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The Fourth Annual Western Workshop in Geometric Topology was held at Oregon State University in Corvallis, Oregon on June 18—20, 1987. The participants were:

Robert Andersen	Oregon State University
Mladen Bestvina	University of California at Los Angeles
Philip Bowers	Florida State University
James Cannon	Brigham Young University
Robert Daverman	University of Tennessee
Helmut Doll	Oregon State University
Robert Edwards	University of California at Los Angeles
Dennis Garity	Oregon State University
Craig Guilbault	University of Tennessee
Jim Hoste	Oregon State University
Isa Jubran	Oregon State University
Jack Lamoreaux	Brigham Young University
Dale Rohm	Oregon State University
Richard Schori	Oregon State University
David Snyder	University of Tennessee
Frederick Tinsley	Colorado College
James Van Buskirk	University of Oregon
John Walsh	University of California at Riverside
David Wright	Brigham Young University

These proceedings contain the notes of two one-hour talks given by the principal speaker, Robert Edwards, and summaries of talks given by other participants. The success of the conference was due in large part to funding provided by the National Science Foundation (DMS-8702818) and the Mathematics Department at Oregon State University. I would like to thank the National Science Foundation and the Mathematics Department for their support.

Dennis Garity

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Some Remarks on the Hilbert — Smith Conjecture

R. D. Edwards

(As transcribed from notes taken by D. Garity)

Hilbert—Smith Conjecture, Version 1:
No p—adic group A_p can act effectively on a manifold M.

In what follows we denote by A_p, where p is a fixed prime, the inverse limit of the inverse system

$$\mathbb{Z}_p \leftarrow \mathbb{Z}_{p^2} \leftarrow \mathbb{Z}_{p^3} \leftarrow \cdots .$$

We say that A_p acts effectively on a topological manifold M if A_p is a topological subgroup of the group of self homeomorphisms of M, Homeo(M), where Homeo(M) is given the compact open topology.

Note that A_p is a compact topological group, and that the underlying space of A_p is a Cantor set. Also, A_p is topologically generated by a certain self homeomorphism h of the Cantor set. That is, there is an element (homeomorphism) h of A_p whose powers are dense in A_p. This homeomorphism is the infinite composition of homeomorphisms, $h = h_1 \circ h_2 \circ h_3 \circ \cdots$, where the i-th homeomorphism interchanges the first two intervals of the i-th stage of the construction of the Cantor set. Further, we note that if G is a closed subgroup of A_p, then either G is trivial, or G is the kernel of one of the natural epimorphisms $A_p \to \mathbb{Z}_{p^n}$, and hence G has finite index in A_p and is isomorphic to A_p.

Some Background on the Hilbert—Smith Conjecture:

Gleason, Montgomery and Zippin have shown that a locally compact topological group that is a manifold is a Lie group. This is often credited as a solution to Hilbert's fifth problem. Hilbert's fifth problem also asked which compact topological groups can act effectively on manifolds. This leads to a second version of the Hilbert—Smith Conjecture.

Hilbert Smith Conjecture, Version 2:
There does not exist a self homeomorphism h of a compact manifold M such that: - 1. Each orbit $\{ h^n(x) \mid n \in \mathbb{Z} \}$ has small diameter with respect to M, and 2. $\{ h^n \mid n \in \mathbb{Z} \} \subset$ Homeo(M) is a relatively compact subgroup. (i.e. This set is uniformly continuous in M and in $\mathbb{Z}$)

The two versions are equivalent. We will outline why Version 2 being false implies that version 1 is false. Let G = closure $\{ h^n \mid n \in \mathbb{Z} \} \subset$ Homeo(M). As input, we know that version 2 is true for compact Lie groups, using a theorem of Newman discussed below. The explanation is completed by applying the fact that a compact topological group G such that G contains no Lie subgroups contains a subgroup A_p for some p. This, like the Gleason - Montgomery - Zippin work, is based upon the fundamental Peter-Weyl theorem, which has as a corollary that any compact topological group is an inverse limit of Lie groups, and hence locally homeomorphic to Euclidean space crossed with some totally disconnected compactum.

The Theorems of Newman and Montgomery.

There is a remarkable pair of theorems of W.H.A. Newman, proved in the early 1930's, which affirmatively answer Version 2 of the conjecture in many cases.

Theorem (Newman). Suppose M is a compact manifold. Then there is an $\epsilon = \epsilon(M) > 0$ such that if G is any finite group of homeomorphisms of M for which each orbit $\{ g(x) \mid g \in G \}$ has diameter less than ϵ, then G must be the trivial group.

Closely related to this we have:

Theorem (Newman). Suppose G is a finite group of homeomorphisms of a manifold M. Then the set of points in M whose orbits are full (i.e. those points left fixed only by the identity in G) is open and dense in M. In particular, the fixed set of G cannot have interior.

I learned the proofs of these theorems from a marvelous article by A. Dress in Topology, volume 8, 1969. (Newman's original article is quite formidable.) For the first theorem, his argument can be summarized as follows. It is relatively easy to reduce to the case where the order of G is some prime p, so we will discuss only this case.

Let $h:M \xrightarrow{\cong} M$ be a generator of G; hence $h^p = id_M$. Let $q:M \to Q$ be the quotient map of M onto the orbit space of h, that is, the points of Q are the orbits of h. Note that each orbit has either p points or just one point, and the latter subset of M, call it F (for "fixed point set" of h), is closed. If the orbits of h have sufficiently small diameter, then one can define a certain "center of gravity" map $\sigma:Q \to M$ such that $\sigma \circ q$ is close to id_M, and hence homotopic to id_M, and also $\sigma \circ q | F = id_F$. For example, $\sigma:Q \to M$ can be defined by regarding M as a subset of some Euclidean space, and hence a retract of some neighborhood there. And so if an orbit of h has sufficiently small diameter, its Euclidean center of gravity $\left[= \frac{1}{p} \sum_{i=1}^{p} h^i(x) \right]$ will lie in this neighborhood and hence can be retracted to M.

Now one argues that on the one hand, the degree of $\sigma \circ q:M \to M$ is one, since $\sigma \circ q$ is homotopic to id_M, whereas on the other hand its degree is a multiple of p. This latter fact follows because h is acting freely on the open set $M \setminus F$, and $\sigma \circ q(F) = F$, and so q is a p-fold covering space over $\sigma^{-1}(M \setminus F)$. Since the degree of a map can be measured over any open set, and since degree is multiplicative, and p-fold covering maps have degree p (we suppress orientation considerations here), the claim follows. Hence p can only be 1, hence the first theorem is established.

As for the second theorem, which is a bit more difficult, it is proved by localizing to the neighborhood of a point the above argument. In more detail: First, it is fairly easy to reduce the theorem to the case as above, where the order of $G = \{ 1, h, h^2, \ldots h^p \}$ is some prime p. Then the problem is to show that if F (as above) has interior, then $p = 1$. Assuming $\mathrm{int}(F) \neq \varnothing$, it is not hard to find a small coordinate chart in M, which we denote by $\mathbb{R}^m$, such that $\mathbb{R}^m_+ \subset F$, where $\mathbb{R}^m_+$ = upper half space of $\mathbb{R}^m$, and such that 0 is a limit point of $M \setminus F$. Now one argues that a center of gravity map $\sigma: V \to M$ (as above) can be defined on a neighborhood V of q(0) in Q, and that $\sigma \circ q$ has local degree at 0 equal to one (since $\sigma \circ q \sim \mathrm{id}$ near 0, keeping the preimage of 0 always just 0 itself), whereas this local degree must be a multiple of p, by the factor-through-the-covering-map argument. Hence $p = 1$, as desired.

It is perhaps also worth remarking here that the A_p version of the second theorem remains open. That is, if A_p acts effectively on a manifold, is it possible that the fixed point set of the action has interior. McAuley has been pursuing this question recently.

We remark that both of the above theorems are true for G an arbitrary compact Lie group. The proofs are similar; one constructs $\sigma: Q \to M$ just as above, and when $\dim(G) \geq 1$, one argues that $(\sigma \circ q)_*: H_m(M) \to H_m(M)$ cannot possibly be the identity, for Q cannot carry homologically the image $q_*[M]$ of the fundamental class of M.

Finally, we recall a result sometimes referred to as the "point wise periodic implies periodic" theorem.

Theorem (Montgomery) Suppose h is a point wise-periodic homeomorphism of a connected manifold. Then h is periodic.

That is, if each orbit of h is finite, then in fact there is a uniform bound on the orbit size. The proof of this (American Journal of Mathematics, 1936) is a clever two-step application of Newman's second theorem above.

Note that this theorem together with Newman's first theorem says that a homeomorphism h such as in Version 2 above cannot be point wise periodic. Similarly, these theorems imply that for any effective A_p-action on a manifold, at least one orbit must be full, i.e. at least one point stabilizer must be trivial. For otherwise all orbits would be finite, and hence uniformly finite, i.e. the A_p-action would reduce to a finite group action.

The Equivalence of Conjecture Versions 1 and 2

The above theorems have relevance to the Hilbert—Smith conjecture. For suppose that G is a compact group of homeomorphisms of a manifold M. If G is not a Lie group, then by the Peter-Weyl Theorem G has a non-finite, totally disconnected subgroup H. Such an H is an inverse limit of finite subgroups, and indeed by passing to a further subgroup if necessary, we may assume that H is of the form $H = \text{inv. lim.} (\mathbb{Z}_{n_1} \leftarrow \mathbb{Z}_{n_2} \leftarrow \mathbb{Z}_{n_3} \leftarrow \ldots)$ where n_i divides n_{i+1} and the homeomorphisms are simply reduction mod n_i . Now by the first theorem of Newman, H cannot have arbitrarily small nontrivial finite subgroups. This forces H to have as a subgroup some A_p. Hence A_p would therefore be acting effectively on a manifold. In other words, the conjecture that the only compact groups which act effectively on manifolds are Lie groups is equivalent to the Hilbert—Smith Conjecture (Version 1).

