Topology Preliminary Exam
August 19, 2011

1. Let Y be a subspace of a topological space X. Let A be a subset of Y. Let $\overline{A}$ be the closure of A in X, and let A° be the interior of A in X.

   (a) Does the closure of A in Y equal $\overline{A} \cap Y$?

   (b) Does the interior of A in Y equal $A^\circ \cap Y$?

2. A function $f : X \to Y$ is called *proper* if for any compact $K \subset Y$, $f^{-1}(K)$ is compact in X.

   (a) Give an example of a continuous function that is *not* proper.

   (b) Show that any proper function from a Hausdorff space X to a compact space Y is continuous.

3. Let (X, d) be a metric space, and let $A \subset X$. Show that A is closed in X if and only if there exists a continuous function $f : X \to \mathbb{R}$ such that A is the zero set of f.

4. Recall that a space X is *limit point compact* if every infinite subset of X has a limit point. Let $\mathbb{Z}^+$ denote the positive integers.

   Recall that the *finite complement topology* is that generated by sets that are complements of a union of a finite number points. The *countable complement topology* is defined similarly.

   (a) Is $\mathbb{Z}^+$ in the finite complement topology limit point compact?

   (b) Is $\mathbb{Z}^+$ in the countable complement topology limit point compact?

5. Let X be the two-sphere $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ with the points $(0, 0, 1)$ and $(0, 0, -1)$ identified. Compute the integral homology groups $H_*(X)$.

6. Define the suspension SX of a topological space X as the quotient of $X \times [0, 1]$ obtained by identifying $X \times \{0\}$ to a point and identifying $X \times \{1\}$ to a (different) point. Show that $\widetilde{H}_{k+1}(SX) \cong \widetilde{H}_k(X)$.

7. Let $X = \mathbb{R}P^2 \vee S^2$, and let x_0 be the join point.

   (a) Compute $\pi_1(X, x_0)$

   (b) Describe the universal cover of X and all possible covering transformations.

8. Let B^n be the closed unit ball in $\mathbb{R}^n$, and let $S^{n-1} \subset B^n$ be the unit sphere. Show there is no continuous map $\phi : B^n \to S^{n-1}$ that restricts to the identity on S^{n-1}.