Pre-calculus Background for Calculus

This handout is intended to be a brief review of basic facts and formulas from pre-calculus mathematics courses. It is not intended to be comprehensive; rather, this should be thought of as providing an indication of the sort of material I will assume you have mastered and that you should be able to use without difficulty within more complex problems. Items with exclamation points (!) are warnings against common errors.

1. Arithmetic and exponentials

   (a) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
   (b) !In general, $\frac{a}{c} + \frac{a}{d} \neq \frac{a}{c+d}$
   (c) $\frac{a}{c} \cdot \frac{b}{d} = \frac{ab}{cd}$
   (d) $\frac{a/c}{b/d} = \frac{ad}{bc}$
   (e) $x^{-a} = \frac{1}{x^a}$
   (f) $x^{a+b} = x^a x^b$
   (g) $(x^a)^b = x^{ab}$
   (h) $(xy)^a = x^a y^a$
   (i) $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$
   (j) $x^{1/n} = \sqrt[n]{x}$
   (k) $x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$
   (l) $\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$
   (m) $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$
   (n) !In general, $\sqrt[n]{x+y} \neq \sqrt[n]{x} + \sqrt[n]{y}$
   (o) $x^0 = 1$ if $x \neq 0$

2. Some basic algebra

   (a) $a^2 - b^2 = (a+b)(a-b)$
   (b) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
   (c) $(a+b)^2 = a^2 + 2ab + b^2$
   (d) $(a-b)^2 = a^2 - 2ab + b^2$
   (e) if $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

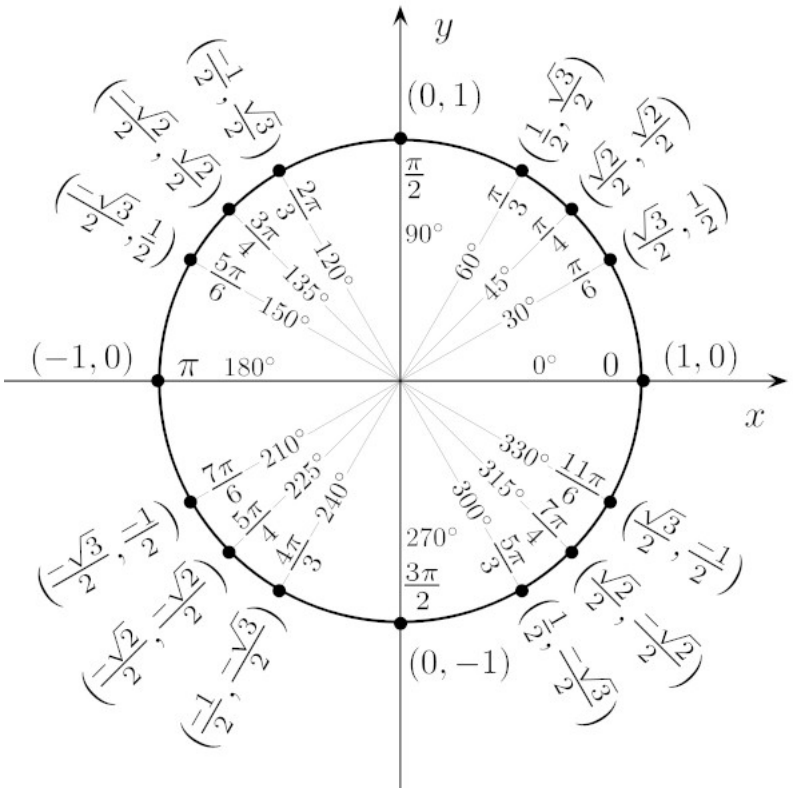
3. Geometry

   (a) Rectangle: Area = base $\cdot$ height
   (b) Triangle: Area = $\frac{1}{2} \cdot$ base $\cdot$ height
   (c) Circle: Area = π(radius)^2; Circumference = $2\pi \cdot$ radius
   (d) Distance between points (x_1, y_1) and (x_2, y_2): $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
   (e) Slope of line including points (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$
   (f) Point-slope form of line through (x_1, y_1) with slope m: $y - y_1 = m(x - x_1)$
   (g) Slope intercept form of line with slope m and y-intercept b: $y = mx + b$
   (h) Equation of circle with center (h, k) and radius r: $(x-h)^2 + (y-k)^2 = r^2$.

4. Trigonometry

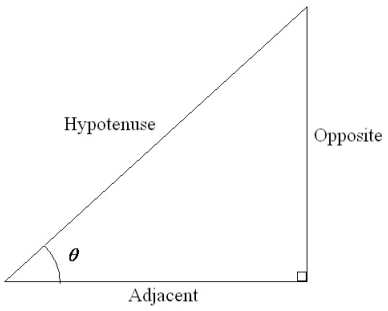
   (a) x degrees $= \frac{\pi x}{180}$ radians; x radians $= \frac{180x}{\pi}$ degrees

   (b) Standard angles. You should be able to compute the values of the trig functions at all these angles *without a calculator*