The equivalence of Versions 1 and 2 (above) of the Hilbert—Smith Conjecture should now be fairly clear. First, suppose Version 1 is false, i.e. suppose some A_p acts effectively on some manifold M. By passing to a small subgroup (which is isomorphic to A_p) we can assume that the orbits of this action are small. Hence a topological generator $h: M \xrightarrow{\cong} M$ of this A_p would offer a counterexample to Version 2.

Conversely, suppose Version 2 is false. Then the closure G of the powers $\{ h^n \mid n \in \mathbb{Z} \}$ in Homeo(M) is a nontrivial compact subgroup. By hypothesis 1 and Newman's first theorem, G can contain no finite subgroups (and hence no Lie subgroups). Such a G must then contain A_p.

<u>Some Current Work.</u>

We ask the question: What sorts of spaces can or can't A_p act effectively on ?

<u>Example:</u> A_2 acts on $I^\infty = \prod_{i=1}^{\infty} c(\mathbb{Z}_{2^i})$ since $\mathbb{Z}_{2^i}$ acts on $\mathbb{Z}_{2^i}$. Hence A_2 acts freely on I^∞ minus the cone point; which is homeomorphic to $I^\infty \times [0,\infty)$.

<u>Question:</u> Can A_2 act freely on a compact Hilbert cube manifold?

<u>Observation:</u> $\mathbb{Z}_p$ cannot act freely on a finite dimensional cell-like set X. However, A_p can act freely on a two-dimensional cell-like set.

<u>Sketch of proof of observation:</u> For $\mathbb{Z}_2$, the proof of the first part of the observation depends on constructing the following diagram for some n, where the map from X to S^n is equivariant. The point is that π is a two to one covering map, and hence is classified by some such map to $\mathbb{R}P^n$. On the other hand, $X \to X/\mathbb{Z}_2$ effectively serves as a classifying space, classifying e.g. $S^{n+1} \to \mathbb{R}P^{n+1}$ by some map $\mathbb{R}P^{n+1} \to X/\mathbb{Z}_2$ which leads to a violation of the Borsuk—Ulam Theorem.

$$\begin{tikzcd} X \arrow[r] \arrow[d, "\pi"'] & S^n \arrow[d] \\ X/\mathbb{Z}_2 \arrow[r] & \mathbb{R}P^n \end{tikzcd}$$

The example that A_p can act freely on a two dimensional set is due to Edwards and Bestvina. First, understand the map $S^3 \xrightarrow{\mathbb{Z}_p} L_p$ which is the join of $\mathbb{Z}_p$ acting on S^1 with itself. Next understand the degrees in the following diagram.

$$
\begin{tikzcd}
S^3 \arrow[d, "\mathbb{Z}_p", "\deg p"'] & S^3 \arrow[l, "\deg q^2"'] \arrow[d, "\mathbb{Z}_{pq}"', "\deg pq"] \\
L_p & L_{pq} \arrow[l, "\deg q"]
\end{tikzcd}
$$

A false start for X would be the following.

$$
\begin{tikzcd}
S^3 \arrow[d] & S^3 \arrow[l] \arrow[d] & S^3 \arrow[l] \arrow[d] \; . \; . \; . & X' = \text{inv. lim.} \\
L_p & L_{p^2} \arrow[l] & L_{p^3} \arrow[l] &
\end{tikzcd}
$$

Note that $\mathbb{Z}_p$ acts freely on X'. Now alter the bonding maps on the lower level to preserve the action on π_1, but so the maps are of degree 0. To do this, take the connected sum of two maps of degree p to get a map of degree 0 and use the fact that $L_{p^2} \# S^3$ is homeomorphic to L_{p^2}. The resulting map acts the same on π_1. The correct diagram to obtain the example is the following where X is now cell-like since the maps are of degree 0. The inverse limit X can be made two dimensional by homotoping the images into the two skeleta.

$$
\begin{tikzcd}
S^3 \arrow[d] & S^3 \arrow[l] \arrow[d] & S^3 \arrow[l] \arrow[d] \; . \; . \; . & X = \text{inv. lim.} \\
L_p & L_{p^2} \arrow[l] & L_{p^4} \arrow[l] &
\end{tikzcd}
$$

This completes the example.

<u>Other Ideas.</u>

One approach for thinking about the conjecture is the following. Suppose there is a free A_p action on $\mathbb{R}^n$.

$$\begin{array}{c} \mathbb{R}^n \\ \big\downarrow \\ \mathbb{R}^n/A_p \end{array}$$

We want to map $\underset{\substack{\ell\text{-fold}\\ \text{join}}}{*} A_p$ equivariantly into $\mathbb{R}^n$. Start with $\ell = 1$. Map A_p onto the orbit of some point x in $\mathbb{R}^n$, mapping the identity to x. Next, $A_p * A_p$ = $\{c(A_p) \times A_p\} \,/\, \{A_p \text{ on base}\}$. First map in the cone, then let A_p act on the cone. Continue this process, obtaining the following.

$$\begin{array}{ccc} \underset{\ell}{*}A_p & \longrightarrow & \mathbb{R}^n \\ \big\downarrow & & \big\downarrow \\ \underset{\ell}{*}A_p/A_p & \longrightarrow & \mathbb{R}^n/A_p \end{array}$$

Assume that $\mathbb{R}^n/A_p$ is finite dimensional. Obtain the following diagram where the right hand side has Menger manifolds.

$$\begin{array}{ccccc} \underset{\ell}{*}A_p & \longrightarrow & \mathbb{R}^n & \longrightarrow & \mu^{n+2} \\ \big\downarrow & & \big\downarrow & & \big\downarrow{\scriptstyle A_p} \\ \underset{\ell}{*}A_p/A_p & \longrightarrow & \mathbb{R}^n/A_p & \longrightarrow & \mu^{n+2}/A_p \end{array}$$

The cohomological dimension of $\mathbb{R}^n/A_p$ is known to be n+2. Try to argue that ℓ cannot be too large.

SHRINKING CERTAIN CELL-LIKE DECOMPOSITIONS QUASI-CONFORMALLY

Mladen Bestvina

Let F be a closed oriented surface of genus >1. Then F supports a hyperbolic structure, and contains a lot of hereditarily indecomposable 1-dimensional continua that can be obtained by passing to the Hausdorff limit of a generic convergent sequence of simple closed geodesics on F. Such continua are called *geodesic laminations* . Generic geodesic laminations have simply-connected complementary components. The universal cover of F is the hyperbolic plane, and it naturally compactifies to a closed disc D^2. The closure of the preimage (under the covering map) of the geodesic lamination determines a cell-like decomposition G of D^2. Now double D^2 to get S^2 with a natural action of $\pi_1(F)$ (double of the covering action), and complete G by points on the other copy of D^2. Now the group action leaves G invariant, so the group acts on the decomposition space $S^2/G \cong S^2$, and the limit set of this new action is a dendrite.

In the talk I proposed a way of shrinking G quasi-conformally to obtain a uniformly quasi-conformal action of $\pi_1(F)$ on S^2 with limit set a dendrite.

Maximal Convex Metrics on some Classical Metric Spaces

Philip L. Bowers

Abstract. It is proved that every convex, complete, two point homogeneous metric for which small spheres are connected has maximal symmetry. This in turn implies that the standard metrics on the classical spaces of geometry are maximally symmetric.

R. Williamson & L. Janos prove the following theorem.

<u>Theorem</u>: For every natural number n, the enclidean metric on $\mathbb{R}^n$ has maximal symmetry.

A metric d on a set X has <u>maximal symmetry</u> provided its isometry group Iso(d) is not properly contained in the isometry group of any metric equivalent to d.

The following theorem generalizes this result of Williamson and Janos.

<u>Theorem</u>: Every convex, complete, two-point homogeneous metric for which small spheres are connected has maximal symmetry.

<u>Definitions</u>: Let (x,d) be a metric space.

1. A point q in X is <u>between</u> points p and r provided $p \neq q \neq r$ and $d(p,q) + d(q,r) = d(p,r)$.
2. d is <u>convex</u> if every pair of distinct points has at least one between point.
3. d is <u>homogeneous</u> if Iso(d) acts transitively on X.
4. d is <u>two-point homogeneous</u> if Iso(d) acts transitively on pairs of equidistant points of X.

The ℓ_1-metric on $\mathbb{R}^2$ is not maximally symmetric even though it possesses many nice qualities. It is convex, complete, homogeneous and spheres are connected. The key to the Theorem is the two-point homogeneity.

<u>Examples:</u> 1. [Wang, 1952] Up to similarity, the only convex, <u>compact</u>, two-point homogeneous spaces are riemannian manifolds with underlying space one of the following:

$$S^n, \underbrace{\mathbb{R}P^n, \mathbb{C}P^n, \mathbb{Q}P^n}_{\text{projective spaces}}, \underbrace{\mathrm{Cay}P^2}_{\text{Cayley projective plane}}$$

2. [Tits, 1955] Up to similarity, the only convex locally compact two-point homogeneous non-compact spaces are riemannian mainfolds with underlying space one of the following:

$$\mathbb{R}^n,\ \mathbb{H}^n,\ \mathbb{C}H^n,\ \mathbb{Q}H^n,\ \mathrm{Cay}\,\mathbb{H}^2\ .$$

3. From 1 and 2, the classical metric spaces of geometry, namely enclidean space E^n, elliptic space $\mathbb{R}P^n$ and the sphere S^n, and hyperbolic space $\mathbb{H}^n$ are maximally symmetric.

