SAMPLE QUESTIONS FOR PRELIMINARY COMPLEX ANALYSIS EXAM

VERSION 4.0

Contents

1. Complex numbers and functions

(1.1) Write all values of i^i in the form $a + bi$.

(1.2) Prove that $\sin z = z^2$ has infinitely many complex solutions.

(1.3) Find $\log\left(\sqrt{3} + i\right)$, using the principal branch.

(1.4) Prove or disprove that there exists a complex number z such that $|z| > \pi^2$ and such that $\frac{\cos(z)}{z} = \frac{1}{2\pi}$.

2. Definition of holomorphic function

(2.1) Find all $v : \mathbb{R}^2 \to \mathbb{R}^2$ such that for $z = x + iy$,

$$f(z) = (x^3 - 3xy^2) + iv(x, y)$$

is analytic.

(2.2) Prove that if $g : \mathbb{C} \to \mathbb{C}$ is a C^1 function, the following two definitions of "holomorphic" are the same:

(a) $\frac{\partial g}{\partial \bar{z}} = 0$

(b) the derivative transformation $g'(z_0) : \mathbb{R}^2 \to \mathbb{R}^2$ is $\mathbb{C}$-linear, for all $z_0 \in \mathbb{C}$. That is, $g'(z_0)m_w = m_w g'(z_0)$ for all $w \in \mathbb{C}$, where m_w is the linear transformation given by complex multiplication by w.

(2.3) True or false: If h is an entire function such that $\frac{\partial h}{\partial z} \neq 0$ everywhere, then h is injective.

(2.4) Find all possible $a, b \in \mathbb{R}$ such that $f(x, y) = x^2 + iaxy + by^2$, $x, y \in \mathbb{R}$ is holomorphic as a function of $z = x + iy$.

(2.5) Prove that the real and imaginary parts of a holomorphic function $f(x + iy)$ are harmonic functions of (x, y).

(2.6) Suppose that $v(x, y)$ is harmonic on $\mathbb{R}^2$. Find all functions $u(x, y)$ defined on $\mathbb{R}^2$ such that the complex-valued function f defined by $f(x - 2iy) = u(x, y) + iv(x, y)$ is holomorphic.

(2.7) Find the maximum value of $|z^4 + ze^z|$ on the closed unit disk.

(2.8) Prove that if $R(x, y)$ and $A(x, y)$ are real-valued functions of $(x, y) \in \mathbb{R}^2$, and if

$$f(x + iy) = R(x, y)\, e^{iA(x,y)},$$

is holomorphic, then

$$\frac{\partial R}{\partial x} = R \frac{\partial A}{\partial y}.$$

3. Complex Integrals and the Cauchy Integral Formula

(3.1) Let n be a positive integer and $0 < \theta < \pi$. Prove that

$$\frac{1}{2\pi i} \int_{|z|=2} \frac{z^n}{1 - 2z\cos\theta + z^2}\, dz = \frac{\sin n\theta}{\sin\theta},$$

where the circle $|z| = 2$ is oriented counterclockwise.

(3.2) Prove or disprove that
- (a) $\log|z|$ is absolutely integrable on Δ (the unit disk).
- (b) z^{-2} is absolutely integrable on Δ.

(3.3) Use the Cauchy Integral Formula to prove the Cauchy Integral Formula for Derivatives.

(3.4) Use the Cauchy Integral Formula to prove the Cauchy Inequalities.

(3.5) Use the Cauchy Integral Formula to prove Liouville's Theorem.

4. Sequences and series, Taylor series, and series of analytic functions

(4.1) Let f be an analytic function in the connected open subset G of the complex plane. Assume that for each point z in G, there is a positive integer n such that the n^{th} derivative of f vanishes at z. Prove that f is a polynomial.

(4.2) Find the radius of convergence of the Taylor series of $(3 - z^2)^{-1}$ at $z = 1 + i$.

(4.3) Prove that

$$\sum_{k \in \mathbb{Z}} \frac{1}{(z - k)^2}$$

is a well-defined meromorphic function on $\mathbb{C}$.

(4.4) Consider the series

$$F(x) = \sum_{n \in \mathbb{Z}} \frac{1}{(x + n)^4 + 1}.$$

- (a) Does the series converge at each $x \in \mathbb{R}$?
- (b) Does the series converge at each $x \in \mathbb{C}$?

