

NOTE: the following problems are NOT in order by section. They are mixed, as they will be on the exam. Answers are at the end of this handout together with the section number of the material.

Remember to **show setup** of each probability problem using **probability notation**, and **all calculator steps** as you would on the exam. You need only write a **summary statement** when requested. Answers are at the end of this handout.

Remember to review definitions for possible true/false or multiple choice questions.

- Heights of male college students are normally distributed with a mean of 69 inches and a standard deviation of 3.1 inches. **(a)** Find the probability that a male college student selected at random is taller than 71 inches. **(b)** Find the probability that the height of a male college student is within 1.5 inches of the population mean. Write each answer as a complete sentence.
- Suppose that $P(A) = 0.260$, $P(B) = 0.150$, and $P(A \text{ and } B) = 0.039$. Are events A and B mutually exclusive? Use a sentence or two to explain your answer.
- A recent study reported that 67% of gifted students are bullied by the time they get to the eighth grade. If a randomly selected group of 20 eighth grade gifted students are surveyed, **(a)** what is the probability that at most 16 of them have been bullied? Write a sentence to interpret your answer. **(b)** What is the mean number of students bullied? **(c)** What is the standard deviation?
- The random variable x is the number of houses sold in one month by a realtor. Its probability distribution is shown below. Write in probability notation and find the probability that the number of houses sold is: **(a)** at least four, **(b)** between one and three, inclusive. **(c)** Find the expected value of the number of houses sold in one month. **(d)** Find the standard deviation of the number of houses sold in one month. **(e)** Find the probability that the number of houses sold in a randomly selected month will be within one standard deviation of the mean.

| x | $P(x)$ |
|-----|--------|
| 0 | 0.09 |
| 1 | 0.10 |
| 2 | 0.14 |
| 3 | 0.17 |
| 4 | 0.20 |
| 5 | 0.16 |
| 6 | 0.14 |

- Suppose that $P(A) = 0.260$, $P(B) = 0.150$, and $P(A \text{ and } B) = 0.039$. Find $P(B | A)$.

6. A machine fills containers with a mean weight per container of 16.0 ounces. Weights are normally distributed. If no more than 5% of the containers weigh less than 15.8 ounces, what is the standard deviation of the weights? Give your answer to the hundredths place.
7. A family has three children. Construct the sample space of genders for the children in this family.
8. Draw a sketch and find $Z_{0.007}$.
9. You are to survey randomly selected students at TCU with regard to the amount of time they studied this week. (a) Identify the random variable of interest, (b) state whether it is discrete or continuous, and (c) list its possible values.
10. A 2003 study found that medical residents' mean number of hours worked in a week is 81.7. Suppose the number of hours worked per week is normally distributed with a standard deviation of 6.9 hours. A large group of medical residents are randomly sampled. (a) What is the probability that a randomly selected medical resident works more than 80 hours in a week? (b) What is the probability that a randomly selected medical resident works less than 60 hours per week? (c) What is the probability that the weekly work time of a randomly selected medical resident is within 4 hours of the reported mean?
11. Suppose that $P(K) = 0.31$, $P(K \text{ and } L) = 0.12$, and $P(K \text{ or } L) = 0.74$. Find $P(L)$.
12. A set of patient records classifies patients by Rh factor and by blood group, as shown below. (a) Suppose a record is selected at random. Find the probability that the patient has an **A+** blood type. (b) Suppose two records are chosen without replacement. Find the probability that both patients have a **B+** blood type.

| | O | A | B | AB | Σ |
|----------|----|----|----|----|----------|
| Rh+ | 41 | 37 | 10 | 6 | 94 |
| Rh- | 9 | 7 | 5 | 3 | 24 |
| Σ | 50 | 44 | 15 | 9 | 118 |

13. You are to survey randomly selected students at TCU with regard to how many classes they are taking this semester. (a) Identify the random variable of interest, (b) state whether it is discrete or continuous, and (c) list its possible values.
14. A kindergarten class has 14 girls and 8 boys. If two children are randomly selected from the class, one at a time without replacement, what is the probability that one of each gender is selected?
15. The amount of time scuba divers stay under water in Lake Clearview is normally distributed, with a mean of 70 minutes. If 5% of divers remain under water for more than 85 minutes, find the standard deviation of diving times at Lake Clearview to the nearest minute.
16. Suppose that $P(M \text{ or } N) = 0.77$, $P(N) = 0.35$, and $P(M \text{ and } N) = 0.14$. Find $P(M)$.

17. It is estimated that 41% of senior citizens did some type of volunteer work last year. If ten senior citizens are selected at random, what is the probability that at least two senior citizens did some type of volunteer work? Write a sentence to interpret your answer.

18. According to the U.S. Department of Agriculture, the mean calorie intake of U.S. males 20 to 39 years old is 2716, with a standard deviation of 72.8. A nutritionist analyzes a sample of U.S. males in this age group. (a) Write in probability notation and find the probability that the calorie intake of one of these men will lie within 20 calories of the population mean. (b) Write in probability notation and find the probability that calorie intake of one of these men will differ from μ by more than 50 calories. Write each answer as a complete sentence.