4. Any hilbert space is maximally symmetric. In fact, the Theorem can be generalized by replacing the hypotheses "convex" and "complete" by the existence of metric sygments between points. This implies that any prehilbert space (real or complex inner product space) with its natural inner product metric is maximal.

What does two-point homogeneity give us?

<u>Exercise</u>: If (X,d) is a two-point homogeneous space and d' is a metric equivalent to d with $\mathrm{Iso}(d) \subset \mathrm{Iso}(d')$, then there is a function $\varphi: \mathrm{im} d \to [o,\infty)$ such that $d' = \varphi \circ d$.

Such a function φ is called a <u>scale change</u> for d. More preisely, given a metric d, a function φ: $\mathrm{im} d \to [o,\infty)$ is a scale change for d provided $\varphi \circ d$ is a metric equivalent to d. The exercise calls us to look for conditions on a metric d that will ensure that no scale change of d can enlarge the isometry group.

<u>Main Proposition</u>: Let (X,d) be a metric space such that
(i) every pair of points can be connected by a d-segment,
(ii) $\exists\, \mu>0$ such that $\forall x,y \varepsilon X$ and $o<\varepsilon<\mu$, $S_d(x,\varepsilon)$ is connected and, if $d(x,y)<\varepsilon$, then $S_d(x,\varepsilon) \cap S_d(y,\varepsilon) \neq \phi$.

Then $\mathrm{Iso}(d) = \mathrm{Iso}(\varphi d)$ for every scale change φ of d.

<u>Outline of proof</u>: Easily $\mathrm{Iso}(\varphi d) \supset \mathrm{Iso}(d)$ and it suffices to show $\mathrm{Iso}(\varphi d) \subset \mathrm{Iso}(d)$. Let $\Lambda \varepsilon\ \mathrm{Iso}(\varphi d)$.

<u>Step 1</u>: Show $\forall \delta>0$, $\exists\, 0<r<\delta$ such that Λ preserves d-distance r; i.e., $\forall x,y \varepsilon X$, $d(x,y) = r \rightleftarrows d(\Lambda x, \Lambda y) = r$.

<u>Step 2</u>: Use Step 1. to show that Λ preserves the d-length of d-recfifiable arcs in X.

For the proof of the Theorem, convexity and completeness imply (i) of the Main Proposition and two-point homogenity and small spheres are connected imply (ii) of the Main Proposition. For details, see [Bowers, 1987].

[Bowers, 1987] Maximal convex metrics on some classical metric spaces, preprint.

[Tits, 1955] Sur certaines classes d'espaces homogènes de groupes de Lie, Memoir, Belgian Acadamy of Sciences.

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Continuation Spaces

J. W. Cannon

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Abstract. For those mathematicians who have worked extensively with covering spaces, orbifolds, and group graphs, it is apparent that all of these notions are essentially equivalent. We show, in fact, how covering space and orbifold questions can be reduced quickly and intuitively to problems about group graphs.

SUBMANIFOLD DECOMPOSITIONS THAT INDUCE APPROXIMATE FIBRATIONS

by R. J. Daverman

The setting involves the following data: a specific closed, orientable n-manifold N; another orientable (n+k)-manifold M, where $n \geq k$; and a usc (i.e., upper semicontinuous) decomposition G of M into copies of N, where $\dim(B=M/G) < \infty$. The subject is the following

QUESTION: When is $p:M \to B$ an approximate fibration?

When it is, Coram and Duvall have an exact sequence relating the homotopy groups of N, M, and B, just like the one for genuine fibrations. It provides the most efficient means available for extracting structural information about M from that of N and B, evidenced in the Corollaries to Theorem 1.

The above question is unsolved for $n=3$ and N arbitrary, even when $k=1$. An impediment exists there because it is not known (at least to me) whether every degree 1 map $N \to N$ induces π_1-isomorphisms.

Some old answers have been given by Daverman-Walsh. The map p must be an approximate fibration if N is the n-sphere. In addition, whenever N is simply connected p restricts to an approximate fibration over its continuity set (consisting of those points $b \in B$ such that every retraction $R:U \to p^{-1}b$ on a neighborhood U of $p^{-1}b$ induces isomorphisms $H_n(p^{-1}b') \to H_n(p^{-1}b)$ for all b' sufficiently close to b).

Here is a related elementary fact: in case N is aspherical with Hopfian fundamental group, then p restricts to an approximate fibration over its continuity set. (A group H is said to be Hopfian provided every epimorphism $\theta: H \to H$ is necessarily an isomorphism.) Every degree 1 map $N \to N$ on an aspherical manifold N with $\pi_1(N)$ Hopfian is a homotopy equivalence, simply because such maps induce π_1-epimorphisms.

Theorem 1. If $n=k=2$ and $\chi(N)<0$, then p is an approximate fibration.

Corollary. Given any (orientable) 2-manifold $N \neq S^1 \times S^1$, there is no usc decomposition of S^4 into copies of N.

Corollary. If $n=k=2$, $\chi(N)<0$, and $\chi(B)\leq 0$, then M is aspherical and $\pi_1(M)$ is the semi-direct product of $\pi_1(N)$ and $\pi_1(B)$.

Sketch of the proof of Theorem 1. By another result of Daverman-Walsh, B is a 2-manifold and the discontinuity set D of p (namely, the complement in B of the continuity set) is locally finite. Thus, the issue reduces to the case where B is an open disk and p is an approximate fibration over $B\backslash b$. Name $g \in G$ such that $p(g)=b$, specify a retraction $R: M \to g$, fix $g' \neq g$, and study $R|: g' \to g$.

CLAIM: $\mathrm{Deg}(R|) \neq 0$ implies $R|$ is a homotopy equivalence. This can be verified by examining

$$\begin{tikzcd} & g^* \arrow[d, "\theta"] \\ g' \arrow[ur, "\lambda"] \arrow[r, "RI"'] & g \end{tikzcd}$$

where $\theta : g^* \to g$ is the covering corresponding to $(RI)_{\#}\pi_1(g')$ and $\theta\lambda = RI$. That $\deg(RI) \neq 0$ implies g^* is compact, with more handles than g and g' unless θ is a homeomorphism. But $\lambda : g' \to g$ induces a π_1-epimorphism, so g^* cannot have more handles than g, and θ must be 1-1. Hence, $\lambda_{\#} : \pi_1(g') \to \pi_1(g^*)$ induces a self-epimorphism of the Hopfian group $\pi_1(N)$, which indicates λ is homotopic to a homeomorphism.

Assume $\deg(RI) = 0$. Here $H_1(M \setminus g) \to H_1(M) \cong H_1(g)$ is surjective and $\pi_1(M \setminus g)$ is given by

$$1 \to \pi_1(g') \to \pi_1(M \setminus g) \to Z \to 1 .$$

Abelianization shows $H_1(M \setminus g)$ is determined by $H_1(g')$ and one additional generator. By a result of [Zieschang-Vogt-Caldeway, p.100],

$$\operatorname{rank}(\text{image } H_1(g')) \leq \operatorname{rank}(H_1(M))/2 = \operatorname{rank}(H_1(g))/2 ,$$

which is impossible.

As a consequence, $RI : g' \to g$ is a homotopy equivalence for all g' sufficiently close to g. The analysis of approximate fibrations by Coram-Duvall shows p is one.

Two remarks are in order. First, nonorientable analogs of Theorem 1 hold. Second, when $k=2$ and n is unrestricted p is not invariably an approximate fibration; counterexamples arise with N the mapping torus of any periodic homeomorphism $h:T \to T$ defined on a closed $(n-1)$-manifold T.

Proposition 2. Whenever $\pi_1(N)$ is finite, p is an approximate fibration over its continuity set.

This is accomplished by (locally) lifting G to the universal cover and examining the induced decomposition into n-spheres, which is known to determine an approximate fibration.

Theorem 3. If $\pi_1(N)$ is finite and $k=2$, then p is an approximate fibration.

The stategy is to show p has no discontinuities and to apply Proposition 2. Details involve algebraic diagram-chasing.

Corollary. If N is elliptic and $k=2$, then p is an approximate fibration.

In closing, here are two obvious unsettled topics.

Question 1. For $n=3$, $k=2$, and N not a bundle over the circle, is p an approximate fibration? What if, in addition, N is aspherical?

Question 2. In Theorem 1 can p be approximated by locally trivial bundle maps?

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STABLE MAPS AND UNIVERSAL MAPS

Dennis J. Garity

This represents joint work with Dale Rohm. We will begin with a summary of results that are known about stable maps into n-cells and into the Hilbert cube. Stable maps are also known as Alexander—Hopf essential maps [N], [GT]. Krasinkiewicz has given a general definition of essential maps into the product of manifolds [K] that coincides with the definition of stable maps in the cases under consideration. All spaces are separable metric spaces and the dimension of a space X, dim(X), means covering dimension [HW].

It is well known that a space has covering dimension greater than or equal to n if and only if it admits a stable map into I^n. If f is stable, then the composition of f with any self homeomorphism of I^n is also stable since self homeomorphisms of I^n take boundary points to boundary points. It is also well known that a compact space is strongly infinite-dimensional if and only if it admits a stable map into the Hilbert cube. This is because if f is a map from a compact space X into the Hilbert cube, the inverse images of pairs of opposite faces of the Hilbert cube form an essential family in X if and only if f is stable [W]. However, self homeomorphisms of the Hilbert cube lack the invariance of domain mentioned above. If f is a stable map from X into the Hilbert cube, the composition of f with self homeomorphisms of the Hilbert cube should preserve stability and should give rise to new essential families in X. We show that this is indeed the case. We use the concept of universal maps introduced by Holsztyński to prove this result.

Definition. A map $f: X \to I^n$ is *stable* if there does not exist a map $g: X \to S^{n-1}$ with $f|_{f^{-1}(S^{n-1})} = g|_{f^{-1}(S^{n-1})}$.