(c) Does the series converge uniformly in x?
(d) Is the limiting function $F(x)$ continuous in x?

(4.5) Find the sixth degree Taylor polynomial of $e^{\sin(z)}$ at the origin.

(4.6) Find all $w \in \mathbb{C}$ such that

$$g(w) = \sum_{j=1}^{\infty} \frac{w + w^{-j}}{(j+1)^j}$$

converges. Is g analytic on this set of w?

(4.7) Let

$$S(z) = \sum_{k=1}^{\infty} \frac{z \sin(kz)}{k^2}.$$

Find the set of all z for which $S(z)$ converges. Is the convergence uniform on this set? Is S analytic on this set?

(4.8) In each case, find the largest open set of $z \in \mathbb{C}$ on which the sum converges, and determine if the sum converges to an analytic function.
(a) $\displaystyle\sum_{n=1}^{\infty} \frac{z^n}{\sqrt{n}(n+1)(z+2)}$
(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + z^4}$

(4.9) Consider the sum $\displaystyle\sum_{n=1}^{\infty} z^n(1-z)^2$.
(a) Does the sum converge uniformly for $|z| \le \frac{1}{2}$?
(b) Does the sum converge uniformly for $|z| < 1$?

(4.10) Find the radius of convergence of the following power series:
(a) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{z}{k+1}\right)^k$
(b) $\displaystyle\sum_{k=1}^{\infty} \left(1 + \frac{1}{2k}\right)^k z^k$.

(4.11) Let $S(z) = \sum_{m=1}^{\infty} \frac{m}{e^{mz}}$.
(a) Find the set of all z such that $S(z)$ converges absolutely.
(b) Does S converge uniformly on this set?
(c) Is S analytic on this set?
(d) Can S be analytically continued to a larger set?

(4.12) From basic principles, prove that if for all $z \in \mathbb{C}$, $f(1+z) = f(1-z)$ for some analytic function f , then the odd coefficients of the Taylor series of f centered at 1 are zero.

(4.13) Find a closed-form formula for

$$\sum_{k=0}^{\infty} a^k \sin(k\theta),$$

where $1 > a > 0$, $\theta \in \mathbb{R}$.

5. Identity Theorem

(5.1) Do there exist functions $f(z)$ and $g(z)$ that are analytic at $z = 0$ and that satisfy
 (a) $f(1/n) = f(-1/n) = 1/n^2$, $n = 1, 2, \ldots$,
 (b) $g(1/n) = g(-1/n) = 1/n^3$, $n = 1, 2, \ldots$?

(5.2) Let E be a connected open set in the complex plane and let $f(z)$ be holomorphic on E. Prove that $f(z)$ is constant if any one of the following conditions hold:
 (a) $f(z)$ is real-valued on E,
 (b) $\mathrm{Re}(f(z))$ is constant on E,
 (c) $|f(z)| = 1$ on E.

(5.3) Prove the identity theorem for holomorphic functions. That is, prove that if f and g are holomorphic functions both defined on a connected open set $U \subseteq \mathbb{C}$, and if $\{z \in U : f(z) = g(z)\}$ has a limit point in U, then $f(z) = g(z)$ for all $z \in U$.

(5.4) Do there exist functions f, g, h that are holomorphic on the open unit disk that satisfy:
 (a) $f\left(\frac{1}{2n}\right) = f\left(\frac{1}{2n+1}\right) = \frac{1}{2n}$ for $n \in \mathbb{Z}_{>0}$?
 (b) $g\left(\frac{1}{n}\right) = \frac{1}{1-\frac{2}{n}}$ for $n \in \mathbb{Z}$, $n \geq 3$?
 (c) $h\left(\frac{1}{n}\right) = \frac{1}{1-\frac{1}{n}}$ for $n \in \mathbb{Z}$, $n \geq 2$?

(5.5) Prove the following version of the identity theorem for holomorphic functions. Let f and g be holomorphic functions defined on a connected open set $U \subseteq \mathbb{C}$. If L is a line such that $L \cap U$ is nonempty, and if $f(z) = g(z)$ for $z \in L \cap U$, then $f(z) = g(z)$ for all $z \in U$.

(5.6) Let f be an analytic function in the connected open subset G of the complex plane. Assume that for each point z in G, there is a positive integer n such that the n^{th} derivative of f vanishes at z. Prove that f is a polynomial.

