Theorems Paved Using Idea From Course Geomt

Definition: A compact Riemannian manifold is hyperbolic if it has constant sectional curvature $-1$.

$M$ hyperbolic $\iff M$ is isometric to $H^n$ for some $n$.

Theorem (Mostow): Let $M, N$ be compact hyperbolic manifolds of dimension $n \geq 3$. If $M \cong N$ are homotopy equivalent, then they are isometric. Moreover, if $f : M \to N$ is a homotopy equivalence, then it is homotopic to an isometry.

Remark: This result is very false for $n = 2$.

Definition: Let $X$ be a coarse space. The quasi-isometry group $Qis(X)$ is the group of closeness classes of coarse equivalences from $X$ to $X$. 
Recall: $X,Y$ metric spaces, $f: X \to Y$ a map, $x \in X$, $a > 0$. Define

$$D_f(x;a) = \frac{\sup \left\{ d(f(x), f(x')) : d(x,x') = a \right\}}{\inf \left\{ d(f(x), f(x')) : d(x,x') = \epsilon \right\}}.$$

Suppose $f$ a constant $K$ such that

$$\limsup_{a \to 0} D_f(x;a) \leq K \quad \forall x \in X.$$

Then we say $f$ is $K$-quasiconformal, and in general say $f$ is quasiconformal if it is $K$-quasiconformal for some $K$.

---

Key result in proving Mostrav Rigidity:

Theorem: for $n \geq 3$, every coarse equivalence $H^n \to H^n$ extends by continuity to a homeomorphism $S^{n-1} \to S^{n-1}$ from the ideal boundary of $H^n$ to itself. This homeomorphism is quasiconformal, and the process of extension to the boundary determines an isomorphism from $\text{Qis}(H^n)$ to the group of quasiconformal homeomorphisms from $S^{n-1}$ to itself.