Theorem 1. [HW] The dimension of X is greater than or equal to n if and only if there exists a stable map $f: X \to I^n$.

The definition of stable maps together with the fact that self homeomorphisms of an n-cell take boundary points to boundary points immediately yields the following result.

Theorem 2. A map $f: X \to I^n$ is stable if and only if $g \circ f: X \to I^n$ is stable for each self homeomorphism g of I^n.

We now turn to stable maps into the Hilbert cube. Stable maps into the Hilbert cube are defined only for compact spaces to avoid the possibility of a map from a disjoint union of n-cells of increasing dimension into the Hilbert cube being stable. See [W] and [B] for a more detailed description of stable maps.

Definition. A map $f: X \to I^\infty$ where X is a compact space is stable if $p_n \circ f: X \to I^n$ is stable for each positive integer n and every projection p_n of I^∞ onto the first (or any) n factors.

Our goal is to prove a result for the Hilbert cube which is analogous to that contained in Theorem 2. The same techniques will not work because there are many self homeomorphisms of I^∞ which do not preserve projections onto the factors. A characterization of stable maps in terms of a property preserved by self homeomorphisms is needed. *Universal maps*, a concept introduced by Holsztyński, is one such property. Using universal maps, we can prove the following Theorem.

Theorem 3. A map $f: X \to I^\infty$ from a compact space X is stable if and only if $g \circ f: X \to I^\infty$ is stable for each self homeomorphism g of I^∞.

Universal maps were introduced by Holsztyński [H1] and have been used in conjunction with the study of fixed point theory and confluent maps. See [H2], [H3], [H4], [H5], [H6], [H7], [GT] and [N]. In order to prove theorem 3, we need results concerning the relation between universal maps and stable maps, and we need results about universal maps into products of spaces.

Definition. [H1] A map $f: X \to Y$ is universal if for every map $g: X \to Y$ there exists a point p in X with $f(p) = g(p)$.

The relationship of universal maps to the fixed point property is made clear in the following theorem.

Theorem 4 [H2]. Given a space Y, the following conditions are equivalent.

1. Y has the fixed point property.
2. The identity map from Y to itself is universal.
3. There exists a space X and a universal map $f: X \to Y$.

Note. A map $f: X \to Y$ is universal if and only if $h \circ f: X \to Y$ is universal for each self homeomorphism h of Y. For if g is any map from X to Y and if f is universal, then there is a point p with $h^{-1} \circ g(p) = f(p)$. So $h \circ f(p) = g(p)$. Conversely, if $h \circ f$ is universal, then there is a point q with $h \circ f(q) = h \circ g(q)$. Thus, $f(q) = g(q)$.

We are now in a position to prove the theorem stated at the end of the previous section.

Proof of Theorem 3. Let $f:X \to I^{\infty}$ be a map and let g be any self homeomorphism of I^{∞}. If J is a finite subset, then we set $I^J = \prod_{j \in J} I_j$ and let $p_J : I^{\infty} \to I^J$ denote projection of I^{∞} onto I^J.

By definition, the map f is stable if and only if $p_J \circ f : X \to I^J$ is stable for each finite subset J of $\mathbb{Z}_+$. By theorem 5, this is equivalent to saying that $p_J \circ f : X \to I^J$ is universal for each finite subset J of $\mathbb{Z}_+$ which, by theorem 6, is equivalent to saying that f is a universal map to the Hilbert cube. By the note after theorem 4, we see that f is universal if and only if $g \circ f$ is universal for each self homeomorphism g of I^{∞}. By theorem 6, this is equivalent to saying that $p_J \circ (g \circ f) : X \to I^J$ is universal for each finite subset J of $\mathbb{Z}_+$ which is then, by theorem 5, equivalent to saying that $p_J \circ (g \circ f) : X \to I^J$ is stable for each finite subset J of $\mathbb{Z}_+$. Finally, by definition, this is equivalent to saying that $g \circ f$ is a stable map to the Hilbert cube. ∎

The next corollary follows immediately from the preceding Theorem and the fact that maps as described in the Corollary are stable.

Corollary Let $\{ (A_i , B_i) \}_{i=1}^{\infty}$ be an essential family for a strongly infinite dimensional space X and let f be a map from X into I^{∞} so that $A_i = (p_i \circ f)^{-1}(-1)$ and so that $B_i = (p_i \circ f)^{-1}(1)$ for each i. If h is any self homeomorphism of I^{∞}, then $\left\{ \left[(p_i \circ h \circ f)^{-1}(-1) , (p_i \circ h \circ f)^{-1}(1) \right] \right\}_{i=1}^{\infty}$ is also an essential family for X.

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Mathematics Department
Oregon State University
Corvallis, Oregon 97331

An Invariant of Dichromatic Links
by

Jim Hoste and Józef H. Przytycki

A 1-trivial dichromatic link in S^3 is a link having at least two components, one of which is unknotted and labeled, or colored, "1" , while all other components are colored "2". By using methods similar to those of Kauffman [K], we define a polynomial invariant of such links which is analogous to the Jones polynomial [J]. This polynomial has since been generalized by Hoste and Kidwell [H-K]. However their approach is far more complicated, just as the establishment of the skein polynomial is more complicated then Kauffman's approach to the Jones polynomial [F-Y-H-L-M-O], [P-T].

If L is a 1-trivial dichromatic link then we may isotope L until the 1-component, that is the component colored "1", is the z-axis union the point at infinity. If we now project the link into the x-y plane we obtain a diagram of the 2-sublink in the punctured plane $\mathbb{R}^2-\{0\}$. Two such punctured diagrams represent isotopic 1-trivial dichromatic links if and only if one can be transformed to the other by a finite sequence of Reidemeister moves in $\mathbb{R}^2-\{0\}$ followed by possibly "flipping over" the plane of projection, that is by viewing the x-y plane from the other side.

If D is a diagram, we denote by sw(D) the self writhe of D. This is the sum of the signs of those crossings between strands belonging to the same component.

Theorem 1 There exists a unique polynomial invariant in $\mathbb{Z}[A^{\pm 1},h]$ of unoriented 1-trivial dichromatic links given by

$$d_L(A,h)=(-A^3)^{-sw(D)}\langle D\rangle$$

where D is any punctured diagram of the link L and $\langle D\rangle$ is the invariant of D determined by the following properties:

1. $\langle \cdot \bigcirc \rangle = 1$
2. $\langle \odot \rangle = h$

3. $\langle \times \rangle = A\langle\,)(\,\rangle + A^{-1}\langle \asymp \rangle$
4. $\langle \cdot \;\; \bigcirc \;\; K \rangle = -(A^2+A^{-2})\langle \cdot \;\; K \rangle \qquad K \neq \varnothing$
5. $\langle \odot \;\; K \rangle \quad = -(A^2+A^{-2})h\langle \cdot \;\; K \rangle \qquad K \neq \varnothing$

Here we follow Kauffman's notation [K] with the additional convention of marking the puncture with a dot. Later, when working with ordinary diagrams, we will also subscript the components with their colors.

<u>Proof</u>: Let D be a punctured diagram. Then, proceeding in a fashion similar to Kauffman [K], one sees that properties 1-5 uniquely determine ⟨D⟩. Moreover, ⟨D⟩ is preserved by Type II and III Reidemeister moves as well as flipping over the plane of projection. However the effect of a Type I Reidemeister move is given by

$$\langle \circlearrowright \rangle = -A^{-3}\langle\,) \,\rangle$$
$$\langle \circlearrowleft \rangle = -A^{3}\langle\,) \,\rangle.$$

From this it follows that d is a well defined isotopy invariant of unoriented 1-trivial dichromatic links. ■

Of course, d behaves similarly to the Jones polynomial with respect to connected sum, mutation, companionship, etc. Therefore we list only a few additional properties of d.

1. Let L be a link represented by the punctured diagram D. Let $\bar{L}$ be represented by the diagram $\bar{D}$ obtained from D by reflecting in the plane of the projection. In other words, $\bar{D}$ is obtained from D by changing every crossing from over to under. Then

$$d_L(A,h) = d_{\bar{L}}(A^{-1},h).$$

2. If one uses ordinary diagrams rather than punctured diagrams then the following "clasp" rule holds .

$$A^2 d_{\underset{i\;\;j}{\text{clasp}}} + A^{-2} d_{\underset{i\;\;j}{\text{clasp}'}} = (A^2+A^{-2})\,h\,d_{\underset{i\;\;j}{)(}} \qquad i \neq j.$$

3. Using ordinary (or punctured) diagrams, one has the following rule

$$(-A^3)^{sw(\times_{2\,2})}\, d_{\times_{2\,2}} = A\,(-A^3)^{sw()(_{2\,2})}\, d_{)(_{2\,2}} + A^{-1}\,(-A^3)^{sw(\asymp_{2})}\, d_{\asymp_{2}}.$$

4. We may define an invariant $\tilde{d}$ of 1-trivial dichromatic links with oriented 2-sublink as follows. In general, if L is any link, some of whose components are oriented, let lk(L) denote the sum of the linking numbers between each pair of oriented components. Now let

$$\tilde{d}_L = (-A^3)^{-2lk(L)} d_{|L|}$$

where $|L|$ denotes L stripped of its orientation. Then, again using ordinary (or punctured) diagrams, the following rule holds

$$A^4 \tilde{d}_{\times_{2\,2}} - A^{-4} \tilde{d}_{\times_{2\,2}} = (A^{-2} - A^2)\, \tilde{d}_{)(_{2\,2}} .$$

5. If L is a 1-trivial dichromatic link, let <u>wrap(L)</u> be the wrapping number of the 2-sublink around the 1-component. That is, the minimal geometric intersection number of the 2-sublink with any disk spanning the 1-component. Then

$$\deg_h d_L \le \mathrm{wrap}(L)$$

where $\deg_h$ is the highest degree of h appearing in d_L.

