4/12 | Calc : Quiz 19 Exam 3 $\rightarrow$ Tuesday

1. $\int x^3 - \cos x \, dx$

$\frac{1}{4}x^4 - \sin x + C$

2. $\int \frac{4}{\sqrt{x}} - 5e^{2x} \, dx$

$= \int 4x^{-1/2} - 5e^{2x} \, dx$

$= 8x^{\frac{1}{2}} - \frac{5}{2}e^{2x} + C$

3. f : $f' = x^4 + 7e^x$

$f(0) = 4$

$f = \int x^4 + 7e^x \, dx$

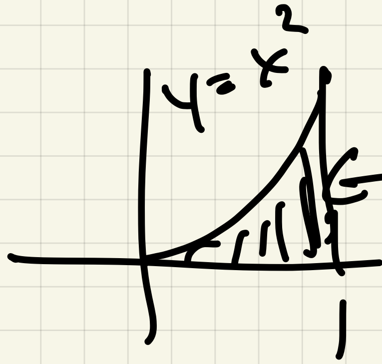
$= \frac{1}{5}x^5 + 7e^x + C$

$$4 = f(0) = 0 + 7 + C$$

$$C = -3$$

$$f = \frac{1}{5}x^5 + 7e^x - 3$$

Last time:



Area $= \frac{1}{3}$

Summation notation:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \dots + a_n$$

$$\sum (a_i + b_i) = \sum a_i + \sum b_i$$

$$\sum c a_i = c \sum a_i$$

Formulas: (a) $\displaystyle\sum_{i=1}^{n} c = nc$

(b) $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(c) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(d) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Ex1 $\sum_{i=1}^{15} (2i-1) =$

$\sum 2i - \sum 1$

$2\sum_{i=1}^{15} i - \sum_{i=1}^{15} 1$

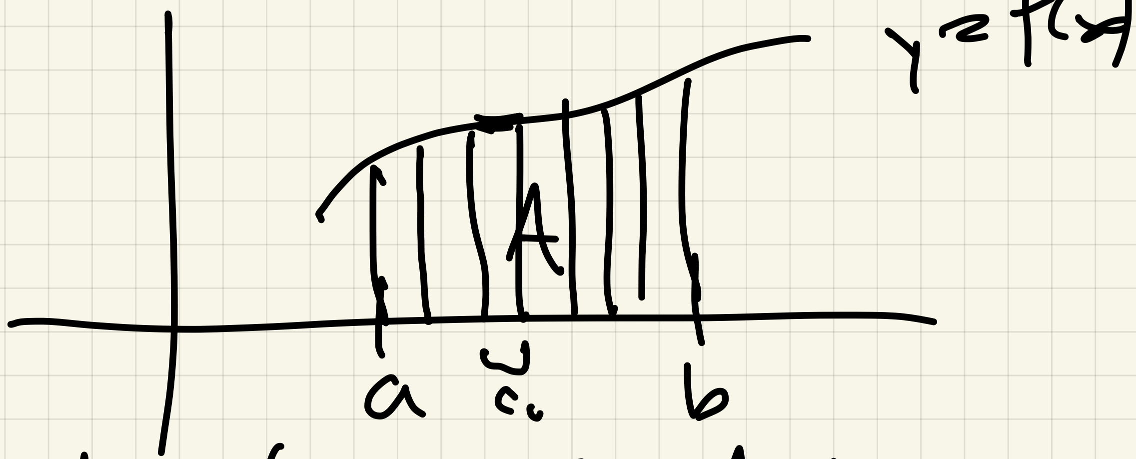
$\cancel{2} \cdot \frac{15(16)}{\cancel{2}} - 15$

$= 15(16) - 15(1)$

$= 15(15) = 15^2 = 225$

$1+3+5+7+9+$

Method to find/estimate areas



To find area A:

1. Subdivide $[a,b]$ into n equal subintervals with width $\Delta x = \dfrac{b-a}{n}$

2. Choose c_i in each subinterval

3. Form sum

$$\sum_{i=1}^{n} \underbrace{f(c_i)}_{\text{ht}} \underbrace{\Delta x}_{\text{width}} \approx \text{Area}$$

(Riemann Sum)

4. $A = \lim\limits_{n \to \infty} \sum_{i=1}^{n} f(c_i) \Delta x$

(Lots of choices ;

