Racing Early in Tournaments

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How much of an edge should you have to race with another player for all your chips early in a poker tournament? Matt Matros suggests that a slight edge is enough in his "To Flip or Not to Flip" article in a recent issue of *Card Player Magazine*. His work is based on "The Theory of Doubling Up," an earlier article in *The Intelligent Gambler* by William Chen and Jerrod Ankenman. I am more reluctant to race early in a tournament than they advocate. In hindsight, that may be because I play a lot of single table sit-and-goes and small- to moderate-sized multi-table tournaments, but few larger ones. I began by altering one of their two assumptions to (what I considered) a more realistic assumption, resulting in the need for unbelievably high edges in some cases. Deciding that a new model was in order, I came up with a generalization of an old standby, the Independent Chip Model, removing its limitation that all players have equal ability. In this article I discuss these models on a theoretical basis and then see how they stack up to some short-handed sit-and-goes that I recently played. I would love to see similar data for several players and a range of tournaments.

Let n be the number of players in the tournament and $E(x)$ be the player's expected value, expressed as a multiple of the first place prize, when his or her fraction of the chips in play is x. Thus, we should have $E(0)=0$ and $E(1)=1$. I implicitly assume the effects of blinds and position on $E(x)$ are negligible, which is a reasonable approximation early in a tournament. Accepting a race to either double up or bust out has positive expected value when its probability of success exceeds $E(x)/E(2x)$.

The main assumption made by Chen and Ankenman is that the probability a player doubles up before busting out is constant, i.e. does not depend on x. If we further assume that the probability a player triples up before busting out is also constant and that $E(x)$ is a continuous function, we obtain Chen and Ankenman's model $E(x)=x^p$ for some positive constant p, where p depends on the player's ability in a particular type of tournament. (The better the player, the smaller the p.) They make a second assumption to calculate p, namely that the player's expected value $E(x)$ is approximately the player's probability of winning the tournament. We can then find p by setting $(1/n)^p$ equal to the pre-start probability of winning the tournament and solving for p. We abbreviate this model PF (Power for First).

This second assumption neglects the payout structure of the tournament. Why not just set $(1/n)^p$ equal to the player's actual expected value at the start of the tournament and solve for p? We abbreviate this model by PEV (Power for Expected Value). As you can see from the first set of tables below, this apparently more reasonable assumption makes one extremely reluctant to race.

Looking at the models theoretically, we see that we would get exactly the same model if we assumed that the player always goes all-in with one other player for any pot he or she enters and has probability of winning that does not depend on stack sizes. It seems dubious whether such a model is appropriate when blinds are small and stacks deep, a circumstance that will lead to many relatively small pots and an occasional large one. Is it time to abandon ship?

Let's take another approach. If a player is average in the sense that his or her expected gain or loss over some sequence of hands is always 0, then the probability the player wins a tournament is just x, the fraction of the total chips he or she holds. The Independent Chip Model assumes that every player in the tournament is average in this sense. It further assumes that the probability of finishing in any position may be modeled by drawing one chip at random for first place, another chip for second, subject to the condition that it can't belong to the player coming in first, and so forth, until all paying positions are chosen. This is equivalent to assuming that the conditional probability of coming in some specific place other than first, given the players who came in ahead of this place, is just the fraction of chips excluding those of higher finishers that are held by the player. For instance, the probability a player with fraction x comes in first, a player with fraction y comes in second, and a player with fraction z comes in third is

$$x \cdot \frac{y}{1-x} \cdot \frac{z}{1-x-y} = \frac{xyz}{(1-x)(1-x-y)}.$$

The obvious drawback to this method is that it does not allow for any differences in players' abilities.

To obtain another estimate for $E(x)$, we could assume our player with chip fraction x has the Chen-Ankenman probability of finishing first, assume all other players have equal probabilities of finishing first, and otherwise use the Independent Chip Model to find probabilities for finishing below first. We obtain a composite model I'll abbreviate by PFIC (Power for First, "Independent Chip" for rest), admittedly bad nomenclature. Though more complicated, it should improve the accuracy of the PF model.

Now let's generalize the Independent Chip Model to players of different abilities. If a player has c chips out of a total of T chips and plays pots where he or she gains one chip with probability $P \neq 1/2$ and loses a chip otherwise, then the probability the player wins all the chips before busting out can be shown to be

$$\frac{1-\left(\dfrac{1}{P}-1\right)^{c}}{1-\left(\dfrac{1}{P}-1\right)^{T}}.$$

For a particular tournament where T is fixed, we can rewrite this in the form

$$\frac{1-b^{x}}{1-b},$$

where x is our player's fraction of the chips, as always, and b is some constant depending on the player's ability. We can find b by setting

$$\frac{1-b^{1/n}}{1-b}$$

equal to the player's probability of winning at the start of the tournament and finding an approximate solution numerically. In the case of an average player, taking the limit as P goes to 1/2, or equivalently as b goes to 1, yields a probability of winning equal to x, which we would anticipate from the Independent Chip Model. We now use the method of the Independent Chip Model method to estimate the probabilities for other finishes as in the PFIC model. We'll abbreviate this estimate by GIC (for Generalized Independent Chip). We could instead work backwards from a player's initial expected value to estimate b. We call this variant GIC2.

Once again, the difference between the PFIC and GIC models is that the PFIC model assumes we play a small number of monster pots, while the GIC model assumes we play lots of small pots. Of course, in reality we play pots of all sizes.