6. a) If the 2-sublink L_2 is oriented then the Jones polynomial of L_2 is

$$V_{L_2}(A^{-1/4}) = \tilde{d}_L(A,1).$$

b) If L is oriented then the Jones polynomial of L is

$$V_L(A^{-1/4}) = -(A^2+A^{-2})\,(-A^3)^{-2lk(L)} d_{|L|}(A,(A^4+A^{-4})/(A^2+A^{-2})).$$

We mention two applications of d.

Suppose L is a link that is both 1-trivial and 2-trivial. In other words the 2-sublink is also an unknot. Hence we may compute d relative to either component. Call these two invariants d^1 and d^2 respectively. If L is interchangeable, that is there is an isotopy exchanging the components, then $d^1_L = d^2_L$.

We may also use d to investigate periodic links.

<u>Theorem 2</u> Let r be prime. Suppose L is a 1-trivial dichromatic link invariant under a $\mathbb{Z}_r$-action on S^3 with fixed point set the 1-component. Then

$$d_L(A,h) \equiv d_L(A^{-1},h) \bmod (A^{4r}-1,\ r).$$

In other words, the two polynomials differ by an element of the ideal generated by $A^{4r}-1$ and r .

<u>Proof:</u> We can find an oriented punctured diagram D having r-fold rotational symmetry and such that |D| represents L. Let $D_{\mathrm{sym}(\;)}$, $D_{\mathrm{sym}(\;)}$ and $D_{\mathrm{sym}(\;)}$ denote three punctured diagrams having r-fold rotational symmetry and which are identical except near the orbit of a single crossing where, at all r crossings, they appear instead with right, left and smoothed crossings respectively. Now using an idea of Murasugi's [M] (see also [P2]) and property 4 we obtain

$$A^{4r}\tilde{d}_{D_{\mathrm{sym}(\;)}} - A^{-4r}\tilde{d}_{D_{\mathrm{sym}(\;)}} \equiv (A^{-2r}-A^{2r})\tilde{d}_{D_{\mathrm{sym}(\;)}} \bmod r.$$

Therefore

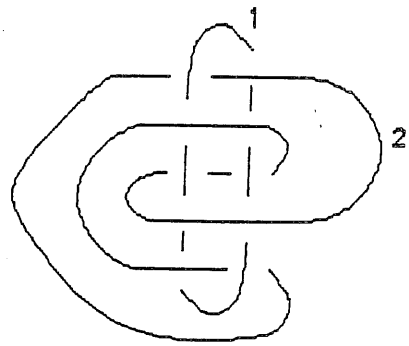
$$\tilde{d}_{D_{\mathrm{sym}(\;)}} \equiv \tilde{d}_{D_{\mathrm{sym}(\;)}} \bmod (A^{4r}-1,\ r)$$

and hence

$$d_{D_{\mathrm{sym}(\;)}} \equiv d_{D_{\mathrm{sym}(\;)}} \bmod (A^{4r}-1,\ r).$$

But this allows one to change $|D|$ to $|\bar{D}|$ without changing d mod $(A^{4r}-1,\ r)$. Now applying property 1 gives the desired result. ∎

Example: Let $L = 7^2_6$ with the components colored as shown below.



The link 7^2_6

Then $d^1=-A^{12}+A^8+A^{-4}+(A^{12}-A^8+1-A^{-4})h^2$. It is laborious to compute d^2, but one can compute the coefficient of h^4 more easily. It equals $A^{16}+2A^{12}-2A^4-1$. Hence 7^2_6 is not interchangeable and the wrapping numbers are 2 and 4 respectively. By Theorem 2, there are no r-fold rotational symmetries about the 1-component with r>3 or about the 2-component with r>2 where r is prime.

Finally, we note that Theorem 1 can be interpreted in the language of skein modules [P1]. In particular, the theorem implies that the skein module

$$\mathcal{S}_{2,\infty}(S^1\times D^2,\ \mathbb{Z}[A^{\pm 1}]\ )(A)$$

is a free module with infinite basis $\{h_i\}_{i=0}^{\infty}$, where h_0 is an unknot and h_i consists of i longitudes.

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Mathematics Department
Oregon State University
Corvallis, OR 97331
USA

Mathematics Department
University of British Columbia
Vancouver, B.C. Canada V6T 1Y4
(and Warsaw University, Poland)

Homogeneous Cantor Sets in E^3

by Jack W. Lamoreaux

If C is a tame cantor set in E, then every homeomorphism f mapping C onto C can be extended to a homeomorphism F of E^3 onto itself. At the other end of the spectrum a rigid cantor set is one such that the only homeomorphism which can be extended is the identity.

We say that a set in B is strongly homogeneous if every homeomorphism of A onto itself can be extended to a homeomorphism of B onto itself. A natural question to ask would be the following: If a cantor set C is strongly homogeneous is it tame? Bob Daverman has answered this question in the negative for E^n $(n \geq 5)$. However, at the present time the answer is not known for E^3.

The standard four-link antoine's necklace can be shown to be 1-homogeneous but not 2-homogeneous. A set A in B is n-homogeneous if $(x_1, \ldots, x_n)$ and $(y_1, \ldots, y_n)$ are each sets of n distinct points in A and there exists a homeomorphism f of B onto itself which when restricted to A is a homeomorphism onto A and for each i; $f(x_i) = y_i$. It can also be shown that a set which uses alternating links of Bing and Whitehead is (for any n) n-homogeneous but not strongly homogeneous. At the present time the following question is unknown: Are there 2-homeogenous cantor sets in E^3 which are not 3 homogeneous?

ALTERNATE CHARACTERIZATIONS OF WEAK INFINITE-DIMENSIONALITY

Dale M. Rohm

The cell-like dimension raising map question is one of the foremost unanswered questions in topology. Because it is known that the image of such a map must be infinite-dimensional, see [S] for a more detailed description of this question, it becomes important to determine exactly what types of infinite-dimensionality such an image could possess. Ancel has extensively studied this aspect of the question [A1] [A2]. In particular, Ancel has shown that a cell-like map defined on a finite-dimensional domain raises dimension if and only if the image does not have property C, a covering property first defined in [H] for metric spaces and later generalized in [AG] for more general spaces. While every space with property C is weakly infinite-dimensional, it is unknown whether or not the converse is true. In [R], a characterization of weak infinite-dimensionality in terms of open covers, similar in form to the definition of property C, was given by the author. This characterization is generalized by the following definitions.

Definitions 1. Let $r \in \{2, 3, 4, \cdots\}$. A space X will be said to have the property C_r if every countable sequence $\{\mathcal{U}_n : n \in \mathbb{N}\}$ of open covers of X, where for each $n \in \mathbb{N}$ the $|\mathcal{U}_n| \leq r$, has for each $n \in \mathbb{N}$ a precise pairwise disjoint open shrinkage $\mathcal{V}_n$ of $\mathcal{U}_n$ such that the $\bigcup\{\mathcal{V}_n : n \in \mathbb{N}\}$ forms an open cover of X. If a space X has property C_r for every $r \in \{2, 3, 4, \cdots\}$, then X will be said to have the property C_∞. A space X will be said to have the property C_ω if every countable sequence $\{\mathcal{U}_n : n \in \mathbb{N}\}$ of open covers of X has for each $n \in \mathbb{N}$ a precise pairwise disjoint open shrinkage $\mathcal{V}_n$ of $\mathcal{U}_n$ such that the $\bigcup\{\mathcal{U}_n : n \in \mathbb{N}\}$ forms an open cover of X.

It is obvious that a space X has property C_2 if and only if the space X is weakly infinite-dimensional. It is also quite easy to show that a space X has property C if and only if the space X has property C_ω. The equivalence of the property C_2 with weak infinite-dimensionality motivates the fairly obvious generalizations of the definitions of essential family and weak infinite-dimensionality contained in the following definitions. The original definition of essential family as given in [RSW] is the case where $r = 2$.

Definitions 2. A closed subset $S \subset X$ of a space X will be said to be a separator of a discrete collection of closed subsets $(A^{\alpha}:\alpha \in \Gamma)$ contained in the space X if $S \subset X$ separates the collection $(A^{\alpha}:\alpha \in \Gamma)$ in X; that is the complement $X \setminus S = \{U^{\alpha}:\alpha \in \Gamma\}$ where $\{U^{\alpha}:\alpha \in \Gamma\}$ is a collection of pairwise disjoint open subsets of X such that for each $\alpha \in \Gamma$ the closed set $A^{\alpha} \subset U^{\alpha}$. A countable family of discrete collections of closed subsets $\{(A_n^{\alpha}:\alpha \in \Gamma_n):n \in \mathbb{N}\}$ of a space X will be called inessential if for each $n \in \mathbb{N}$ there exists a closed set $S_n \subset X$ which separates $(A_n^{\alpha}:\alpha \in \Gamma_n)$ in X with the $\bigcap\{S_n:n \in \mathbb{N}\} = \varnothing$. Let $r \in \{2, 3, 4, \cdots\}$, then a space X will be said to be weakly infinite-dimensional with respect to r-tuples, and denoted by WID_r, if any countable family of r-tuples of pairwise disjoint closed subsets $\{(A_n^k:k = 1, 2, \cdots, r):n \in \mathbb{N}\}$ of X is *inessential. If the space X is* WID_r for every $r \in \{2, 3, 4, \cdots\}$, then the space X will be said to be WID_{∞}. If every countable family of discrete collections of closed subsets $\{(A_n^{\alpha}:\alpha \in \Gamma_n):n \in \mathbb{N}\}$ of a space X is inessential, then the space X will be said to be WID_{ω}.