right / left end point

midpoint

max f / min f

<u>Ex3</u> $y = f(x) = 5 - x^2$ on

$[a,b] = [-2,2]$

$n = 4$, estimate area with

(a) left end point sum

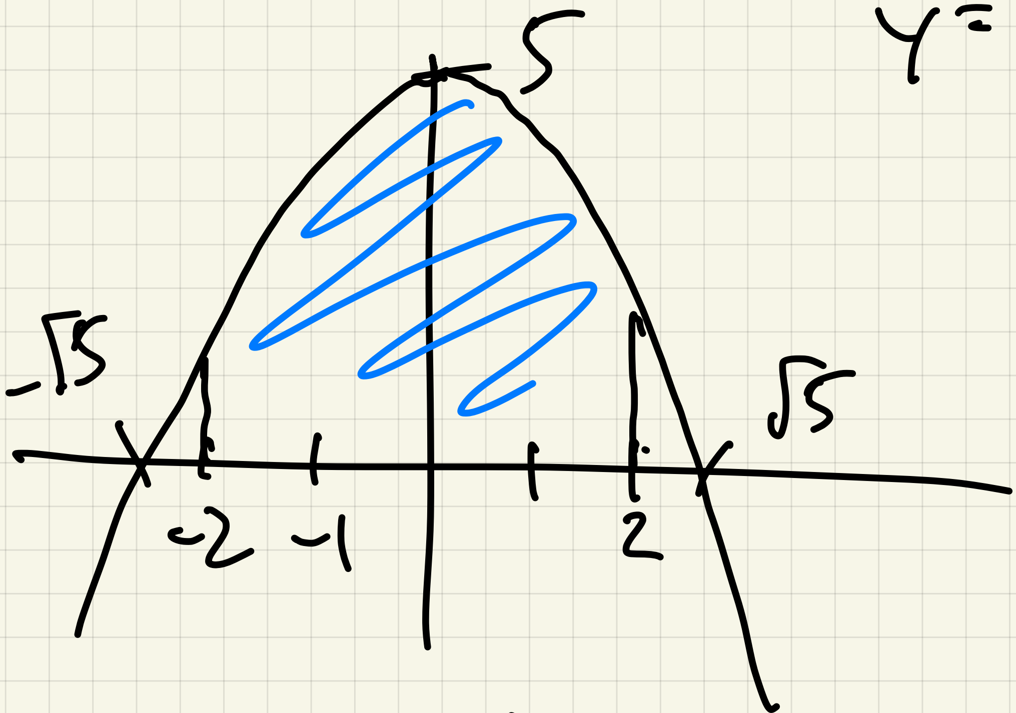
(b) right end point sum

(c) lower sum

(d) upper sum

(e) exact area

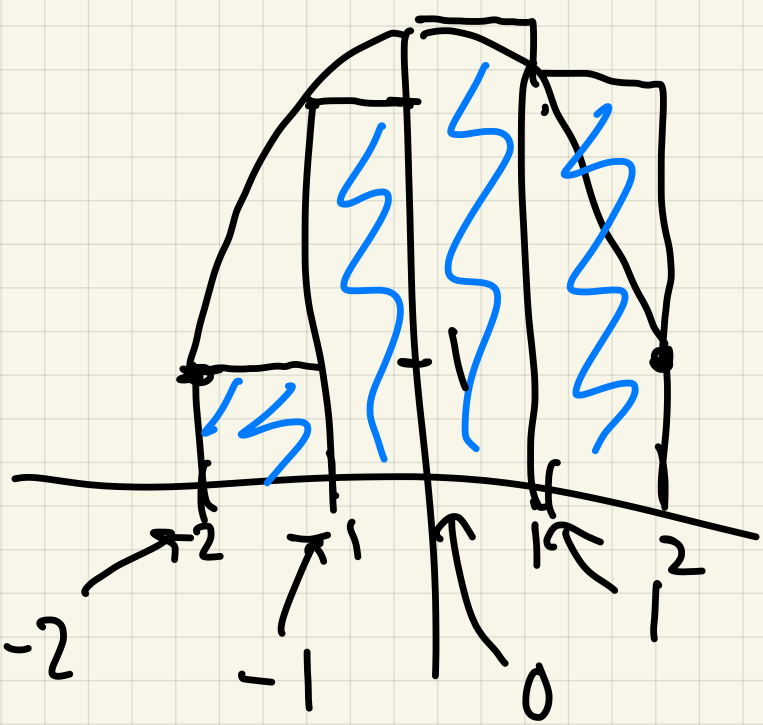
$y = 5 - x^2$



$$\Delta x = \frac{2-(-2)}{4} = \frac{4}{4} = 1$$

(a) left endpoints

$f = 5 - x^2$



$$\sum_{i=1}^{4} f(c_i)\,\Delta x =$$

$f(-2)(1) + f(-1)(1) + f(0)(1) + f(1)(1)$

$1 + 4 + 5 + 4 = 14$

(b) right end point

$c_1 = -1, \quad c_2 = 0, \quad c_3 = 1, \quad c_4 = 2$

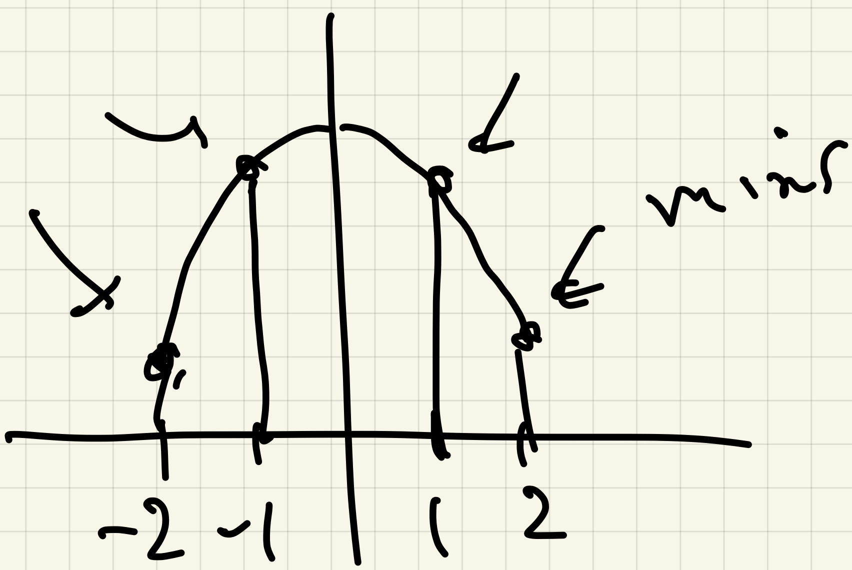
$\sum f(c_i)\Delta x = f(-1) + f(0) + f(1) + f(2)$

$= 4 + 5 + 4 + 1$

$= 14$

(c) lower sum

c_i so $f(c_i)$ is minimal



$c_1 = -2, \ c_2 = -1, \ c_3 = 1, \ c_4 = 2$

$$\sum f(c_i)\Delta x = f(-2) + f(-1) + f(1) + f(2)$$
$$= 1 + 4 + 4 + 1 = 10$$

(d) Upper sum

c_i $f(c_i)$ maximal

$c_1 = -1,\ c_2 = c_3 = 0,\ c_4 = 1$

$$\sum f(c_i)\Delta x = f(-1) + f(0) + f(0) + f(1)$$
$$= 4 + 5 + 5 + 4 = 18$$

(e) Exact area:

① Break $[-2, 2]$ into n subintervals, $\Delta x = \frac{4}{n}$

② Choose c_i to be the right endpoint

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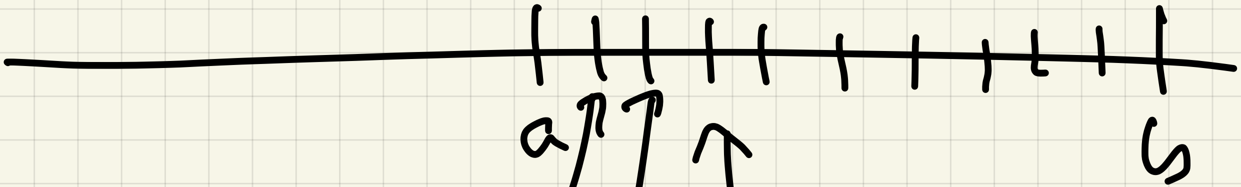

