2015 Calculus Bee

Tuesday, April 21, 2015

1st Place	Dave Thompson
2nd Place	An Vu
3rd Place	Nick Vreeburg

1. Compute $f'(x)$ for
$$f(x) = \frac{x}{x+2015}.$$
Your answer should be a simplified fraction, with numerator and denominator factored.

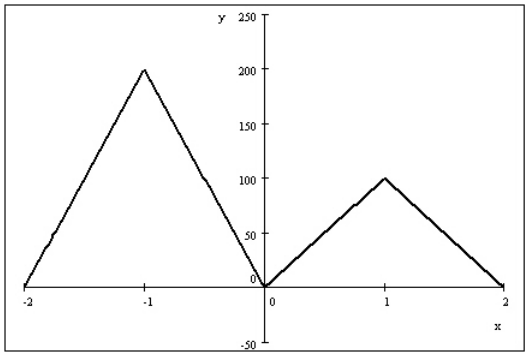
2. Evaluate
$$\int \frac{x^2 - 4x + 5}{x}\ dx.$$

3. Find the maximum value of $x^3 e^{-2x}$.

4. Evaluate
$$\int_0^{\pi/2} (\sin x + \cos x)^2\ dx.$$

5. Find all constants a such that the curve $y = ae^x$ is tangent to the line $y = x$.

6. If g is a function whose second derivative is continuous and such that $g(0) = 2$, $g(1) = 0$, $g'(0) = 1$, $g'(1) = 4$, simplify
$$\int_0^1 sg''(s)\ ds \quad .$$



7. Above is the graph of $y = \frac{dg}{dx}$.
   (a) Find the value of x in $[-2, 2]$ at which the maximum of $g(x)$ occurs.
   (b) Find $g(1) - g(-2)$.

8. Find
$$\frac{1}{2015} - \frac{1}{2015^2} + \frac{1}{2015^3} - \frac{1}{2015^4} + \frac{1}{2015^5} - \cdots.$$

9. Find a positive number H so that the area between $y = x^2$ and $y = H^2$ is 4.

10. Evaluate
$$\int \frac{1}{\sec 3x}\ dx.$$