HOMEWORK 7

(1) (a) What is \[ \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^2 \] ? What is \[ \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^3 \] ? What is the pattern?

(b) Answer the same question, but for the matrix \[ \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \].

(c) Answer the same question, but for the matrix \[ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \].

(2) Why are the statements that “any 2 sides of a triangle together are longer than the 3rd side” and “|x+y| ≤ |x|+|y|” both called the triangle inequality, even though they look quite different?

(3) Farey addition of fractions involves adding their numerators and denominators. For example, the ‘Farey sum’ of \( \frac{3}{5} \) and \( \frac{2}{3} \) is \( \frac{5}{8} \). What can you say about the behaviour of this kind of addition, especially in relation to ‘ordinary’ addition of fractions?

(4) We discussed in class the definition \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) of the derivative of a function \( y = f(x) \).

(a) Is \( \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \) also a correct definition of the derivative? Why or why not?

(b) Is \( \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{h} \) also a correct definition of the derivative? Why or why not?

(5) Matrix multiplication tends to be very difficult for students to understand. Catie’s class offered one approach to helping them over this hurdle.

(a) Compare and contrast Catie’s approach to the one in the text (see Chapter 5).

(b) Matrices are meant to model real-life data. What real-life situation(s) is (are) modelled by matrix multiplication? (You may find the examples in Chapter 5 useful, but make sure you create a new one on your own!)

(c) Alissa mentioned in class the value of having students discover concepts and patterns for themselves. How would you guide students to a discovery of matrix multiplication?