   (c) Definitions of trig functions

Given a right triangle:



i. $\sin\theta = \frac{O}{H}$

ii. $\cos\theta = \frac{A}{H}$

iii. $\tan\theta = \frac{O}{A}$

iv. $\csc\theta = \frac{H}{O}$

v. $\sec\theta = \frac{H}{A}$

vi. $\cot\theta = \frac{A}{O}$

(d) Basic identities

i. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
ii. $\cot\theta = \frac{\cos\theta}{\sin\theta}$
iii. $\csc\theta = \frac{1}{\sin\theta}$
iv. $\sec\theta = \frac{1}{\cos\theta}$

(e) Pythagorean identities

i. $\sin^2\theta + \cos^2\theta = 1$
ii. $\tan^2\theta + 1 = \sec^2\theta$
iii. $1 + \cot^2\theta = \csc^2\theta$

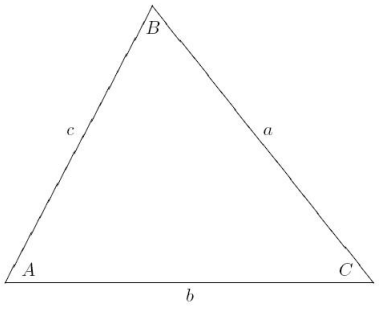
(f) Sign identities

i. $\sin(-\theta) = -\sin\theta$
ii. $\cos(-\theta) = \cos\theta$
iii. $\tan(-\theta) = -\tan\theta$
iv. $\csc(-\theta) = -\csc\theta$
v. $\sec(-\theta) = \sec\theta$
vi. $\cot(-\theta) = -\cot\theta$

(g) Angle addition

i. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
ii. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
iii. !In general, $\sin(\alpha+\beta) \neq \sin\alpha + \sin\beta$ and $\cos(\alpha+\beta) \neq \cos\alpha + \cos\beta$

(h) Laws of sines and cosines for an arbitrary triangle



i. Law of sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
ii. Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

5. Exponents and logarithms

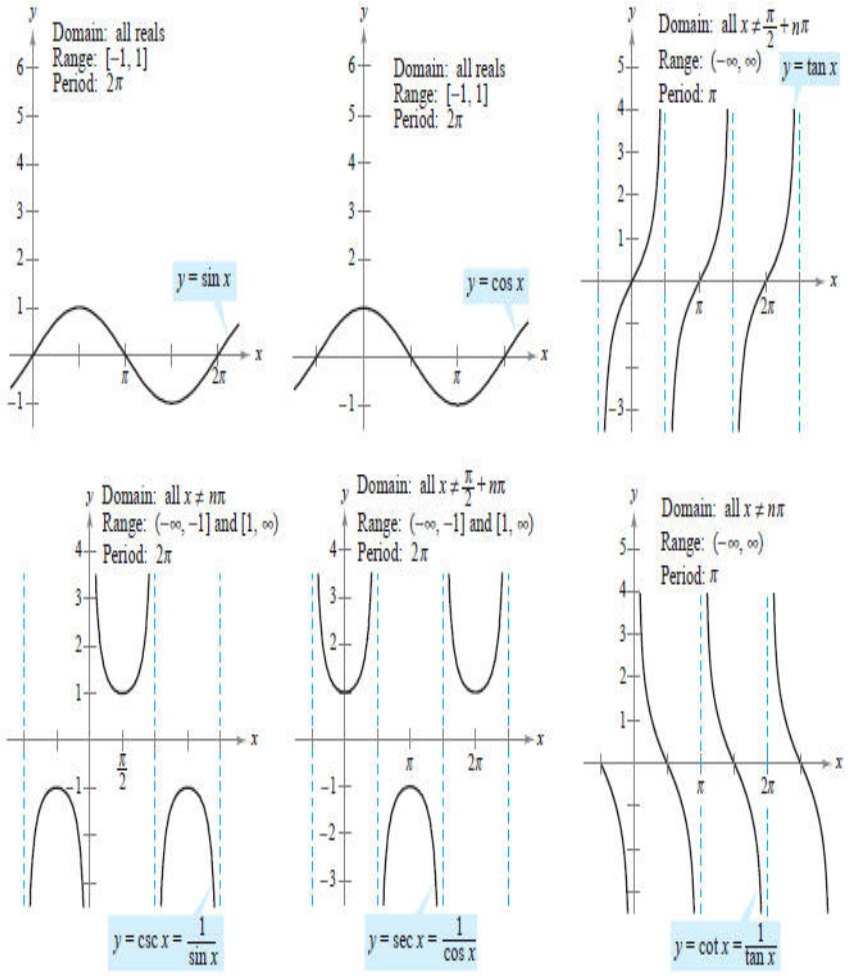
   (a) Definition of log: $y = \log_a x \leftrightarrow a^y = x$ $(x, a > 0,\ a \neq 1)$

   (b) Properties

   i. $a^{\log_a x} = x$
   ii. $\log_a(a^x) = x$
   iii. $\ln x = \log_e x$
   iv. $\log_a(xy) = \log_a(x) + \log_a(y)$
   v. $\log_a(x/y) = \log_a(x) - \log_a(y)$
   vi. $\log_a x^r = r \log_a x$
   vii. $\log_a b = \frac{\log_c b}{\log_c a} = \frac{\ln b}{\ln a}$
   viii. !In general, $\log_a(x + y) \neq \log_a x + \log_a y$
   ix. $\log_a 1 = 0$

6. Graphs

(a) Trig functions



(b) e^x and $\ln x$