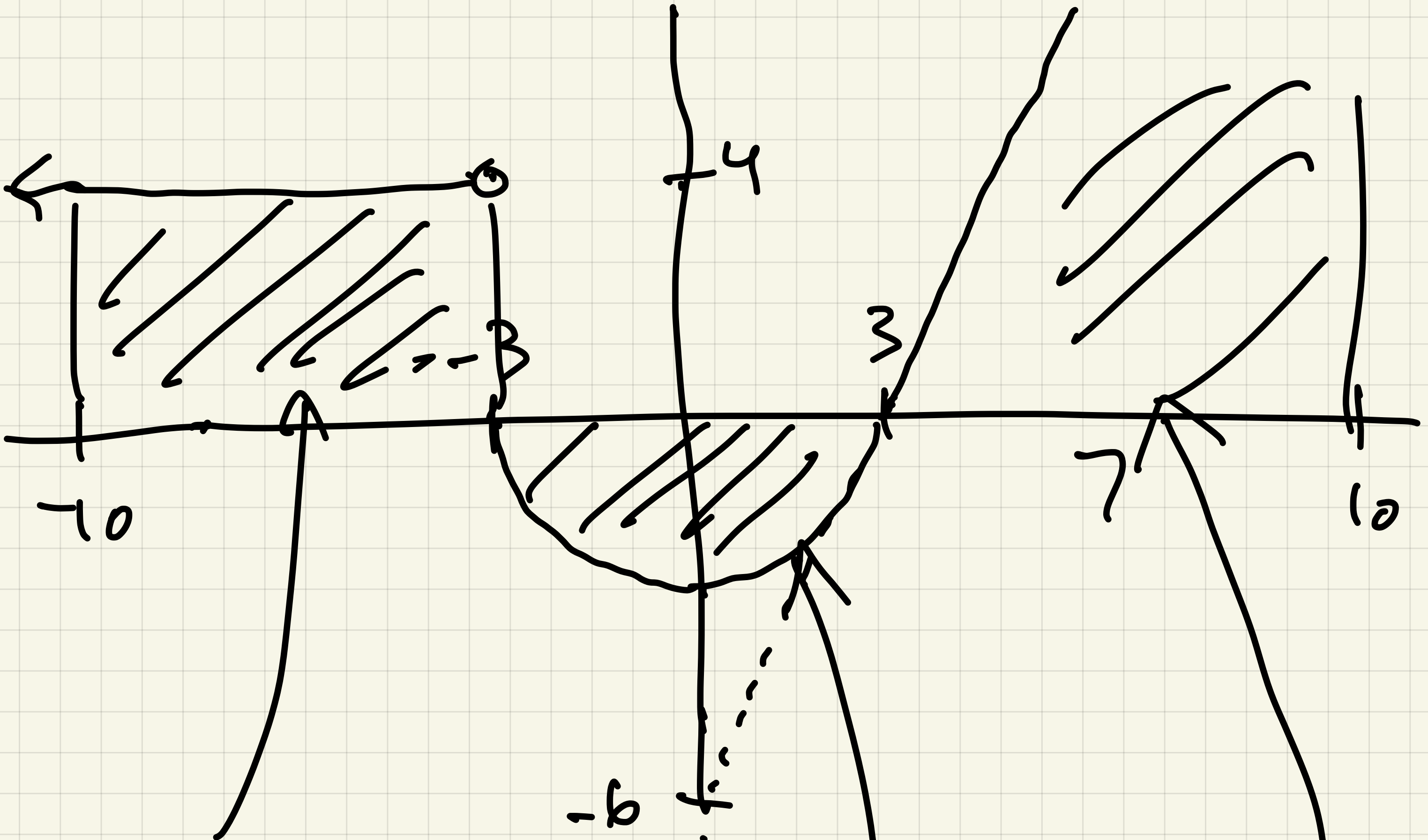
__Ex 2__

$$f(x) = \begin{cases} 4 & x < -3 \\ -\sqrt{9-x^2} & -3 \le x \le 3 \\ 2x-6 & x \geq 3 \end{cases}$$