The ungainly definition of WID_{ω} may be simplified by assuming separability, for then we need only consider whether or not all countable families of discrete sequences of closed subsets $\{(A_n^k:k \in \mathbb{N}):n \in \mathbb{N}\}$ are inessential. In any event, these last properties are seen to be equivalent through an application of the following lemma.

Lemma. Let $r \in \{2, 3, 4, \cdots\} \cup \{\omega\}$ and let $(A^{\alpha}:\alpha \in \Gamma)$ be a discrete collection, an r-tuple when $r \in \{2, 3, 4, \cdots\}$, of closed subsets of a space X. Let $S \subset X$ be a closed subset of X which separates $(A^{\alpha}:\alpha \in \Gamma)$ in X. If $T \subset X$ is a closed subset of X which separates the pair $(\bigcup\{A^{\alpha}:\alpha \in \Gamma\}, S)$ in X, then the closed subset T is also a separator of the original collection $(A^{\alpha}:\alpha \in \Gamma)$ in X.

Theorem 1. Let $r \in \{2, 3, 4, \cdots\} \cup \{\omega\}$ be fixed but arbitrary. A space is WID_r if and only if the space is weakly infinite-dimensional.

When we combining these results with more basic results we obtain the following diagram of implications.

Summary 1. Let $r \in \{2, 3, 4, \cdots\}$. Metric spaces satisfy the following implications of properties.

$$
\begin{array}{ccccccccccccc}
 & & C_2 & \Longleftarrow & C_r & \Longleftarrow & C_{r+1} & \Longleftarrow & C_\infty & \Longleftarrow & C_\omega & \Longleftrightarrow & \text{property } C \\
 & & \Updownarrow & & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow & & \\
\mathrm{WID} & \Longleftrightarrow & \mathrm{WID}_2 & \Longleftrightarrow & \mathrm{WID}_r & \Longleftrightarrow & \mathrm{WID}_{r+1} & \Longleftrightarrow & \mathrm{WID}_\infty & \Longleftrightarrow & \mathrm{WID}_\omega & &
\end{array}
$$

The remaining reverse implications are harder to analyze. A "generic" proof, similar to the proof of theorem 1, was not found. However, there is an inductive proof, giving most of the reverse implications, which also provides some insight into possible essential differences between property C and weak infinite-dimensionality.

Theorem 2. Let $r \in \{2, 3, 4, \cdots\}$ be fixed but arbitrary. If a space has the property C_r, then the space also has the property C_{r+1}. Thus, if a space has property C_r for some $r \in \{2, 3, 4, \cdots\}$, then the space has property C_r for every $r \in \{2, 3, 4, \cdots\}$; that is the space has property C_∞.

Corollary. Let $r \in \{2, 3, 4, \cdots\}$. If a space is WID_r, then the space also has property C_r.

After combining these results with previous results, we obtain the following summary.

Summary 2. Let $r \in \{2, 3, 4, \cdots\}$. A space X satisfies the following implications of properties.

$$
\begin{array}{ccccccccccccc}
 & & C_2 & \Longleftrightarrow & C_r & \Longleftrightarrow & C_{r+1} & \Longleftrightarrow & C_\infty & \Longleftarrow & C_\omega & \Longleftrightarrow & \text{property } C \\
 & & \Updownarrow & & \Updownarrow & & \Updownarrow & & \Updownarrow & & \Downarrow & & \\
\mathrm{WID} & \Longleftrightarrow & \mathrm{WID}_2 & \Longleftrightarrow & \mathrm{WID}_r & \Longleftrightarrow & \mathrm{WID}_{r+1} & \Longleftrightarrow & \mathrm{WID}_\infty & \Longleftrightarrow & \mathrm{WID}_\omega & &
\end{array}
$$

Finally, we obtain characterizations of property C and weak infinite-dimensionality which, while very similar in form, have clearly delineated differences, particularly when applied to compacta.

Theorem 3. A compactum X has property C if and only if every countable collection of finite open covers $\{\mathcal{U}_n : n \in \mathbb{N}\}$ of X has precise pairwise disjoint open shrinkages $\mathcal{V}_n$ of U_n for each $n \in \mathbb{N}$ such that the $\bigcup\{\mathcal{V}_n : n \in \mathbb{N}\}$ forms a cover of X.

Theorem 4. A compactum X is weakly infinite-dimensional if and only if every countable collection of finite open covers $\{\mathcal{U}_n : n \in \mathbb{N}\}$ of X, with the $\sup\{|\mathcal{U}_n| : n \in \mathbb{N}\} < \infty$, has precise pairwise disjoint open shrinkages $\mathcal{V}_n$ of U_n for each $n \in \mathbb{N}$ such that the $\bigcup\{\mathcal{V}_n : n \in \mathbb{N}\}$ forms a cover of X.

We end by asking two related questions.

Question 1. Can we always write a weakly infinite-dimensional space X as $X = K \cup Z$ where K is a compact weakly infinite-dimensional subspace and Z is a subspace which has property C?

Question 2. Let $f: X \to Y$ be a proper open and closed surjective mapping. If Y has property C and if $f^{-1}(y)$ has property C for every $y \in Y$, then must X also have property C?

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HOMOLOGY SPHERE DECOMPOSITIONS YIELDING GENERALIZED MANIFOLDS

David Snyder

Let G denote an uppersemicontinuous decomposition of an $(n + k) -$ manifold M into compacta each having the shape of a homology n-sphere. What can be said about the decomposition space M/G? Is it an ANR; k-gm; k-manifold?

PAST RESULTS:

1) (Coram-Duvall) If $M=S^3$ and $n=1$, then $M/G \cong S^2$.

2) (Daverman) If $k=1$, then M/G is a 1-manifold.

3) (Daverman-Walsh) If $k=2$ and M is orientable then M/G is a 2-manifold.

4) (Daverman-Walsh) If $2<k\leq n$ and M is orientable, then M/G is a k-gm.

N.B. These last two results hold for more general manifold decompositions.

5) (Lacher) If $f:S^{2n+1} \rightarrow N$ is a n-sphere mapping onto the manifold N, then N is a homotopy n-sphere (if no fiber of f is trivial).

6) (Coram-Duvall) Such a k-sphere mapping is an approximate fibration off a finite set of points $F \subset N$.

n-winding functions:

Given $b \in B = M/G$, let U be a nbhd of b such that $p^{-1}U$ retracts to $p^{-1}b$ (where p is the decomposition map) via $r_b\colon p^{-1}U \rightarrow p^{-1}b$. For each $y \in U$ define $\alpha_b(y) = |(r_b|p^{-1}y)^*(1)|$ where $(r_b|p^{-1}y)^*\colon \underset{\mathbb{Z}}{\underset{\parallel}{H^n(p^{-1}b;\mathbb{Z})}} \rightarrow \underset{\mathbb{Z}}{\underset{\parallel}{H^n(p^{-1}y;\mathbb{Z})}}$. We say the decomposition

G is non-degenerate if for every $b \in B = M/G$, $\alpha_b \neq 0$ on a nbhd of b.

The Leray Sheaf $\mathcal{H}^n[p]$ of the decomposition map is a topological space with map $\Pi:\mathcal{H}^n[p] \to B$ satisfying:

1) $\Pi^{-1}b = \mathbb{Z}$ with the discrete topology $= H^n(p^{-1}b;\mathbb{Z}) = \varinjlim H^n(p^{-1}U;\mathbb{Z})$.

2) The group operations are continuous.

3) Π is a local homeomorphism.

Given $Y \subset B$, a section over Y into $\mathcal{H}^n[p]$, is a continuous map $\sigma:Y \to \mathcal{H}^n[p]$ so that $\Pi \circ \sigma = \mathcal{I}dy$.

The set of sections over Y forms a group, by 1) and 2).

$\mathcal{H}^n[p]$ is locally constant at $b \in B$ if $\exists$ nbhd U of b $\Pi^{-1}U \cong U \times \mathbb{Z}$ and $\Pi =$ projection to the first factor.

Proposition G a homology n-sphere decomposition of the manifold M^{n+k}.

Then:

1) G is nondegenerate $\Leftrightarrow$ $\mathcal{H}^n[p]$ is T_2.

2) α_b is continuous at $b \Leftrightarrow \mathcal{H}^n[p]$ is locally constant at b.

Proposition (Dydak-Walsh)

$\mathcal{H}^n[p]$ is locally constant over an open dense subset C of B.

Set $F = B \backslash C$. The Leray spectral sequence yields

Proposition 1 For each $U^{open} \subset B$ there is a l. e. s. $\cdots \to \check{H}^i(U;\mathbb{Z}) \to \check{H}^i(p^{-1}U;\mathbb{Z}) \to \check{H}^{i-n}(U;\mathcal{H}^n[p]) \to \cdots$

Proposition 2 For each nbhd U of $x \varepsilon B$ there is a l.e.s. $\cdots \to \check{H}^i(U,U - x;\mathbb{Z}) \to \check{H}^i(p^{-1}(U,p^{-1}(U - x);\mathbb{Z}) \to \check{H}^{i-n}(U,U - x; \mathcal{H}^n[p]) \to \cdots$

Corollary 1

TFAE:

1. B is $i-\mathrm{clc}_{\mathbb{Z}}$ at b.

2. B is $(i - n - 1) - \mathrm{clc}_{\mathcal{H}^n[p]}$ at b.

Corollary 2

B is $\mathrm{clc}_{\mathbb{Z}}^{n}$

Corollary 3

If $F = \phi$ then B is a generalized manifold. (if dim $B < 0$)

Corollary 4

$B - F$ is a generalized manifold (if dim $B < \infty$)

Proposition

If G is a non-degenerate homology n-sphere decomposition of the $(2n + 1)$-manifold M and dim $B<\infty$ then $\check{H}^i(B,B-\{b\};\mathbb{Z}) = \begin{cases} \mathbb{Z} & i = n+1 \\ 0 & \text{otherwise} \end{cases}$ for each $b \varepsilon B$.

