2005 Calculus Bee

Winners:
1st place: Pathikrit Bhowmick
2nd Place: John Rhoads
3rd Place: Nathan Wenneker

1. Find \( \int (3x + 5)^{2005} \, dx \).
2. Find \( \frac{d}{d\theta} \left( \sin^{2005} (\theta^{2005}) \right) \).
3. Find \( \int_{0}^{2005} 2005^{2005} \, dx \).
4. The graph of \( y = g(x) \) is pictured below. Put the following four numbers in increasing order:
   2005, -2005, \( g(-2005) \), \( g'(2005) \)

5. Evaluate \( \int_{-2005}^{2005} \left( 1 + xe^{x^4} \cos x \right) \, dx \).
6. Find the area between the curves \( y = x + 2 \) and \( y = x^2 \).
7. For which value of \( x \) is the quantity \( \frac{1}{\sqrt{x^2 + 2005x + 2005^{2005}}} \) the greatest?
8. Evaluate \( \lim_{x \to 0^+} \frac{9}{x^2} + \frac{7}{x} - \frac{3}{x} \).
9. Suppose that you are given the following information about a function \( g \):
    \[ g(0) = 3; \quad g(1) = 5 \]
    \[ g'(0) = 11; \quad g'(1) = 31 \]
    \[ g''(0) = 23; \quad g''(1) = 19 \]
    \( g''(x) \) is continuous.

Find \( \int_{0}^{1} xg''(x) \, dx \).
10. For \( f(x) = 1 + 3x^2 + 9x^4 + 27x^6 + \ldots + 3^n x^{2n} + \ldots \), express \( f'(1/2) \) as a simplified fraction.