__Find__ $\displaystyle\int_{-10}^{10} f(x)\,dx$

$y = 2x - 6$





Area $= 4 \cdot 7 = 28$

area $= \frac{1}{2} \pi \cdot 3^2$

area $\frac{1}{2} bh = \frac{1}{2} \cdot 7 \cdot 14 = 49$

So

$$\int_{-10}^{10} f(x)\,dx = 28 - \frac{9\pi}{2} + 49$$

$$= 77 - \frac{9\pi}{2}.$$

Rules for definite integrals

① $\int_a^a f(x)\,dx = 0$

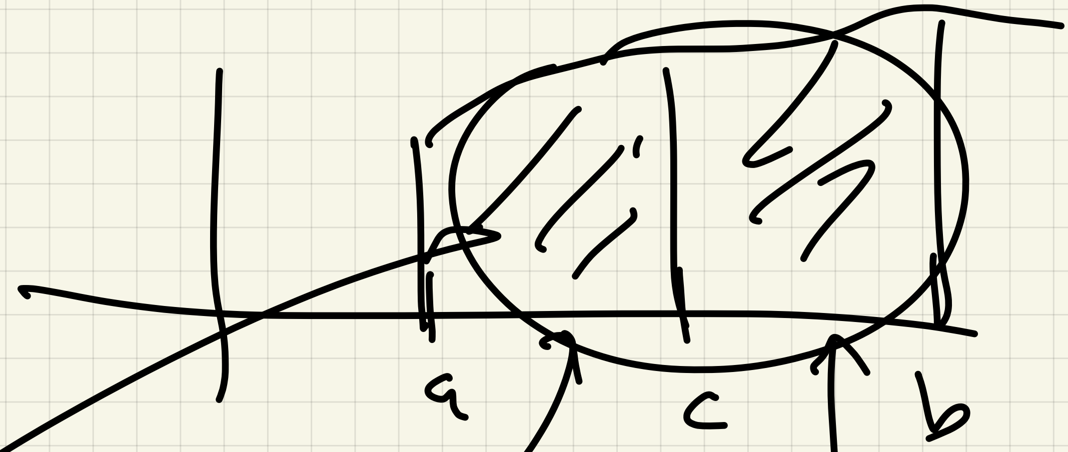
② $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

③ $\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$

④ $\int_a^b f(x) \pm g(x)\,dx$

$= \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

⑤ $\int_a^b f(x) = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$



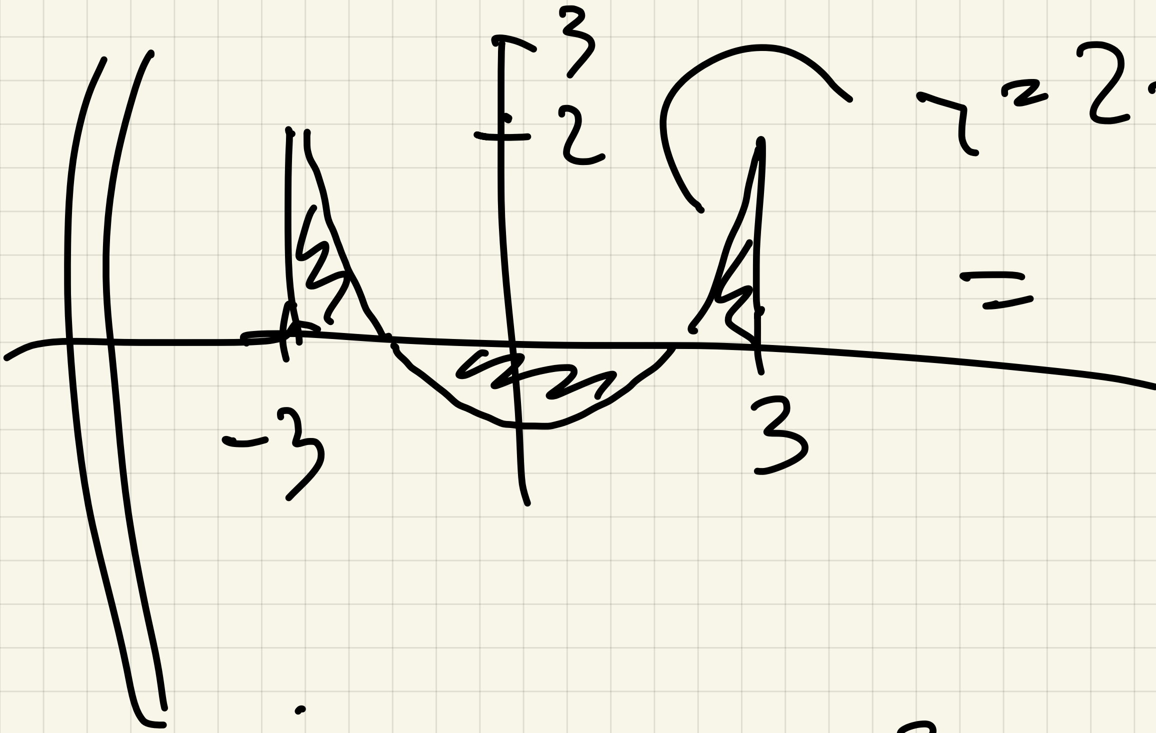
$$\int_a^b f = \int_a^c f + \int_c^b f$$