Theorem

If G is a non-degenerate homology n-sphere decomposition of M^{2n+1} and dim $B<\infty$, then F is locally finite and B is a generalized manifold.

This theorem extends a result of Daverman-Walsh [DW Theorem 3.1].

Questions

1) Is the theorem true if the hypothesis "non-degenerate" is omitted?

2) Is there a u.s.c. decomposition of a orientable manifold into [compacta having the shape of] close orientable n-manifolds so that the decomposition space is not an ANR?

3) Is there a u.s.c. decomposition of a closed $(2n + 2)$-manifold into homology n-spheres so that the decomposition space is not a generalized manifold?

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Perfect Subgroups of Locally Indicable Groups

by

F. C. Tinsley

Background:

Daverman and Tinsley showed that if G is a finitely presented group and $P<G$ is a non-trivial finitely generated perfect subgroup, then there exists an acyclic map of closed n-manifolds ($n \geq 5$) $f:M^n \to N^n$ such that $\pi_1(M^n) \cong G$, $\pi_1(N^n) \cong G/\langle P \rangle$, and Map($f$), the mapping cylinder of f, embeds in an $(n+1)$-manifold (Da-Ti, Theorem 5.2). We investigate whether the condition that P be finitely generated is, indeed, necessary.

Group Theoretic Formulation:

The topology of acyclic maps yields the following group theoretic characterization. It depends, in part, on the notion of movability of a compactum (see Da-Ti, p.345, for example).

Fact: Suppose $G=\pi_1(M)$ where M is a closed n-manifold ($n \geq 5$). There exists an acyclic map of closed manifolds $f:M \to N$ with $\ker(f_\#) \neq 1$ and with $f^{-1}(y)$ nearly 1-movable for each y N if and only if there is an infinite sequence of homomorphisms:

$$G \leftarrow G_1 \leftarrow G_2 \leftarrow G_3 \leftarrow \dots \leftarrow \dots \leftarrow$$

where each G_i is finitely presented, each $G_i \leftarrow G_{i+1}$ is non-trivial, each $G_i \leftarrow G_{i+1}$ induces the trivial map $H_k(G_i;Z) \leftarrow H_k(G_{i+1};Z)$ for $k=1,2$, and for each i there is a j such that for every k G_j is contained in the normal closure of G_k in G_i.

Question: Is there such a group G which contains no finitely generated perfect subgroups?

Whitehead's Conjecture:

In a sequence of papers from 1979 to the present Howie investigat[e] the behav or of perfect subgroups under certain group theoretic operations while working on the following:

Question: (Whitehead, 1941) Is every subcomplex of an aspherical 2-complex itself aspherical?

Howie modifies a result of Adams (Ad) to obtain:

Theorem: (Ho, Cor. to Theorem A) Let L be a subcomplex of the aspherical 2-complex M, and let $i:L \to M$ be the inclusion map. If z is a 0-cell of L such that $\pi_2(L,z)\neq 0$, then the kernel of the induced map $i_\#:\pi_1(L,z) \to \pi_1(M,z)$ contains a non-trivial, finitely generated, perfect subgroup P.

Howie considers the following classes of groups.

$\mathfrak{F}$ = {G|G contains no non-trivial perfect subgroups}
$\mathfrak{U}$ = {G|G contains no non-trivial finitely generated perfect subgroups}

Clearly, $\mathfrak{F} \subset \mathfrak{U}$. He reformulates Whitehead's question in this context.

Question: If $G=\pi_1(K,z)$ where K is an aspherical 2-complex, then is $G\in\mathfrak{U}$?

The Class $\mathfrak{L}$:

We formulate a third class of groups.

Definition: G has property * if there is a closed n-manifold M with $\pi_1(M)=G$ and an acyclic map $f:M \to N$ with N an n-manifold, $\ker(f_\#)\neq 1$, and $f^{-1}(y)$ nearly 1-movable for each $y\in N$.

$\mathfrak{L}$ = {G|G does not satisfy property *}

If we restrict our attention to finitely presented groups, then clearly $\mathfrak{F}\subset\mathfrak{L}$. Also, from the result of Daverman and Tinsley, $\mathfrak{L} \subset \mathfrak{U}$.

Question: Is $\mathfrak{U} \subset \mathfrak{L}$?

If so, then the only acyclic maps of manifolds arise when the fundamental group of the source contains a finitely generated perfect subgroup.

Standard Group Constructions:

To better understand these classes, we consider how they behave under the standard constructions of amalgamated procucts and HNN extensions. Howie proves the following for $\mathcal{F}$ and $\mathcal{U}$.

Theorem: (Ho, Theorem E) The classes $\mathcal{F}$ and $\mathcal{U}$ are closed under split amalgamated free products.

Corollary: The class $\mathcal{U}$ is closed under split HNN extensions.

He gives an example to show that neither is closed under general amalgamated free products even when the amalgamated subgroup is free.

Example: Take two copies of the group

$$\langle x_i, y_i, z_i \mid x_i = x_i^{-1} y_i^{-1} x_i y_i,\ y_i = y_i^{-1} z_i^{-1} y_i z_i \rangle \quad i = 0,1$$

and amalgamate $x_0 \simeq z_1$ and $z_0 \simeq x_1$.

The original group contains no perfect subgroups. However, the result is Higman's group, a finitely generated perfect group.

Locally Indicable Groups:

A locally indicable group is one for which each finitely generated subgroup admits an epimorphism to Z. It follows easily that a locally indicable group cannot contain any finitely generated perfect subgroups.

Question: If G is a finitely presented locally indicable group, then is $G \in \mathcal{L}$?

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THE LOCAL CONSTANCY OF THE ORIENTATION SHEAF OF A HOMOLOGY MANIFOLD: AN EASY PROOF

by

J. Dydak and J. Walsh

A result that establishes that locally finitely generated presheaves, on complete spaces, with mutually isomorphic and finitely generated stalks induce sheaves that are locally constant on a dense open set is combined with standard arguments involving the Mayer–Vietoris sequence to prove the following.

Theorem. Let n be a positive integer and suppose that X is a locally compact metrizable space satisfying:

a) X is homologically locally connected with respect to a principal ideal domain R;

b) there is a finitely generated R–module M such that, for each $x \in X$,

$$H_n(X, X - \{x\}; R) \simeq M;$$

c) the homology sheaves $\mathcal{H}_{n+1}$ and $\mathcal{H}_{n-1}$ are locally constant and the stalks of $\mathcal{H}_{n+1}$ are finitely generated.

Then the n–th homology sheaf $\mathcal{H}_n$ is locally constant.

A corollary is the result originally proved by Bredon that the orientation sheaf of a homology manifold is locally constant.

A NOTE ON THE WHITEHEAD CONTRACTIBLE 3-MANIFOLD

David G. Wright

This paper is the summary of an expository talk given at the Fourth Annual Western Regional Miniconference on Geometric Topology held at Oregon State University, Corvallis, Oregon on June 18-20, 1987.

1. Introduction.

Let M be a closed, orientable, irreducible 3-manifold such that $\pi_1(M)$ is infinite. One easily checks that the univeral covering space is a contractible open 3-manifold. For all known M, the universal covering space must be R^3. McMillan and Thickstun [M-T] pointed out that there must be contractible open 3-manifolds that are not universal covering spaces for such manifolds because there are uncountably many contractible open 3-manifolds and only countably many closed 3-manifolds. Recently, Robert Myers [M] gave specific examples of contractible open 3-manifolds that are not covering spaces. In particular he showed that the Whitehead contractible 3-manifold [Wh] is not a covering space. We do not prove this result, but we give a proof of what Myers calls his "key insight" for the Whitehead manifold.

2. Some Basic Facts.

The Whitehead manifold W is the union $\bigcup T_i$, $-\infty<i<\infty$, of tori such that T_i is a Whitehead link in T_{i+1} as shown in Figure 1. Let $X = \bigcap T_i$, $-\infty<i<\infty$. We call X a *core* for the Whitehead manifold.

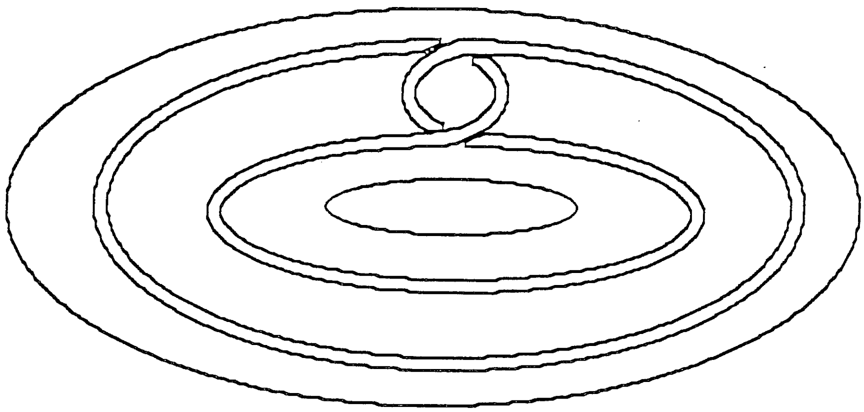

Figure 1

We list 3 basic facts about the Whitehead manifold. The proofs appear in [W]. The proofs for 1 and 2 can be found in [M].

Fact 1. The manifold W - X is irreducible; i.e., every bicollared 2-sphere bounds a 3-ball.

Fact 2. For each i, the boundary of T_i is incompressible in W - X; i.e., the inclusion from $\pi_1(\text{Bd } T_i)$ into $\pi_1(W - X)$ is a monomorphism.

Fact 3. If loops γ_1 and γ_2 are contained in Bd T_i and Bd T_j, respectively for $i \neq j$ and if γ_1 and γ_2 are homotopic in W - X, then γ_1 and γ_2 are both inessential in W - X.

