# Greg Friedman
# Abstracts of papers

***Alexander polynomials of non-locally-flat knots***
**Indiana University Mathematics Journal 52 (2003), 1479-1578**

We generalize the classical study of Alexander polynomials of smooth or PL locally-flat knots to PL knots that are not necessarily locally-flat. We introduce three families of generalized Alexander polynomials and study their properties. For knots with point singularities, we obtain a classification of these polynomials that is complete except for one special low-dimensional case. This classification extends existing classifications for PL locally-flat knots. For knots with higher-dimensional singularities, we further extend the necessary conditions on the invariants. We also construct several varieties of singular knots to demonstrate realizability of certain families of polynomials as generalized Alexander polynomials. These constructions, of independent interest, generalize known knot constructions such as frame spinning and twist spinning.

***Intersection Alexander polynomials***
**Topology 43 (2004), 71-117**

By considering a (not necessarily locally-flat) PL knot as the singular locus of a PL stratified pseudomanifold, we can use intersection homology theory to define *intersection Alexander polynomials*, a generalization of the classical Alexander polynomial invariants for smooth or PL locally-flat knots. We show that the intersection Alexander polynomials satisfy certain duality and normalization conditions analogous to those of ordinary Alexander polynomials, and we explore the relationships between the intersection Alexander polynomials and certain generalizations of the classical Alexander polynomials that are defined for non-locally-flat knots. We also investigate the relations between the intersection Alexander polynomials of a knot and the intersection and classical Alexander polynomials of the link knots around the singular strata. To facilitate some of these investigations, we introduce spectral sequences for the computation of the intersection homology of certain stratified bundles.

***Stratified fibrations and the intersection homology of the regular neighborhoods of bottom strata***
**Topology and Its Applications 134 (2003), 69-109**

In this paper, we develop Leray-Serre-type spectral sequences to compute the intersection homology of the regular neighborhood and deleted regular neighborhood of the bottom stratum of a stratified PL-pseudomanifold. The $E^2$ terms of the spectral sequences are given by the homology of the bottom stratum with a local coefficient system whose stalks consist of the intersection homology modules of the link of this stratum (or the cone on this link). In the course of this program, we establish the properties of stratified fibrations over unfiltered base spaces and of their mapping cylinders. We also prove a folk theorem concerning the stratum-preserving homotopy invariance of intersection homology.

***All frame-spun knots are slice***
**Proceedings of the American Mathematical Society 132 (2004), 3103-3109**

Frame-spun knots are constructed by spinning a knot of lower dimension about a framed submanifold of $S^n$. We show that all frame-spun knots are slice (null-cobordant).

***Groups of locally-flat disk knots and non-locally-flat sphere knots***
**Journal of Knot Theory and Its Ramifications 14 (2005), 189-215**

The classical knot groups are the fundamental groups of the complements of smooth or piecewise-linear (PL) locally-flat knots. For PL knots that are not locally-flat, there is a pair of interesting groups to study: the fundamental group of the knot complement and that of the complement of the "boundary knot" that occurs around the singular set, the set of points at which the embedding is not locally-flat. If a knot has only point singularities, this is equivalent to studying the groups of a PL locally-flat disk knot and its boundary sphere knot; in this case, we obtain a complete classification of all such group pairs in dimension $\geq 6$. For more general knots, we also obtain complete classifications of these group pairs under certain restrictions on the singularities. Finally, we use spinning constructions to realize further examples of boundary knot groups.

***Intersection homology of regular and cylindrical neighborhoods***
**Topology and Its Applications 149 (2005), 97-148**

We develop a spectral sequence for the intersection homology of regular neighborhoods in PL stratified pseudomanifolds and of certain cylindrical neighborhoods of pure subsets in homotopically stratified spaces. We also develop a theory of simplicial homology with stratified systems of local coefficients and show that the $E^2$ terms of our spectral sequences are computable in terms of the homology of the "base" space with a suitably defined stratified system of local coefficients.

***Triangulations of 3-dimensional pseudomanifolds with an application to state-sum invariants*** **(with Markus Banagl, Universitat Heidelberg)**
**Algebraic and Geometric Topology 4 (2004), 521-542**

We demonstrate the triangulability of compact 3–dimensional topological pseudomanifolds and study the properties of such triangulations, including the Hauptvermutung and relations by Alexander star moves and Pachner bistellar moves. We also provide an application to state–sum invariants of 3–dimensional topological pseudomanifolds.