(5.7) Suppose that a holomorphic function F is defined on a neighborhood that contains the closed unit disk $\overline{D}$. Find and example of such a function F with the property that $F(z) = -i\overline{F(z)}$ for all z on $\partial\overline{D}$, or show that no such function exists.

6. Schwarz Lemma and Cauchy Inequalities

(6.1) Suppose a and b are positive.
 (a) Prove that the only entire functions f for which $|f(z)| \leq a|z|^{1/2} + b$ for all z are constant.
 (b) What can you prove if $|f(z)| \leq a|z|^{5/2} + b$ for all z?

(6.2) Let f be an analytic function in the open unit disc of the complex plane such that $|f(z)| \leq C/(1 - |z|)$ for all z in the disc, where C is a positive constant. Prove that $|f'(z)| \leq 4C/(1 - |z|)^2$.

(6.3) Let the function f be analytic in the entire complex plane, and suppose that $f(z)/z \to 0$ as $|z| \to \infty$. Prove that f is constant.

(6.4) Find all entire functions f in the plane satisfying $f(0) = 1$ and $|f(z)| \leq 2|z|^{3/2} - 1$ for $|z| \geq 2$.

(6.5) If $f(z)$ is an entire function that is not a polynomial, prove that, given given arbitrary $C > 0$, $R > 0$, and integer $m > 0$, there exists a z in $|z| > R$ such that $|f(z)| > C|z|^m$.

(6.6) Let $f(z)$ be an analytic function that maps the open disc $|z| < 1$ into itself. Show that $|f'(z)| \leq 1/(1 - |z|^2)$.

(6.7) Prove or disprove that there exists a holomorphic function on the punctured unit disk Δ^* such that

$$\lim_{z\to 0} zf(z) = 0, \ \lim_{z\to 0} |f(z)| = \infty.$$

(6.8) Let h be an analytic function on a neighborhood of the open unit disk such that $|h(z)| \leq 2$ for all z such that $|z| = 1$. Suppose that $\left|h\left(\frac{1}{3}\right)\right| = 0$. Find all possible values of $\left|h\left(\frac{1}{2}\right)\right|$, for any choice of h with the mentioned properties.

7. Liouville's Theorem

(7.1) Suppose that $f(z)$ and $g(z)$ are entire functions such that $|f(z)| \leq |g(z)|$ for all z. Show that $f(z) = cg(z)$ for some constant $c \in \mathbb{C}$.

(7.2) True or False:
- (a) If g is an entire function that is bounded on $\mathbb{R}$, then g is a constant function.
- (b) If h is an entire function such that $h(z) \to \infty$ as $z \to \infty$, then h is a polynomial.

(7.3) Prove the fundamental theorem of algebra using Liouville's Theorem.

(7.4) Let h be an entire function such that $\operatorname{Re}(h(z)) > -1$ for all $z \in \mathbb{C}$. Prove that h is a constant function.

8. Laurent series and singularities

(8.1) Find the Laurent expansions of $1/(z^2+4)^2$ at i and at $2i$.

(8.2) Find the first three non-zero terms of the Laurent expansion of $(e^z - 1)^{-1}$ at $z = 0$. Find the largest number R such that the Laurent series converges in $0 < |z| < R$.

(8.3) Compute the Laurent series of $\frac{z-1}{z(z-2)}$ (centered at the origin).

(8.4) Compute the Laurent series of $\frac{e^z}{z^2}$ (centered at the origin).

(8.5) Identify the type of singularity at $z = 0$:
- (a) $\frac{\sin(z)}{z}$
- (b) $z^2 e^{1/z}$

(8.6) Prove this part of the Riemann removable singularity theorem: Let D be a domain in $\mathbb{C}$, and let $z_0 \in D$. Suppose that g is a holomorphic function on $D \setminus \{z_0\}$ with an isolated singularity at z_0. Prove that if g is bounded on $D \setminus \{z_0\}$, then there exists a holomorphic function h on D such that $h(z) = g(z)$ for $z \in D \setminus \{z_0\}$.

(8.7) Let $L(w) = \sum_{m\in\mathbb{Z}} a_m w^m$ be a Laurent series. Let Δ^* denote the punctured open unit disk. Find necessary and sufficient conditions on the constants a_m such that L is analytic on Δ^*.

