(1) Define the term ‘relation’ in your own words. (There is a technical, mathematical definition of the term, but you need not give that. Your example can involve examples, or analogies, or anything that would help you explain it to a high-school student.)

(2) (a) Define the term ‘function’ in your own words. (It may help to pay attention to the difference between the terms ‘relation’ and ‘function’.)
(b) Discover the history of the term ‘function’. It has meant different things to mathematicians over the years. Describe at least one historical definition of the term ‘function’. (The names Dirichlet, Fourier, and Weierstrass, among other, are often involved.)

(3) We saw in class that, if \( x = 0.5 \), then the solutions to \( x^2 + y^2 = 1 \) are \( y \approx \pm 0.866 \). Is it possible to find a solution \((x, y)\) to \( x^2 + y^2 = 1 \) with both \( x \) and \( y \) terminating decimals?

(4) In class, we saw that one can piece together many copies of a small equilateral triangle to form larger equilateral triangles. One could call the number of copies of the small equilateral triangle inside the bigger one a ‘triangular number’.
(a) The numbers defined this way are not what we usually call triangular. What are they?
(b) We have seen triangular numbers (those of the form \( n(n+1)/2 \)) and square numbers (those of the form, well, \( n^2 \)). Can we define other ‘shaped’ numbers? Do they have formulas?

(5) \((\star)\) Prove that the maximum number of bounded or unbounded regions formed by \( n \) lines in the plane is \( n^2 + n + 2 \). \((\text{Hint}: \use induction.\))

(6) In class, we defined the ‘projective plane’ to be a disc (in what we usually think of as the plane). ‘Lines’ are then either diameters of the disc, or arcs (of a different circle) passing through antipodal points on the circumference of the disc. \textbf{NOTE:} The entire circumference of the disc itself counts as a line!

We insist that lines (and regions) wrap around the circle, so that heading towards the far east edge of the circle makes one come out the far west edge.
(a) Argue why it is reasonable to call these objects ‘lines’, even though they are not lines in the familiar sense.
(b) Make a table of the maximum number of projective regions formed by \( n \) lines in the projective plane.
(c) \((\star\star)\) Explain the similarity between your table and the table formed in class of the maximum number of bounded or unbounded regions in the
'ordinary’ plane. (When we want to distinguish it from the projective plane, we call the ordinary plane affine.)

(7) Find an application of the ideas of absolute value and distance in undergraduate-level mathematics. (Natural places to look are calculus or linear algebra; or, if you have encountered them before, the theory of metric spaces.) Does the material on absolute values and distances in the text prepare students for this application? If so, how? If not, how could it be adjusted to do so?