Poster Titles and Abstracts

Taylor Brysiewicz
Necklaces Count Polynomial Parametrized Osculants
We consider the problem of geometrically approximating a complex analytic curve in the plane by the image of a polynomial parametrization of bidegree (d_1, d_2). We count the number of such curves to be the number of primitive necklaces on d_1 white beads and d_2 black beads. We show that this number is odd when $d_1 = d_2$ is squarefree and use this to give a partial solution to a conjecture by Rababah. Our results naturally extend to a generalization regarding hypersurfaces in higher dimensions. There, the number of parametrized curves of multidegree $(d_1, \ldots, d_n)$ which optimally osculate a given hypersurface are counted by the number of primitive necklaces with d_i beads of color i.

James Collins
Singular Value Homotopy for Determining Critical Parameter Values
Often in applications, a polynomial system is parameterized in some way. This can arrive when finding solutions of partial differential equations, or when finding steady state solutions for dynamic systems. These parameter values can determine not just the quantitative nature of the solution, but also the qualitative nature. In particular, the value of the parameter can determine the number of solutions to a polynomial system. We have developed a homotopy continuation method to determine the critical parameter value at which the number of solutions to a polynomial system changes. This poster will describe this homotopy and give some examples of how it is used in applications.

Christopher Eur
Divisors on matroids and their volumes
The classical volume polynomial in algebraic geometry measures the degrees of ample (and nef) divisors on a smooth projective variety. We introduce an analogous volume polynomial for matroids, and give a complete combinatorial formula. For a realizable matroid, we thus obtain an explicit formula for the classical volume polynomial of the associated wonderful compactification. We then introduce a new invariant called the volume of a matroid as a particular specialization of its volume polynomial, and discuss its algebro-geometric and combinatorial properties in connection to graded linear series on blow-ups of projective spaces.

Aida Maraj
Algebraic Properties of Hierarchical Models
One can study hierarchical models via their corresponding toric ideals. We present a formula for the size of a model in terms of its Krull dimension. Additionally, we give a recursive formula for the Hilbert Series for the class of Independent Models, which gives more fine-grained data. We conclude the poster with a sketch of an application of these algebraic models to Statistics.

Nida Obatake

Hopf bifurcations and oscillations in two dual-site phosphorylation networks

Chemical Reaction Network theory is an area of mathematics that analyzes the behaviors of chemical processes. A major problem in this area is stability in these networks. This poster focuses on bifurcations in a particular network, the irreversible fully distributive dual-site phosphorylation network. Experimental results suggest that this network does not exhibit bifurcations, but as far as we know, there are no theoretical results to support this conjecture. In this work we examine the capacity for Hopf bifurcations, by analyzing the steady state locus of the ODE system obtained from the network. To reduce the number of variables, we compute a parameterization of the steady state locus. We use Maple to compute the corresponding Hurwitz matrix and its minors, so that we may apply a generalization of the Routh-Hurwitz criterion for Hopf bifurcations. Using SAGE, we examine the Newton polytope to understand the signs that the Hurwitz determinants take. We show for the first time that the relevant Hurwitz determinants change sign, and discuss the implications for bifurcations and oscillations in the network. We also show that the irreversible mixed-mechanism dual-site phosphorylation network admits oscillations. Joint work with Anne Shiu and Xiaoxian Tang.

Kaitlyn Phillipson

Gröbner Bases of Neural Ideals

The neural ideal was introduced as an algebraic object used to better understand the combinatorial structure of neural codes. Every neural ideal has a particular generating set, called the canonical form, that encodes a minimal description of the receptive field structure intrinsic to the neural code. On the other hand, for a given monomial order, any polynomial ideal is also generated by its unique (reduced) Gröbner basis with respect to that monomial order. How are these generating sets - canonical forms and Gröbner bases - related? We show that when the canonical form of the neural ideal is a Gröbner basis, it is the union of all reduced Gröbner bases for the ideal (i.e. the universal Gröbner basis). A natural question to pursue, then, is under what conditions will the canonical form be a Gröbner basis? We give some partial answers to this question. In addition, the Gröbner basis elements gives new information about the structure of the receptive field.

Fazle Rabby

Double Structures on Conics in $\mathbb{P}^3$

Let Y be a smooth connected curve in $\mathbb{P}^3$. A multiplicity structure on Y is some curve Z that as a topological space has the same set of points as Y, but has more functions defined on it than Y. Using Ferrand's construction, we will describe all double structures on conics in $\mathbb{P}^3$ and give their total ideals.

Margaret Regan

polyTop: Software for computing topology of smooth real surfaces

A common computational problem is to compute topological information about a real surface defined by a system of polynomial equations. Our software, polyTop, leverages numerical algebraic geometry computations from Bertini and Bertini_Real with topological computations in javaPlex to compute the Euler characteristic, genus, Betti numbers, and generators of the fundamental group of a smooth real surface. This poster will highlight several examples that demonstrate this new software.

Jose Rodriguez

Multiprojective witness sets and a trace test

In the field of numerical algebraic geometry, positive-dimensional solution sets of systems of polynomial equations are described by witness sets. When these systems have a multivariable group structure, it is natural to consider the solution sets as multiprojective varieties. In our work we develop algorithms to study these varieties. This is joint work with Jonathan Hauenstein.