(8.8) Prove this part of the Riemann removable singularity theorem: Let D be a domain in $\mathbb{C}$, and let $z_0 \in D$. Suppose that g is a holomorphic function on $D \setminus \{z_0\}$ with an isolated singularity at z_0. Prove that if $\lim_{z\to z_0} (z - z_0)\, g(z) = 0$, then there exists a holomorphic function h on D such that $h(z) = g(z)$ for $z \in D \setminus \{z_0\}$.

9. Residue Theorem

(9.1) Let C be the positively oriented contour consisting of the vertical diameter and right half of the circle $|z - 1| = 3$. An entire function $f(z)$ is known to have no zeros on C

and satisfies the two conditions

$$\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}\, z\, dz = 3,$$
$$\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}\, z^2\, dz = \frac{5}{2}.$$

Determine the zeros of $f(z)$ inside C.

(9.2) Let D be a domain which contains in its interior the closed unit disk $|z| \le 1$. Let $f(z)$ be analytic in D except at a finite number of points $z_1, \dots, z_k$ on the unit circle $|z| = 1$ where $f(z)$ has first order poles with residues $s_1, \dots, s_k$. Let the Taylor series of $f(z)$ at the origin be $f(z) = \sum_{n=0}^{\infty} a_n z^n$. Prove that there exists a positive constant M such that $|a_n| \le M$.

(9.3) Compute

$$\int_{\partial\Delta} \frac{1+z^2+z^4}{\left(z-\frac{1}{2}\right)^6}\, dz,$$

where Δ is the open unit disk and $\partial\Delta$ is the unit circle, oriented counterclockwise.

(9.4) Evaluate

$$\int_{\partial\Delta} \frac{\cos\left(e^{-z}\right)}{z^2}\, dz,$$

where $\partial\Delta$ is the boundary of the unit disk, oriented counterclockwise.

(9.5) Find

$$\int_{\partial\Delta} \frac{dz}{(2z-1)\,(z+3)^2},$$

where $\partial\Delta$ is the boundary of the unit disk, oriented counterclockwise.

(9.6) Find

$$\int_{\alpha} \left(1+z^2\right) \tan(z)\; dz,$$

where α is the path parametrized as

$$\alpha(t) = \left(2\sin(t), 2\cos(t)\right), \;\; -\pi \le t \le 7\pi.$$

10. Contour Integrals

(10.1) Evaluate $\displaystyle\int_0^{\infty} \frac{\log 3x}{x^2+1}\, dx$.

(10.2) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x}{x^4+1}\, dx$.

(10.3) Show that $\int_\gamma e^{iz} e^{-z^2}\, dz$ has the same value on every straight line path γ parallel to the real axis.

(10.4) Compute

$$\int_0^{\infty} \frac{1}{1+x^4} dx.$$

(10.5) Compute

$$\int_{-\infty}^{\infty} \frac{\cos(ax)}{\left(1+x^2\right)^2} dx.$$

(10.6) For any $c > 0$, find

$$\int_0^\infty \frac{\sqrt{x}}{(x^2+c^2)^2}dx.$$

(10.7) Find

$$\int_0^\infty \frac{(\log(x))^2}{4x^2+1}dx.$$

11. Argument Principle

(11.1) (a) Show that there is a complex analytic function defined on the set $U = \{z \in \mathbb{C} \mid |z| > 4\}$ whose derivative is

$$\frac{z}{(z-1)(z-2)(z-3)}.$$

(b) Is there a complex analytic function on U whose derivative is

$$\frac{z^2}{(z-1)(z-2)(z-3)}\ ?$$

(11.2) Let $f(z)$ be analytic in the annulus $\Omega = \{1 < |z| < 2\}$. Assume that f has no zeros in Ω. Show that there exists an integer n and an analytic function g in Ω such that, for all $z \in \Omega$, $f(z) = z^n e^{g(z)}$.

(11.3) Rouché's Theorem and generalizations.

(a) Prove Rouché's Theorem. That is, let C be a simple closed contour in the complex plane, and let D be the interior of C. If f and g are two functions that are holomorphic on a neighborhood of the closure of D and if $|g(z)| < |f(z)|$ for $z \in C$, then f and $f+g$ have the same number of zeros in D, counted with multiplicities.