***Knot spinning***
**in the Handbook of Knot Theory, edited by William Menasco and Morwen Thistlethwaite, Elsevier Science (July 9, 2005)**

This exposition provides an introduction to higher-dimensional knot theory via concrete examples arising through various knot spinning constructions.

***Superperverse intersection cohomology: stratification (in)dependence***
**Mathematische Zeitschrift 252 (2006), 49 - 70**

Within its traditional range of perversity parameters, intersection cohomology is a topological invariant of pseudomanifolds. This is no longer true once one allows *superperversities*,

perversities with $\bar{p}(2) > 0$. In this case, intersection cohomology may depend on the choice of the stratification by which it is defined. Topological invariance also does not hold if one allows stratifications with codimension one strata. Nonetheless, both errant situations arise in important situations, the former in the Cappell-Shaneson superduality theorem and the latter in any discussion of pseudomanifold bordism. We show that while full invariance of intersection cohomology under restratification does not hold in this generality, it does hold up to restratifications that fix the the top stratum.

***Singular chain intersection homology for traditional and super-perversities***
**Transactions of the American Mathematical Society 359 (2007), 1977-2019**

We introduce a singular chain intersection homology theory which generalizes that of King and which agrees with the Deligne sheaf intersection homology of Goresky and MacPherson on any topological stratified pseudomanifold, compact or not, with constant or local coefficients, and with traditional perversities or superperversities (those satisfying $\bar{p}(2) > 0$). For the case $\bar{p}(2) = 1$, these latter perversities were introduced by Cappell and Shaneson and play a key role in their superduality theorem for embeddings. We further describe the sheafification of this singular chain complex and its adaptability to broader classes of stratified spaces.

***A multiperversity generalization of intersection homology*** **(with Gil Kalai, Hebrew University and Yale University)**
**Pure and Applied Mathematics Quarterly 3 (2007), Special Issue: In honor of Robert MacPherson, Part 3 of 3, 205-224**

We define a generalization of intersection homology, based on considering a set of perversities rather than a single perversity, and explore some of its properties. The question of whether these homology groups are independent of the stratification is left open, however some steps in this direction are made following known proofs of the topological invariance of the classical intersection homology groups.

***There exist non-trivial PL knots whose complements are homotopy circles***
**Fundamenta Mathematicae 193 (2007), 1-6**

We show that there exist non-trivial piecewise-linear (PL) knots with isolated singularities $S^{n-2} \subset S^n$, $n \geq 5$, whose complements have the homotopy type of a circle. This is in contrast to the case of smooth, PL locally-flat, and topological locally-flat knots, for which it is known that if the complement has the homotopy type of a circle, then the knot is trivial.

***Cobordism of disk knots***
**Israel Journal of Mathematics 163 (2008), 139-188**

We study cobordisms and cobordisms rel boundary of PL locally-flat disk knots $D^{n-2} \hookrightarrow D^n$. Any two disk knots are cobordant if the cobordisms are not required to fix the boundary sphere knots , and any two even-dimensional disk knots with isotopic boundary knots are cobordant rel boundary. However, the cobordism rel boundary theory of odd-dimensional disk knots is more subtle. Generalizing results of J. Levine on the cobordism of sphere knots, we define disk knot Seifert matrices and show that two higher-dimensional disk knots with

isotopic boundaries are cobordant rel boundary if and only if their disk knot Seifert matrices are algebraically cobordant. We also ask which algebraic cobordism classes can be realized given a fixed boundary knot and provide a complete classification when the boundary knot has no 2-torsion in its middle-dimensional Alexander module.

In the course of this classification, we establish a close connection between the Blanchfield pairing of a disk knot and the Farber-Levine torsion pairing of its boundary knot (in fact, for disk knots satisfying certain connectivity assumptions, the disk knot Blanchfield pairing will determine the boundary Farber-Levine pairing). In addition, we study the dependence of disk knot Seifert matrices on choices of Seifert surface, demonstrating that all such Seifert matrices are *rationally* S-equivalent, but not necessarily integrally S-equivalent.

***Intersection homology of stratified fibrations and neighborhoods***
**Advances in Mathematics 215 (2007), 24-65**

We derive spectral sequences for the intersection homology of stratified fibrations and approximate tubular neighborhoods in manifold stratified spaces. The neighborhoods include regular neighborhoods in PL stratified spaces.