Let's look at some probabilities needed to race under the models for the first hand of a variety of tournaments. An ability of 20% means a player places first 20% more than the average player or has an expected value 20% greater than the average player, depending on the model. The probabilities shaded yellow are those falling between 50% and 60%, which includes typical probabilities for the player with an edge in a race.

			10 Players		
		1st: 50%	2nd: 30%	3rd: 20%	
Ability	**PF**	**PEV**	**PFIC**	**GIC**	**GIC2**
-50%	40.6%	50.0%	42.9%	48.6%	48.2%
-20%	46.8%	57.6%	50.3%	52.2%	52.0%
0	50.0%	61.6%	54.2%	54.2%	54.2%
10%	51.5%	63.4%	56.0%	55.2%	55.3%
20%	52.8%	65.1%	57.6%	56.1%	56.3%
40%	55.3%	68.2%	60.7%	57.9%	58.4%
100%	61.6%	75.9%	68.2%	62.7%	64.4%

			50 Players		
	1st: 40%	2nd: 25%	3rd: 16%	4th: 11%	5th: 8%
Ability	**PF**	**PEV**	**PFIC**	**GIC**	**GIC2**
-50%	44.2%	52.0%	44.9%	50.0%	50.0%
-20%	48.1%	56.5%	49.1%	50.8%	50.8%
0	50.0%	58.8%	51.3%	51.3%	51.3%
10%	50.9%	59.8%	52.2%	51.5%	51.5%
20%	51.6%	60.7%	53.1%	51.7%	51.7%
40%	53.1%	62.4%	54.8%	52.1%	52.2%
100%	56.5%	66.5%	58.8%	53.3%	53.5%

			500 Players		
	1st: 25%	2nd: 15.4%	3rd: 10.5%	…	45th: 0.4%
Ability	**PF**	**PEV**	**PFIC**	**GIC**	**GIC2**
-50%	46.3%	54.0%	46.7%	50.3%	50.3%
-20%	48.8%	56.9%	49.4%	50.6%	50.6%
0	50.0%	58.4%	50.7%	50.7%	50.7%
10%	50.5%	59.0%	51.3%	50.8%	50.8%
20%	51.0%	59.6%	51.9%	50.9%	50.9%

40%	51.9%	60.6%	52.9%	51.0%	51.0%
100%	54.0%	63.1%	55.3%	51.5%	51.5%

			2005 WSOP Main Event 5619 Players		
	1st: 14.2%	**2nd: 8.0%**	**3rd: 4.7%**	**…**	**560 th: 0.024%**
Ability	**PF**	**PEV**	**PFIC**	**GIC**	**GIC2**
-50%	47.3%	55.3%	47.7%	50.3%	50.3%
-20%	49.1%	57.4%	49.7%	50.5%	50.5%
0	50.0%	58.5%	50.7%	50.7%	50.7%
10%	50.4%	58.9%	51.1%	50.7%	50.7%
20%	50.7%	59.3%	51.5%	50.8%	50.8%
40%	51.4%	60.1%	52.3%	50.9%	50.9%
100%	52.9%	61.8%	54.1%	51.3%	51.3%

My own feeling from looking at these tables, once my glassy eyes returned to normal, was that the PFIC model, where the probability of finishing first is estimated by the Chan and Ankenman model and the expected value is then computed as in the Independent Chip Model, may be closest to reality. It's just a guess, however.

When I started thinking about modeling expected value in poker tournaments, I also wanted to test the models on some real data. I chose short-handed Sit-and-Goes because I enjoy them and because I could generate data quickly. I played 81 tournaments for five players; they paid 70% for first place and 30% for second. I recorded my chip counts at the end of level II and my result in the tournament. (I recorded chip counts at the end of level III as well, but too many players were eliminated by then for the data to be useful.) The initial stack was 50 big blinds at level II.

The table below gives my actual expected value, as a multiple of the first place prize, and the expected value predicted by the five models. I also divided those tournaments where I still had chips into thirds by the number of chips I had at that point. For simplicity, I assumed all four opponents remained at the end of level II. (Had I kept track of the number of opponents remaining, the predictions for the PFIC, GIC, and GIC2 models would have been at most very marginally higher.)

	Actual	**PF**	**PEV**	**PFIC**	**GIC**	**GIC2**
End of Level II Chips	0.38	0.31	0.36	0.39	0.40	0.37
End of Level II Chips, Highest 1/3	0.59	0.52	0.58	0.64	0.67	0.63
End of Level II Chips, Middle 1/3	0.38	0.32	0.39	0.42	0.43	0.39
End of Level II Chips, Lowest 1/3	0.29	0.19	0.25	0.25	0.24	0.21

As one would expect, the PF model of Chen and Ankenman consistently underestimates my expected values. On the other hand, the PFIC model, which augments their model with estimates for the probability of finishing second, and my three models give reasonably similar predictions that are fairly consistent with the actual expected values. In spite of this agreement, these models may give distinctly different probabilities needed for a race to have positive expected value for a very good or a very bad player. In my mind, the jury on early races is still out, but the truth probably lies within the range of probabilities given by these four models.

My test of the models was expedient, but not the best type of tournament for such tests. The validity of these or any other models needs substantial testing by many people, each playing a large number of one of a variety of tournaments. I hope it happens and would be glad to work with any frequent players who would like to participate in such a study.