(b) Prove or disprove that if $<$ is replaced by $\leq$ in the hypothesis, then the conclusion is still valid.

(c) Prove or disprove the following variant of the theorem. Suppose that f and g are holomorphic on unit disk. Suppose that there exists $R \in (0,1)$ such that if $R < r < 1$, then $|g(z)| < |f(z)|$ for all z on the circle of radius r centered at 0. Then f and $f+g$ have the same number of zeros in the unit disk.

(11.4) The stronger symmetric version of Rouché's Theorem.

(a) Prove the stronger symmetric version of Rouché's Theorem. That is, let C be a simple closed contour in the complex plane, and let D be the interior of C. If f and g are two functions that are holomorphic on a neighborhood of the closure of D and if $|f(z)-g(z)| < |f(z)|+|g(z)|$ for $z \in C$, then f and g have the same number of zeros in D, counted with multiplicities.

(b) Prove that the standard Rouché's Theorem is a corollary of the symmetric version given above (see previous problem).

(c) Prove or disprove that if $<$ is replaced by $\leq$ in the hypothesis, then the conclusion is still valid.

(d) Prove or disprove that if $|f(z)-g(z)|$ is replaced by $|f(z)+g(z)|$ in the hypothesis, then the conclusion is still valid.

12. Rouché's Theorem

(12.1) How many zeros does the complex polynomial

$$3z^9 + 8z^6 + z^5 + 2z^3 + 1$$

have in the disk $1 < |z| < 2$?

(12.2) Let f be a holomorphic map of the unit disc $\mathbb{D} = \{z \mid |z| < 1\}$ into itself, which is not the identity map $f(z) = z$. Show that f can have, at most, one fixed point.

(12.3) Prove that there does not exist a polynomial of the form $p(z) = z^n + a_{n-1}z^{n-1} + ... + a_0$ such that $|p(z)| < 1$ for all z such that $|z| = 1$.

(12.4) Let f be a holomorphic function whose domain includes the closed unit disk. Suppose that f maps the boundary of the unit disk into the open disk of radius R centered at the origin. Show that f maps the closed unit disk into the open disk of radius R centered at the origin.

(12.5) Let A be a simply-connected open set in $\mathbb{C}$, and let α be a closed, Jordan, rectifiable curve in A with interior $I(\alpha)$. Suppose that f is a holomorphic function on A such that the restriction $f|_\alpha$ is one-to-one. Prove that f has at most one zero in $I(\alpha)$.

(12.6) Prove the fundamental theorem of algebra by using Rouché's Theorem.

13. Conformal maps

(13.1) Describe the region in the complex plane where the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(\frac{nz}{z-2}\right)$$

converges. Draw a sketch of the region.

(13.2) Let $\mathcal{D}$ be the domain exterior to the unit circle and with a slit along the negative real axis from $-\infty$ to -1. Find a biholomorphic mapping from $\mathcal{D}$ to the upper half-plane so that ∞ is a fixed point and $z = 4$ maps to i. Prove the mapping is unique.

(13.3) Find a conformal mapping of the exterior of the semi-circle $|z| = 1$ with $\operatorname{Im}(z) < 0$ onto the interior of the unit circle $|w| < 1$.

(13.4) Determine the largest disk about the origin whose image under the mapping $z \mapsto z^2 + z$ is one-to-one.

(13.5) Find a conformal map from the set $\left\{z = (x, y) : x > 0, x^2 + \left(y - \frac{1}{2}\right)^2 < \frac{1}{4}\right\}$ to the open unit disk.

(13.6) Suppose the f is an entire holomorphic function such that f maps the real axis to itself and the imaginary axis to itself. Prove that f is an odd function.

(13.7) Find the set of all possible orientation-preserving conformal maps from $\{z \in \mathbb{C} : 0 < \Im(z) < \pi\}$ to $\{z \in \mathbb{C} : \Im(z) > 0\}$, and prove that no other maps are possible.

(13.8) Prove or disprove the existence of a holomorphic map that maps
  (a) the set $P = \{z : \Im(z) > \Re(z)^2\}$ onto the entire complex plane $\mathbb{C}$.
  (b) $\mathbb{C}$ onto P.
  (c) $P \setminus \{i\}$ onto $\mathbb{C} \setminus \{0\}$.
  (d) $\mathbb{C} \setminus \{0\}$ onto $P \setminus \{i\}$.

