2019 Calculus Bee
Monday, April 22, 2019

1st Place	Cong Minh Quang Truong
2nd Place	Bao Thach
3rd Place	Lauren Nagel

(1) Let a be a real number. Find and simplify

$$\frac{d}{dx}\left(e^{ax} + a - 2019\right).$$

(2) Evaluate

$$\int \cos^2(5x)\ dx.$$

(3) Find $\lim_{x\to\infty} \dfrac{e^x}{x^x}$.

(4) Suppose that $4x^2 + y^2 = 20$. Find all points (x, y) where the tangent line of this curve has slope 1.

(5) Find the area in the xy-plane below the curve $y = xe^{-x^2}$, above the x-axis, and to the right of $x = 1$.

(6) Let $f(x) = (x-1)(x-2)(x-3)\cdots(x-2018)(x-2019)$. What is $f'(2019)$?

(7) The line $y = 2x - 4$ is tangent to the curve $y = x^4 - 2x^3 + ax^2 + bx$ at $x = -1$ and at $x = 2$. Find the value of a.

(8) Find $\lim_{x\to 0^+} \dfrac{e^x + e^{-1/x} - 1}{\sin(2019x)}$.

(9) Find a positive value c such that the volumes generated by revolving the region bounded by $y = cx$, $y = 0$, and $x = 1$ about the x- and y- axes are equal.

(10) Compute

$$\sum_{k=1}^{\infty} \frac{k}{3^{k-1}}.$$