Preliminary Topology Exam
August 2015

1. Show that a closed subset of a compact topological space is compact.

2. Suppose U, V are subsets of a space X such that the union of the interior of U and the interior of V is X. Now let A be an arbitrary subset of X, let $C = A \cap U$ and $D = A \cap V$. Is A the union of the interiors of C and D in A?

3. Let $\mathbb{R}$ be the real line with its usual ordering. We define the Alexandrov topology on $\mathbb{R}$ to be generated by the open sets $U_x = [x, \infty)$ for $x \in \mathbb{R}$.

   (a) show that this is really a basis for a topology

   (b) for $x \in \mathbb{R}$, determine the closure of the set $\{x\}$ in the Alexandrov topology.

4. Show that if X is regular, every pair of points of X have neighborhoods whose closures are disjoint.

5. Prove that $\mathbf{R}^2$ is simply connected, using basic definitions only.

6. Compute the fundamental group of a surface of genus two, and show that it is not abelian.

7. Let $\mathbb{C}P^n$ be the complex projective space of (complex) dimension n. Compute the Euler characteristic $\chi(\mathbb{C}P^n)$.

8. Let X and Y be CW complexes whose 1-skeleta are homeomorphic to the circle S^1 and such that $H_1(X) \cong \mathbb{Z}$ and $H_1(Y) \cong \mathbb{Z}_2$. Let Z be the space obtained by gluing X and Y together along their distinguished copies of S^1. Compute $H_1(Z)$.