$a+\Delta x = c_1$

$a+2\Delta x = c_2$

$a+3\Delta x = c_3$

$$\boxed{c_k = a + k\Delta x}$$

In our case

$$c_i = -2 + i\left(\frac{4}{n}\right)$$

form sum: & $f(x) = 5 - x^2$

$$\sum_{i=1}^{n} f(c_i)\,\Delta x =$$

$\|$

$$\sum_{i=1}^{n} f\Big(\underbrace{-2+\frac{4i}{n}}_{c_i}\Big)\cdot\frac{4}{n}$$

$$\sum_{i=1}^{n}\Big(5-\Big(\underbrace{-2+\frac{4i}{n}}\Big)^2\Big)\frac{4}{n}$$

$$\sum_{i=1}^{n}\Big(5-\Big(4-\frac{16i}{n}+\frac{16i^2}{n^2}\Big)\Big)\frac{4}{n}$$

$$\sum_{i=1}^{n}\Big(1+\frac{16i}{n}-\frac{16i^2}{n^2}\Big)\frac{4}{n}$$

$$\sum_{i=1}^{n}\frac{4}{n}+\sum\frac{64i}{n^2}-\sum\frac{64i^2}{n^3}$$

$$\boxed{\sum_{i=1}^{n}\frac{4}{n}}+\frac{64}{n^2}\boxed{\sum_{i=1}^{n} i}-\frac{64}{n^3}\boxed{\sum i^2}$$

$$\parallel$$

$$4+\frac{64}{\cancel{n^2}}\,\frac{\cancel{n}(n+1)}{2}-\frac{64}{n^3}\,\frac{n(n+1)(2n+1)}{6}$$

$$= 4 + 32\left(\frac{n+1}{n}\right) - \frac{64}{6}\,\frac{(n+1)(2n+1)}{n^2}$$

$$= 4 + 32\left(\frac{n+1}{n}\right) - \frac{32}{3}\,\frac{2n^2+3n+1}{n^2}$$

$$\lim_{n\to\infty} 4 + 32\left(\frac{n+1}{n}\right) - \frac{32}{3}\left(\frac{2n^2+3n+1}{n^2}\right)$$

$$4 \qquad 1$$

$$4 + 32 - \frac{32}{3}(2)$$

$$= 4 + 32 - \frac{64}{3}$$

$$36 - \frac{64}{3} = \frac{108 - 64}{3} = \frac{44}{3}$$

$$\| \quad 14^{2/3}$$