(6) $f(x) \geq g(x)$ on $[a,b]$,

then $$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$

<u>Ex 3</u> $$\int_{-3}^{3} 2 - \sqrt{9-x^2}\,dx$$

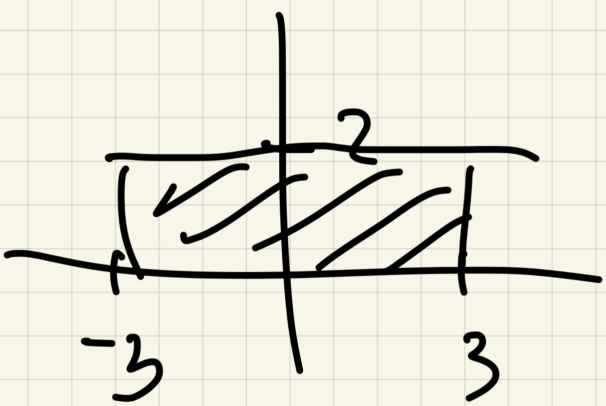
(4)

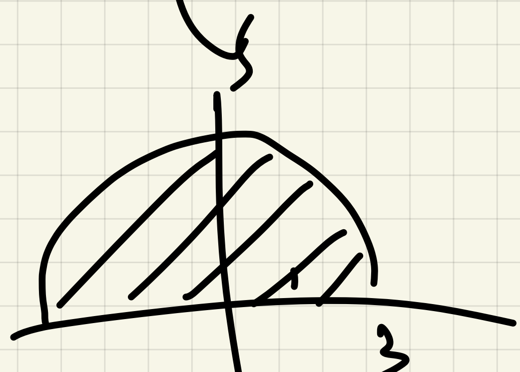


$y = 2 - \sqrt{9-x^2}$

= signed area ??