3. The Key Insight.

Theorem. If C is a continuum in the interior of T_i that can be isotoped into the complement of T_{i+1} by an isotopy of W so that the track of C misses the core X, then C lies in a 3-ball of W.

Proof. We suppose that C does not lie in a 3-ball of W. Let F_t be an isotopy where F_0 equals the identity and $F_1(C)$ misses T_{i+1}. By the Covering Isotopy Theorem [E-K], [C], we may assume that X is fixed under the isotopy. Furthermore, we assume that $F_1(\text{Bd } T_i)$ is in general position with Bd T_{i+1}. Let J be a simple closed curve in $F_1(\text{Bd } T_i) \cap \text{Bd } T_{i+1}$. Then the isotopy shows that J is homotopic to the curve $F_1^{-1}(J)$ which lies in T by a homotopy which misses X. Hence by Facts 2 and 3, we see that J must bound a disk in both $F_1(\text{Bd } T_i)$ and Bd T_{i+1}. Now choose a curve of $F_1(\text{Bd } T_i) \cap \text{Bd } T_{i+1}$ that is an innermost curve in Bd T_{i+1}. The union of the disks bounded by J in $F_1(\text{Bd } T_i)$ and Bd

T_{i+1} forms a 2-sphere that bounds a 3-ball in W - X by Fact 1. If C fails to lie in a 3-ball, then so does $F_1(C)$. Hence, there is an isotopy, fixing $F_1(C)$ and X, that reduces the number of intersection curves of $F_1(\mathrm{Bd}\ T_i) \cap \mathrm{Bd}\ T_{i+1}$ By an inductive argument, we may assume that $F_1(\mathrm{Bd}\ T_i) \cap \mathrm{Bd}\ T_{i+1} = \emptyset$. So $F_1(\mathrm{Bd}\ T_i)$ is either contained in the interior of T_{i+1} or $F_1(\mathrm{Bd}\ T_i)$ is contained in the complement of T_{i+1}. We now show that both of these cases are impossible.

If $F_1(\mathrm{Bd}\ T_i)$ is contained in the interior of T_{i+1}, then $F_1(T_i)$ is contained in the interior of T_{i+1}. But this contradicts the fact that $F_1(C)$ is contained in the complement of T_{i+1}.

If $F_1(\mathrm{Bd}\ T_i)$ is contained in the complement of T_{i+1}, then the isotopy shows that any curve on Bd T_i is homotopic to a curve in the complement of T_{i+1} by a homotopy that misses X. Let $H: S^1 \times I \to W - X$ be a homotopy between a non-trivial curve H_0 in Bd T_i and a curve H_1 in the complement of T_{i+1}. We may suppose that the homotopy is in general position with respect to Bd T_{i+1}. One of the simple closed curve components of $H^{-1}(\mathrm{Bd}\ T_{i+1})$ must separate the boundary components of the domain of H. Restricting H to the proper subset of this annulus gives a homotopy in W - X between H_0 and a loop in Bd T_{i+1}. Now Fact 2 implies that H_0 is a trivial curve H_0 in Bd T_i, contradicting the choice of H_0.

The above contradiction stemmed from the supposition that C does not lie in a 3-ball of W, and our theorem is proved.

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Department of Mathematics
Brigham Young University
Provo, Utah 84604

PROBLEMS
1987 WESTERN WORKSHOP IN GEOMETRIC TOPOLOGY

1. The cell-like dimension raising map problem. Is there a cell-like map defined on a finite-dimensional compact metric space which raises dimension?

2. The Hilbert-Smith conjecture. A_p cannot act effectively on any topological manifold. (See the write up of R. Edwards' talk for more information.)

3. The Resolution Problem. Do all ENR homology manifolds have cell-like resolutions ?

4. (R. Edwards) Suppose that M is an open contractible n-manifold that admits degree 1 maps arbitrarily close to infinity. Is M homeomorphic to $\mathbb{R}^n$?

5. (D. Wright) Sticky Cantor Set Problem. Is there a Cantor set C in E^n and an $\epsilon>0$ such that for every ϵ-homeomorphism $h:E^n \to E^n$, $C \cap h(C) \neq \varnothing$?

6. (R. Daverman) Are homotopy equivalent homology spheres homeomorphic?

7. (R. Edwards) Recall that A_p acts freely (indeed principally) on $\underset{k}{*}A_p$, the k-fold join of A_p. If $\ell > k$, is it possible to have an A_p equivariant map from $\underset{\ell}{*}A_p$ to $\underset{k}{*}A_p$? (Surely not, but how about a nice proof?)

8. (R. Edwards) For fixed n and for ℓ sufficiently large with respect to n, is it true that for any map $\phi:\underset{\ell}{*}A_p \to \mathbb{R}^n$, ϕ necessarily fails to embed some A_p orbit, or that ϕ necessarily maps some A_p orbit to a point?

9. (R. Edwards) Suppose E is any finite dimensional space on which A_p acts freely with finite dimensional quotient B. Suppose X is a compactum and f is any map from X to B. Let W be the pullback of E by f; hence A_p acts freely on W with quotient X. If X is locally 1-connected, then this A_p action on W is in fact principal, i.e., X has an open cover $\{\mathcal{U}_1, \mathcal{U}_2, \ldots \mathcal{U}_p\}$ such that over each $\mathcal{U}_i$, W looks like $\mathcal{U}_i \times A_p$ with the obvious product action. Can one bound p solely in terms of the dimension of B? (For example, p = dimension(B) + 1.) It turns out that one may as well assume that E is a Bestvina (Menger) manifold of some finite dimension, on which A_p acts freely, since such spaces serve as classifying spaces for free A_p actions on compacta having finite dimensional quotients.

10. (R. Edwards) Can A_p act freely on

a. some compact ANR, that is (after crossing with I^∞), on some compact Hilbert cube manifold? (Recall that A_p acts on the Hilbert cube fixing a single point, and freely off of that point. Hence A_p acts freely on $I^\infty \setminus$ point which is homeomorphic to $I^\infty \times [0, \infty)$.)

b. some compact contractible (possibly finite dimensional or with finite dimensional quotient) space ? (Recall the Bestvina-Edwards example shows that A_p acts freely on some two-dimensional cell-like compactum.)

c. some compactum which is locally contractible or perhaps locally n-connected for all n?

11. (R. Edwards) Can one have an open map from a compact manifold onto a space such that all of the point inverses are one-dimensional solenoids, and such that the map raises dimension by any pre-specified amount? (If solenoid is replaced by Cantor set, then there are such maps by work of Walsh and Wilson.)

12. (R. Edwards) Suppose that X is a cell-like compactum such that ΣX is contractible. Must X be contractible? Note that X has the singular homology of a point by a Mayer-Vietoris argument, so X must be path connected.

13. (J. Lamoreaux) If C is a Cantor set which is strongly homogeneously embedded in E^3, then must C be tame?

14. (J. Lamoreaux) Is there a Cantor set in E^3 which is 2-homogeneously embedded, but not 3-homogeneously embedded?

15. (J. Cannon) Consider two figure eight knots in S^3 separated by a 2-sphere. Perform surgery on each to obtain a connected sum of two hyperbolic homology spheres. Can the group be killed by a single element ?

16. (J. Hoste) Can every homology sphere be obtained by surgery on a knot ?

17. (D. Wright) If M is a contractible 3-manifold, $M \neq \mathbb{R}^3$, then can M cover a closed manifold (or any manifold not equal to itself) ?

18. (M. Bestvina) Suppose that M is a K(G,1)-manifold where G is finitely generated. Does M have finitely many ends? What if M is covered by $\mathbb{R}^n$?

19. (J. Hoste) Can a connected sum of three different manifolds be obtained by surgery on a knot?

20. (R. Edwards) Can you construct open maps having 1-dimensional solenoids for point inverses which raise dimension?

21. (R. Daverman) If G is a decomposition of a manifold M into k-dimensional submanifolds, then is the dimension of M/G less than infinity ? This is not known when k=1, even for continuous decompositions.

22. (R. Daverman, attributed to S. Ferry) If complexes P and Q are cell-like equivalent through LC^1 spaces, the are they simple homotopy equivalent ?

24. (R. Daverman) Given any closed orientable n-manifold N, does there exist a usc decomposition g of some (n+k)-manifold M (with k unrestricted) such that $p:M \rightarrow M/G$ fails to be an approximate fibration?

26. (R. Daverman) If G is an usc decomposition of a 5-manifold M into copies of a 3-manifold N other than a surface bundle over S^1, is $p:M \rightarrow M/G$ an approximate fibration? What if N is known to be aspherical?

27. (R. Daverman) If G is an usc decomposition of a 4-manifold M into copies of a 2-manifold $N \not\approx S^1 \times S^1$, can $p:M \rightarrow M/G$ be approximated by fibrations?

28. (R. Daverman) If $f:S^4 \rightarrow S^4$ is a surjective map for which the closure of the image of the nondegeneracy set is 0-dimensional, is f cell-like?

29. (P. Bowers) Is Borsuk's conjecture true? Can every bonded subset of $\mathbb{R}^n$ be partitioned into n+1 pieces with strictly smaller diameter? This is known for $n \leq 3$. Is there a polynomial bound on the number of such subsets needed?

30. (J. Walsh) Is there a finite-dimensional homogeneous AR?

31. (D. Snyder) Other than I^∞, is there a compact homogeneous space which is homeomorphic to its cone?

32. (D. Snyder) Is there a usc decomposition of a orientable manifold into [compacta having the shape of] close orientable n-manifolds so that the decomposition space is not an ANR?

33. (D. Snyder) Is there a usc decomposition of a closed (2n + 2)-manifold into homology n-spheres so that the decomposition space is not a generalized manifold?