***Intersection homology Künneth theorems***
**Mathematische Annalen 343 (2009), 371-395**

Cohen, Goresky and Ji showed that there is a Künneth theorem relating the intersection homology groups $I^{\bar{p}}H_*(X \times Y)$ to $I^{\bar{p}}H_*(X)$ and $I^{\bar{p}}H_*(Y)$, provided that the perversity $\bar{p}$ satisfies rather strict conditions. We consider biperversities and prove that there is a Künneth theorem relating $I^{\bar{p},\bar{q}}H_*(X \times Y)$ to $I^{\bar{p}}H_*(X)$ and $I^{\bar{q}}H_*(Y)$ for all choices of $\bar{p}$ and $\bar{q}$. Furthermore, we prove that the Künneth theorem still holds when the biperversity $p, q$ is "loosened" a little, and using this we recover the Künneth theorem of Cohen-Goresky-Ji.

***Intersection homology with field coefficients: K-Witt spaces and K-Witt bordism***
**Communications on Pure and Applied Mathematics 62 (2009), 1265-1292**
see also the Corrigendum, **Communications on Pure and Applied Mathematics 65 (2012), 16391640**

We construct geometric examples of pseudomanifolds that satisfy the Witt condition for intersection homology Poincaré duality with respect to certain fields but not others. We then compute the bordism theory of $K$-Witt spaces for an arbitrary field $K$, extending results of Siegel for $K = \mathbb{Q}$.

***Intersection homology and Poincare duality on homotopically stratified spaces***
**Geometry and Topology 13 (2009), 2163-2204**

We show that intersection homology extends Poincaré duality to manifold homotopically stratified spaces (satisfying mild restrictions). These spaces were introduced by Quinn to provide "a setting for the study of purely topological stratified phenomena, particularly group actions on manifolds." The main proof techniques involve blending the global algebraic machinery of sheaf theory with local homotopy computations. In particular, this includes showing that, on such spaces, the sheaf complex of singular intersection chains is quasi-isomorphic to the Deligne sheaf complex.

***On the chain-level intersection pairing for PL pseudomanifolds***
**Homology, Homotopy and Applications 11 (2009), 261-314**

James McClure recently showed that the domain for the intersection pairing of PL chains on a PL manifold $M$ is a subcomplex of $C_*(M)\otimes C_*(M)$ that is quasi-isomorphic to $C_*(M)\otimes C_*(M)$ and, more generally, that the intersection pairing endows $C_*(M)$ with the structure of a partially-defined commutative DGA. We generalize this theorem to intersection pairings of PL intersection chains on PL stratified pseudomanifolds and demonstrate the existence of a partial restricted commutative DGA structure. This structure is shown to generalize the iteration of the Goresky-MacPherson intersection product. As an application, we construct an explicit "roof" representation of the intersection homology pairing in the derived category of sheaves and verify that this sheaf theoretic pairing agrees with that arising from the geometric Goresky-MacPherson intersection pairing.

***An introduction to intersection homology with general perversity functions***
**Topology of Stratified Spaces, Greg Friedman, Eugnie Hunsicker, Anatoly Libgober, Laurentiu Maxim (editors) Mathematical Sciences Research Institute Publications 58, Cambridge University Press 2011, 177-222**

We provide an expository survey of the different notions of perversity in intersection homology and how different perversities require different definitions of intersection homology theory itself. We trace the key ideas from the introduction of intersection homology by Goresky and MacPherson through to the recent and ongoing work of the author and others.

***Intersection homology with general perversities***
**Geometria Dedicata 148 (2010), 103-135**

We study intersection homology with general perversities that assign integers to stratum components with none of the classical constraints of Goresky and MacPherson. We extend Goresky and MacPherson's axiomatic treatment of Deligne sheaves, and use these to obtain Poincaré and Lefschetz duality results for these general perversities. We also produce versions of both the sheaf-theoretic and the piecewise linear chain-theoretic intersection pairings that carry no restrictions on the input perversities.

***An elementary illustrated introduction to simplicial sets***
**Rocky Mountain Journal of Mathematics 42 (2012), 353-424**

This is an expository introduction to simplicial sets and simplicial homotopy theory with particular focus on relating the combinatorial aspects of the theory to their geometric/topological origins. It is intended to be accessible to students familiar with just the fundamentals of algebraic topology.