$$\int_{-3}^{3} 2\,dx - \int_{-3}^{3} \sqrt{9-x^2}\,dx$$





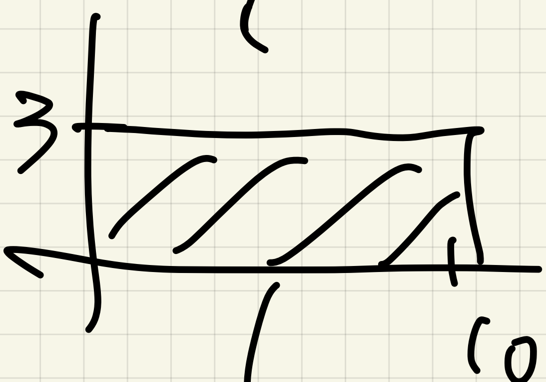
12

$\frac{1}{2} 9\pi$

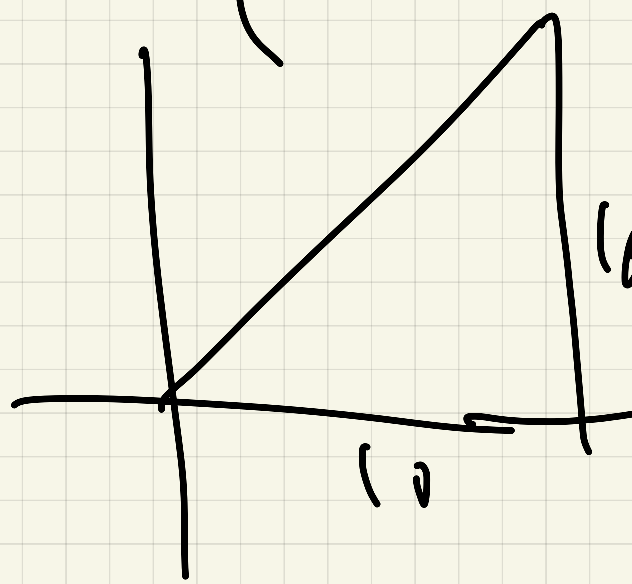
$12 - \frac{9\pi}{2}$

(b) $\int_0^{10} (3-x)\,dx$

$= \int_0^{10} 3\,dx - \int_0^{10} x\,dx$



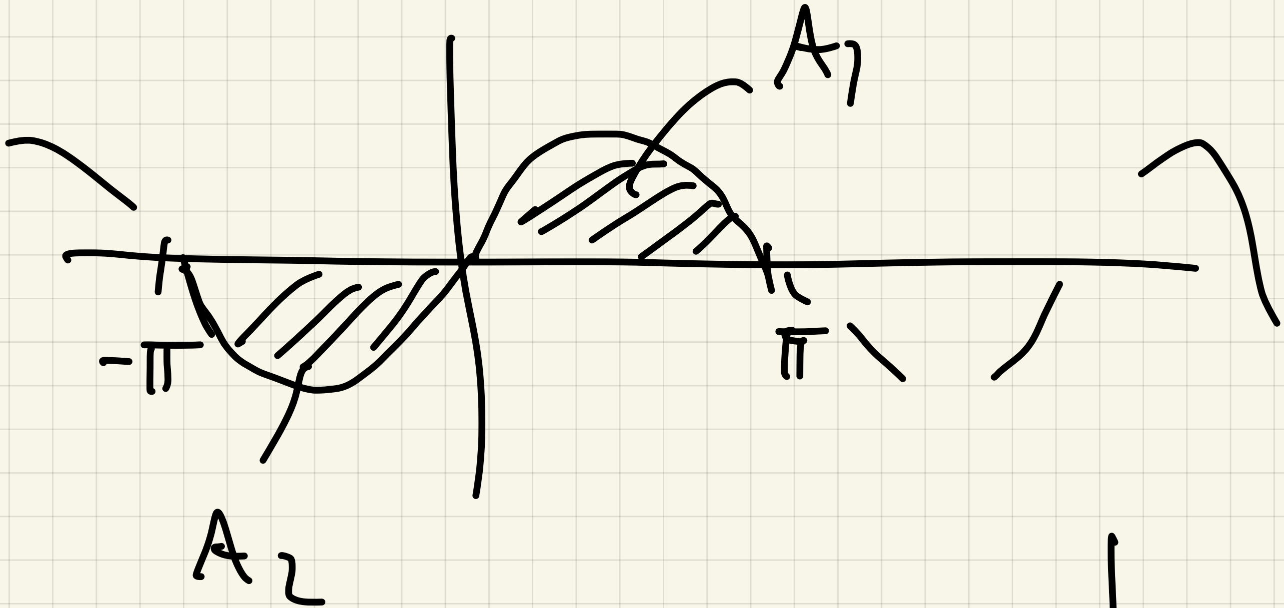
30



$\frac{1}{2} 10 \cdot 10 = 50$

$30 - 50 = -20$

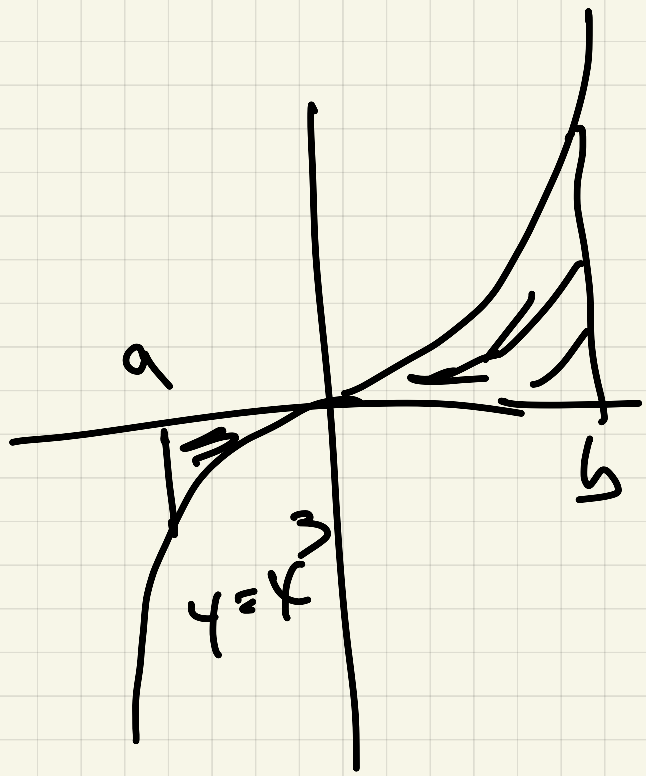
(c) $\int_{-\pi}^{\pi} \sin x\,dx = 0$



Ex 4 Compute

$$\int_a^b x^3\,dx$$

$a < b$



No formulas:

Long way;

Can use definition:

~~Δx = width~~

① break ~~into~~ $[a,b]$ into n intervals width $\Delta x = \frac{b-a}{n}$

② c_k = right end point

$$= a + k\left(\frac{b-a}{n}\right)$$

③ Estimating sum:

$$\sum_{k=1}^{n} \left(a + k\left(\frac{b-a}{n}\right)\right)^3 \cdot \left(\frac{b-a}{n}\right)$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \underbrace{\left(a + k\left(\frac{b-a}{n}\right)\right)^3}_{} \left(\frac{b-a}{n}\right)$$

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$\sum_{k=1}^{n} \left(a^3 + 3a^2 k\left(\frac{b-a}{n}\right) + 3a k^2\left(\frac{b-a}{n}\right)^2 + k^3\left(\frac{b-a}{n}\right)^3\right.$$

$$\left(\frac{h-a}{n}\right)$$