(13.9) Let F be a holomorphic function that maps the unit disk D into $\overline{D}$. Suppose that $F(\frac{1}{2}) = 0$.
  (a) Give one example of such a map F that is invertible.

(b) Give the inverse of the map found in (a).
(c) Find the greatest possible value of $|F(-\frac{1}{2})|$ among all such maps.

(13.10) Let F be a holomorphic function whose domain contains a point z_0 where $F'(z_0) \neq 0$. Let $C_\varepsilon(z_0)$ denote the circle of radius ε centered at z_0, oriented counterclockwise.
(a) Prove that there exists $\varepsilon > 0$ such that

$$\int_{C_\varepsilon(z_0)} \frac{1}{F(z) - F(z_0)}\, dz = \frac{2\pi i}{F'(z_0)}.$$

(b) Prove or disprove that the equation above is true for any $\varepsilon > 0$.

(13.11) Let $\Delta_\delta(z_0)$ denote the disk of radius $\delta > 0$ centered at $z_0 \in \mathbb{C}$, and let $\Delta_\delta^*(z_0) = \Delta_\delta(z_0) \setminus \{z_0\}$. Suppose that $p(z)$ and $q(z)$ are polynomials such that at some $z_0 \neq 0$, $p(z_0) \neq 0$ and $q(z_0) = 0$.
(a) Prove that if z_0 is a zero of multiplicity one for q, then there exists $\delta > 0$ such that $\frac{p(z)}{q(z)}$ is a conformal map restricted to $\Delta_\delta^*(z_0)$.
(b) Prove that if z_0 is a zero of multiplicity $k > 1$ for q, then for all $\delta > 0$, the restriction of $\frac{p(z)}{q(z)}$ to $\Delta_\delta^*(z_0)$ is not a conformal map.

(13.12) Find the image of the set $\{z : 3 < |z| < 4\}$ under the map $f(z) = \log(z)$, where the branch $-\frac{\pi}{2} < \theta(z) \leq \frac{3\pi}{2}$ is chosen.

14. Analytic Continuation

(14.1) Let

$$f(z) = \int_{-1}^{1} \frac{e^{-x^2}}{x - z}\, dx.$$

(a) Show that $f(z)$ is analytic in $\mathbb{C} - [-1, 1]$.
(b) Show that $f(z)$ may be continued analytically across the open segment $(-1, 1)$.
(c) Show that the analytic continuations of f from above $(-1, 1)$ and from below $(-1, 1)$ are different. What is their difference on the cut $(-1, 1)$?

(14.2) Find the largest open set on which the each series converges to an analytic function. Prove or disprove that the functions can be analytically continued to be defined on a larger open set, and describe that set if possible.

(a) $a(z) = \sum_{n=10}^{\infty} n z^{2n}$.
(b) $b(z) = \sum_{n=10}^{\infty} (-1)^n z^{n^2}$.
(c) $c(z) = \sum_{n=10}^{\infty} \frac{(-1)^n}{n} z^n$.
(d) $d(z) = \sum_{n=10}^{\infty} z^{n!}$.
(e) $e(z) = \sum_{n=10}^{\infty} z^{7^{n-2}}$.
(f) $f(z) = \sum_{n=10}^{\infty} z^{g(n)}$, where $g(n) = 2n$ or $g(n) = 2n + 1$, depending on n.

(14.3) Let
$$F(z) = \int_{-\infty}^{\infty} \frac{e^{zx}}{1+e^x}\, dx.$$

(a) Determine the set of z for which the integral converges.
(b) Show that F can be analytically continued, and find the largest possible domain of its analytic continuation.

(14.4) Let
$$G(z) = \int_{0}^{\infty} e^{(1+2z)x}\, dx.$$

(a) Determine the set of z for which the integral converges.
(b) Show that F can be analytically continued, and find the largest possible domain of its analytic continuation.

15. Suggested Practice Exams

(A) Do the following problems:
(2.5),(4.10b),(6.4),(9.2),(10.2),(12.5),(13.5),(14.2cd)

(B) Do the following problems:
(4.5),(5.2),(6.6),(8.3),(9.1),(11.3),(13.7),(14.3)

(C) Do the following problems:
(4.7),(5.4),(7.3),(8.1),(10.4),(11.1),(12.6),(14.1)