Corrigendum to “Intersection homology with field coefficients: \(K\)-Witt spaces and \(K\)-Witt bordism”

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The author’s paper [2] concerns \(K\)-Witt spaces and, in particular, a computation of the bordism theory of such spaces. However, there is an error in the computation of the coefficient groups in dimensions \(4k+2\) when char\((K) = 2\). In this corrigendum, we state, as far as possible, the correct results. Details can be found in [1].

If we consider \(K\)-Witt spaces and \(K\)-Witt bordism using \(K\)-orientations, then for char\((K) = 2\), this is unoriented bordism, which we denote \(\mathcal{N}_{i}^{K-Witt}\).

**Theorem 1.** For a field \(K\) with char\((K) = 2\) and for \(i \geq 0\),

\[
\mathcal{N}_{i}^{K-Witt} \cong \begin{cases} 
\mathbb{Z}_2, & i \equiv 0 \mod 2, \\
0, & i \equiv 1 \mod 2.
\end{cases}
\]

This result is also provided without detailed proof by Goresky in [3, page 498].

If we consider \(K\)-Witt spaces and \(K\)-Witt bordism using \(\mathbb{Z}\)-orientations, then we denote the bordism theory by \(\Omega_{i}^{K-Witt}\). In this case, there remains one ambiguity in the computation, but we can show the following:

**Theorem 2.** For a field \(K\) with char\((K) = 2\) and \(k \geq 0\),

1. \(\Omega_{0}^{K-Witt} \cong \mathbb{Z}\),
2. \(\Omega_{4k}^{K-Witt} \cong \mathbb{Z}_2\),
3. \(\Omega_{4k+1}^{K-Witt} = \Omega_{4k+3}^{K-Witt} = 0\),
4. Either
   (a) \(\Omega_{4k+2}^{K-Witt} = 0\) for all \(k\), or
   (b) there exists some \(N > 0\) such that \(\Omega_{4k+2}^{K-Witt} = 0\) for all \(k < N\) and \(\Omega_{4k+2}^{K-Witt} \cong \mathbb{Z}_2\) for all \(k \geq N\).

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1Since these are geometric bordism groups, they vanish in negative degree.

Independent of the existence or value of $N$ in condition [4] of the theorem, the computations from [2] Section 4.5 of $\Omega^*_K \cdot$ as a generalized homology theory on CW complexes continue to hold and to imply that for $\text{char}(K) = 2$, 

$$\Omega_n^K \cdot (X) \cong \bigoplus_{r+s=n} H_r(X; \Omega_s^K \cdot).$$

Similarly,

$$\mathcal{N}_n^K \cdot (X) \cong \bigoplus_{r+s=n} H_r(X; \mathcal{N}_s^K \cdot).$$

**Other minor errata.** In [2] it should not be part of the definition of a $K$-Witt space that the space be irreducible as a pseudomanifold. However, as every $K$-Witt space of dimension $> 0$ is bordant to an irreducible $K$-Witt space [4, page 1099], this error does not affect the bordism group computations of [2]. Not every 0-dimensional $K$-Witt space is bordant to an irreducible one, but the computation of $\Omega^*_0 \cdot$ reduces to the manifold theory and gives the result of [2] if one removes irreducibility from the definition.

The argument that $\Omega^*_{4k+2} = 0$ given in [2] does not hold when $k = 0$. However, in this dimension it is not difficult to prove the result directly; details are provided in [1].

**References**


