LECTURE TOPICS FOR TEACHING OF MATHEMATICS
MATH 40970-080/60003-080
SPRING 2010

HIGH-SCHOOL LEVEL TALKS

(1) Why is the product of 2 negative numbers positive? Tests for divisibility. (Chapter 1.)
(2) Solving absolute-value equations and inequalities. Square roots. Midpoint and distance in the coordinate plane. (Chapter 2.)
(3) Function notation. Transforming functions. (Chapter 2.)
(4) Factoring quadratic expressions. Completing the square. The quadratic formula. Factoring polynomials. (Chapter 3.)
(5) Complex numbers. Complex numbers and quadratic equations. (Chapter 4.)
(6) Systems of linear equations and matrices.
(7) Multiplying matrices. (Chapter 5.)
(8) Matrices as transformations. (Chapter 5.)
(9) Similarity of triangles. Predicting similarity relationships. (Chapter 6.)
(10) Trigonometric ratios. Trigonometry and circles. (Chapter 7.)
(11) Polar coordinates. (Chapter 7.)
(12) Conic sections. Hyperbolas. (Chapter 8.)

UNDERGRADUATE-LEVEL TALKS

(1) One approach to computing the greatest common divisor of two numbers is the one that's often taught in schools: Factor each number as a product of primes, and then just multiply together the primes that occur in both numbers. This works well for small numbers, but gets slow and difficult very quickly for larger numbers. However, a work-around has been known for thousands of years: The Euclidean algorithm. Describe and illustrate this algorithm. (Chapter 1: “Algorithma”.)
(2) We’ve all factored integers and polynomials in our time, but, for most of us, we’ve probably thought of these two kinds of factorisation as essentially different operations. In fact, they are closely related, and there is also a Euclidean algorithm to allow one to find the greatest common divisor of two polynomials. Describe and illustrate this algorithm. (Chapter 3: “Factoring polynomials”, “The Euclidean algorithm for polynomials”.)
(3) What is a complex number, really? Many answers tell us what it looks like (a number of the form \(a + bi\), with \(a\) and \(b\) real), but not what it is—that mysterious quantity \(i\)? There are many answers to this problem; two common ones involve the coordinate plane, and matrices. Describe these, or other, realisations. (Chapter 4: “The birth of the complex numbers”; Chapter 5: “Transformations”; Chapter 7: “Addition, multiplication, and absolute value”.)
(4) A $2 \times 2$ matrix gives rise to a transformation of points in the plane, and a $3 \times 3$ matrix gives rise to a transformation of points in space. (This goes on as the size increases, but it gets harder to see the objects that are being transformed.) It’s easy to write down the formula for this transformation, but that’s not the same as understanding it. What distortions are, or are not, possible? How can you tell, just by looking at the matrix, what the transformation will be? (Chapter 5: “Scalings, reflections, rotations, shears, projections”.)

(5) As discussed above, a $2 \times 2$ matrix gives rise to a transformation of points in the plane. Unlike the most familiar transformations, which take numbers to numbers, this one takes points in the plane to points in the plane; and yet we can boil down a lot of information about this complicated set-up to two numbers, called the determinant and trace. What are these numbers, and where do they come from? (Chapter 5: “Areas and linear maps”. This will be a difficult lecture.)

(6) If you’ve dealt with complex numbers, you probably think of them as having real and imaginary parts—2 coordinates. Another way to think of it is to say that they have magnitude and argument—also 2 coordinates, but very different from the original ones. This turns out to be related to polar coordinates. What does all this have to do with sums of squares? How can it help us to remember trigonometric identities? (Chapter 6: “Sums of squares”; Chapter 7: “Addition, multiplication, and absolute value”, “Argument and the square root of $-1$”. This will probably be at least 2 lectures.)

(7) Out of all the curves that one can draw in the plane, 4 classes, the circles, the ellipses, the parabolas, and the hyperbolas, get singled out and called ‘conic sections’. Why these curves and no others? How are they related to each other, and what makes them interesting objects to study? (Chapter 8: “Properties of conic sections”.)

(8) Most people are at least familiar with the terms ‘sine’ and ‘cosine’, but it would not be at all surprising for someone with a solid college (non-mathematics) education who did not know the terms ‘hyperbolic sine’ and ‘hyperbolic cosine’. Moreover, the definitions of these functions involve exponential functions, so they look very different from the ordinary trigonometric functions. Where did these functions come from, and how did they get their names? (Chapter 8: “The hyperbolic functions”.)