Zvi Rosen

Geometry of the Sample Frequency Spectrum

The sample frequency spectrum (SFS), which describes the distribution of mutant alleles in a sample of DNA sequences, is a widely-used summary statistic in population genetics. The expected SFS has a strong dependence on the historical population demography and this property is exploited by popular statistical methods to infer complex demographic histories from DNA sequence data; however, these inference methods exhibit pathological behavior. We use tools from algebraic geometry and convex geometry to characterize the geometry of the expected SFS for piecewise-constant demographies and show how these results explain the pathological behavior. As a bonus, this characterization implies that the expected SFS of a sample of size n under an *arbitrary* population history can be recapitulated by a piecewise-constant demography with only κ_n epochs, where κ_n is between $n/2$ and $2n-1$.

Alex Ruys de Perez

A Canonical Form for Neural Codes

A neural code on n neurons is a set of binary strings of length n. Biologically speaking, these strings represent all possible firing combinations of an individual's n place cells for a physical location, a place cell being a neuron which will fire if and only if the individual is in a particular area within that location. I will explain what it means for two neural codes to be isomorphic to each other. This isomorphism gives rise to a canonical form, i.e. a choice of a unique representative for each isomorphism class. I will introduce one possible canonical form, as well as an algorithm to compute that canonical form for an arbitrary neural code.

Aleksandra Sobieska

Counterexamples for Cohen-Macaulayness of Lattice Ideals

Let L in $\mathbb{Z}^n$ be a lattice, I its corresponding lattice ideal, and J the toric ideal arising from the saturation of L. We produce infinitely many examples, in every codimension, of pairs I, J where one of these ideals is Cohen–Macaulay but the other is not.

Tanner Strunk

Likely Topology of Random Real Curves and Surfaces

In their paper, "How Many Zeros of a Random Polynomial Are Real?", Alan Edelman and Eric Kostlan investigate the expected number of real roots of a polynomial of degree d. They compute this value as square root of d when using a seemingly more natural distribution than simply independent standard normal distributions for each of the coefficients of the polynomial. Using tools from numerical algebraic geometry and some simple data analysis tools (e.g. R or MATLAB), we can ask similar questions about zero loci of random real hypersurfaces in projective space. Using bertini and persistent homology–specifically javaplex–we can plot and examine curves in the real unit 2-sphere in R^3 to make conjectures about properties of random curves in RP^2. We can similarly use bertini and bertini_real to make conjectures about properties of random real surfaces in RP^3.

Simon Telen

Truncated Normal Forms for Solving Polynomial Systems

Let $f_1, \ldots, f_s \in R = \mathbb{C}[x_1, \ldots, x_n]$ define a zero-dimensional ideal $I = \langle f_1, \ldots, f_s \rangle \subset R$. We consider the problem of computing numerical approximations of the points in the variety $\mathbb{V}(I)$ of I. It is well known that the $\mathbb{C}$-algebra R/I is finite dimensional as a $\mathbb{C}$-vector space and the eigenstructure of the matrices of multiplication in R/I can be used to compute the points defined by I. This poster is about computing the multiplication maps corresponding to the coordinate functions x_i in a numerically stable way. We introduce Truncated Normal Forms (TNFs) to present a general algebraic framework. In this framework it is possible for algorithms to adapt the choice of basis for R/I to stabilize the computations. The resulting bases do not correspond to Groebner or border bases and they do not necessarily consist of residue classes of monomials. We work out specific constructions for the cases where $n = s$ and the f_i are generic with respect to their Newton polytopes. Numerical results show that this approach is very successful for systems in low dimensions (small n) of challenging degrees. For instance, we compute good numerical approximations of all 28900 intersections of two generic plane curves of degree 170 using only double precision arithmetic.

Angelica Torres

Computational aspects of stability of steady states and algebraic parameterizations

Criteria such as Routh-Hurwitz and Linard-chipart are used to establish whether a steady state of a system of ordinary differential equations is asymptotically stable, by computing the Jacobian of the system and studying the sign of the real part of its eigenvalues. I am interested in determining the stability properties of the steady states of reaction networks (using mass-action kinetics), when the values of the reaction rate constants are unknown. To this end, I combine the Routh-Hurwitz criterion (see [1]) and the Linard-Chipart criterion, with the use of algebraic parameterizations of the steady states (see [3]). In the poster I will present some examples where this approach can be succesfully applied (taken from [2] and [3]) and some computational aspects that arise when the approach is used in big networks.

References:
[1] Linda J.S. Allen. Introduction to mathematical Biology, Pearson. First edition. 2006
[2] E, Feliu & C, Wiuf C. Enzyme sharing as cause of multi-stationarity in signaling systems. J. R. Soc. Interface published online 2 November 2011
[3] C. Conradi, E. Feliu, M. Mincheva & C. Wiuf. Identifying parameter regions for multistationarity https://arxiv.org/abs/1608.03993

Anyu Zhang

Partitioning Data Using Monomial Bases to Improve Network Inference in Systems Biology

Network inference in systems biology is plagued by too few input data and too many candidate models which fit the data. When the data are discrete, models can be written as a linear combination of *finitely* many monomials. The problem of selecting a model can be reduced to selecting an appropriate monomial basis.

Recently affine transformations were used to partition input data into equivalence classes with the same basis. We wrote a Python package to build the equivalence classes for small networks. We propose a "standard position" for data sets and developed a metric to measure how far a set is from being in standard position. We used this metric to define the representative of an equivalence class. The implication of this work is guidance for systems biologists in designing experiments to collect data that result in a unique model (set of predictions), thereby reducing ambiguity in modeling and improving predictions.