***Additivity of perverse signatures*** **(with Eugénie Hunsicker)**
**Journal fr die reine und angewandte Mathematik (Crelle's Journal) 676 (2013), 51-95**

A well-known property of the signature of closed oriented 4n-dimensional manifolds is Novikov additivity, which states that if a manifold is split into two manifolds with boundary along an oriented smooth hypersurface, then the signature of the original manifold equals

the sum of the signatures of the resulting manifolds with boundary. Wall showed that this property is not true of signatures on manifolds with boundary and that the difference from additivity could be described as a certain Maslov triple index. Perverse signatures are signatures defined for any oriented stratified pseudomanifold, using the intersection homology groups of Goresky and MacPherson. In the case of Witt spaces, the middle perverse signature is the same as the Witt signature. This paper proves a generalization to perverse signatures of Walls non-additivity theorem for signatures of manifolds with boundary. Under certain topological conditions on the dividing hypersurface, Novikov additivity for perverse signatures may be deduced as a corollary. In particular, Siegels version of Novikov additivity for Witt signatures is a special case of this corollary.

***Cup and cap product in intersection (co)homology*** **(with James McClure, Purdue University)**
**Advances in Mathematics 240 (2013), 383-426**

We construct cup and cap products in intersection (co)homology with field coef- ficients. The existence of the cap product allows us to give a new proof of Poincare duality in intersection (co)homology which is similar in spirit to the usual proof for ordinary (co)homology of manifolds.

***K-Witt bordism in characteristic 2***
**Archiv der Mathematik 100 (2013), 381-387**

This note provides a computation of the bordism groups of $K$-Witt spaces for fields $K$ with characteristic 2. We provide a complete computation for the unoriented bordism groups. For the oriented bordism groups, a nearly complete computation is provided as well a discussion of the difficulty of resolving a remaining ambiguity in dimensions equivalent to 2 mod 4. This corrects an error in the $char(K) = 2$ case of the authors prior computation of the bordism groups of $K$-Witt spaces for an arbitrary field $K$.

***The symmetric signature of a Witt space*** **(with James McClure, Purdue University)**
**Journal of Topology and Analysis 5 (2013), 121-159**

Witt spaces are pseudomanifolds for which the middle-perversity intersection homology with rational coefficients is self-dual. We give a new construction of the symmetric signature for Witt spaces which is similar in spirit to the construction given by Mischenko for manifolds. Our construction has all of the expected properties, including invariance under stratified homotopy equivalence.

***Unitary equivalence of normal matrices over topological spaces*** **(with Efton Park, TCU)**
**Journal of Topology and Analysis 8 (2016), 313-348**

Let $A$ and $B$ be normal matrices with coefficients that are continuous complexvalued functions on a topological space $X$ that has the homotopy type of a CW complex, and suppose these matrices have the same distinct eigenvalues at each point of $X$. We use obstruction theory to establish a necessary and sufficient condition for $A$ and $B$ to be unitarily

equivalent. We also determine bounds on the number of possible unitary equivalence classes in terms of cohomological invariants of $X$.

***Stratified and unstratified bordism of pseudomanifolds***
**Topology and Its Applications 194 (2015), 51-92**

We study bordism groups and bordism homology theories based on pseudomanifolds and stratified pseudomanifolds. The main seam of the paper demonstrates that when we use classes of spaces determined by local link properties, the stratified and unstratified bordism theories are identical; this includes the known examples of pseudomanifold bordism theories, such as bordism of Witt spaces and IP spaces. Along the way, we relate the stratified and unstratified points of view for describing various classes of pseudomanifolds.

***Generalizations of intersection homology with duality over the integers***
**submitted**

We provide a generalization of the Deligne sheaf construction of intersection homology theory and a corresponding generalization of Poincaré duality on pseudomanifolds such that the Goresky-MacPherson, Goresky-Siegel, and Cappell-Shaneson duality theorems all arise as special cases. Unlike classical intersection homology theory, our duality theorem holds with ground coefficients in an arbitrary PID and with no locally torsion free conditions on the underlying space.

***The chain-level intersection pairing for PL pseudomanifolds revisited***
**submitted**

We revisit the construction of the PL intersection pairing for PL chains on PL manifolds and for PL intersection chains on PL stratified pseudomanifolds. We provide a correction to the Goresky-MacPherson proof of a version of Poincare duality on pseudomanifolds that is necessary for the construction of the intersection pairing on pseudomanifolds, and we provide a construction of the intersection product for locally finite intersection chains on non-compact pseudomanifolds that reduces to the pre-existing (corrected) construction on compact pseudomanifolds. As an application of the techniques developed, we provide a direct proof that the Goresky-MacPherson homology intersection product is Poincare dual to the cup product pairing on compact oriented PL manifolds, but we leave as an open question, with discussion of the diffi- culties involved, whether an analogous proof can be formulated to demonstrate such a duality between intersection and cup product pairings for intersection homology of pseudomanifolds.