19. An allergy study gave a sample of patients an allergy drug, a placebo, or no treatment (the control group). The following table shows whether or not the patient reported improvement in allergy symptoms over the following two weeks. If a patient is selected at random, find the probability that the patient (a) received the placebo and showed improvement, (b) is not in the control group, (c) is in the control group or showed NO improvement.

| | Allergy drug | Placebo | Control | Σ |
|----------------|--------------|---------|---------|----------|
| IMPROVEMENT | 65 | 42 | 31 | 138 |
| NO IMPROVEMENT | 55 | 58 | 49 | 162 |
| Σ | 120 | 100 | 80 | 300 |

20. An allergy study gave a sample of patients an allergy drug, a placebo, or no treatment (the control group). The following table shows whether or not the patient reported improvement in allergy symptoms over the following two weeks. (a) If a patient is selected at random, find the probability that the patient received the placebo given that the patient showed NO improvement. (b) Given that the patient is not in the control group, find the probability that the patient showed improvement.

| | Allergy drug | Placebo | Control | Σ |
|----------------|--------------|---------|---------|----------|
| IMPROVEMENT | 65 | 42 | 31 | 138 |
| NO IMPROVEMENT | 55 | 58 | 49 | 162 |
| Σ | 120 | 100 | 80 | 300 |

21. When M&M/Mars surveyed children in Japan to learn what color they would like to have added to packages of M&Ms candies, 16% of the children chose purple. Suppose 3 Japanese children are selected at random. Let the random variable x be the number of children who vote for purple M&Ms. (a) Construct a probability distribution for x . (b) Find the mean and standard deviation of the probability distribution.

22. Suppose that $P(D) = 0.33$, $P(E) = 0.45$, and $P(D \text{ or } E) = 0.69$ (a) Find $P(D \text{ and } E)$. (b) Are events D and E mutually exclusive? JUSTIFY YOUR ANSWER.

23. (a) What z-scores bound the middle 75% of a standard normal distribution? (b) Given that $P(z > K) = 0.23$, find the value of K . (c) Given that $P(M < z < 0) = 0.48$, find the value of M .

PROBLEMS 24 – 32 ARE MULTIPLE CHOICE:

24. Given events A and B with $P(A) = 0.69$, $P(B) = 0.52$, and $P(A \text{ and } B) = 0.38$. Find the probability of A or B.

- (a) 0.359 (b) 0.731 (c) 0.830 (d) 1.210 (e) None of these

25. Find the area under the z curve to the right of -0.05.

- (a) -0.520 (b) 0.520 (c) 0.408 (d) 0.692

26. Find the area under the z curve to the left of -1.76.

- (a) -0.539 (b) 0.539 (c) 0.039 (d) 0.961

27. Find the area under the z curve between -0.58 and 3.04.

- (a) 0.718 (b) 0.493 (c) 0.280 (d) 0.282

28. Find the area under the z curve between 0.31 and 3.23.

- (a) 0.621 (b) 0.498 (c) -0.378 (d) 0.378 (e) 0.379

29. Find the following z value: $z_{0.06}$.

- (a) 0.0239 (b) 0.4761 (c) 1.55 (d) 1.56

30. In testing a new drug, researchers found that 94% of all patients using it have no side effects. If a random sample of 55 patients using the drug is selected, what is the expected number of patients who will experience side effects.

- (a) About 2 patients (b) About 3 patients (c) About 28 patients
(d) About 52 patients (e) None of these

31. In testing a new drug, researchers found that 6% of all patients using it have side effects. If a random sample of 55 patients using the drug is selected, what is the standard deviation of patients who will experience side effects.

- (a) About 2 patients (b) About 3 patients (c) About 28 patients
(d) About 52 patients (e) None of these

32. If two events have no outcomes in common, they are:

- (a) independent. (b) complements (c) dependent (d) mutually exclusive
(e) None of these

33. Suppose of the patients having a certain disease, four are chosen at random. The probability that a single patient is cured by a new medication is 0.8.

- (a) Find the probabilities that zero through four of the patients (out of four) are cured.
(b) Find the mean and standard deviation of the number of patients who are cured.
(c) What is the probability that the number of patients who are cured by this new medication is within one standard deviation of the mean (inclusive).

34. Two cold tablets are accidentally put into a bottle with fifteen aspirin tablets. The two types of tablets have the same shape. (a) If two tablets are selected, one at a time without replacement, find the probability that both are the SAME type of tablet. Now suppose the tablets are put back in the bottle. (b) If two tablets are selected, one at a time without replacement, find the probability that one of EACH type of tablet is selected.

- 35.** Records show that 91.6% of Topper's Pizza deliveries are on time, i.e. they take less than 30 minutes. If seventeen pizzas are ordered at random times to different locations, find the probability that at most fifteen pizzas are delivered on time. Write a sentence to interpret your answer.
- 36.** A radar unit is used to measure the speed of automobiles on an expressway during rush-hour traffic. The speeds of the automobiles are normally distributed with a standard deviation of 6.9 miles per hour. Find the mean of all speeds if 5% of the automobiles travel faster than 75 miles per hour. Give your answer to the tenths place.
- 37.** The random variable x represents the number of students who are absent on a given day from a class of 25 students. The probability distribution of x is shown below. **(a) Write in probability notation and find** the probability that at most three students are absent. **(b)** Find the mean number of absent students. **(c)** Find the standard deviation of absent students.

| x | $P(x)$ |
|-----|--------|
| 0 | 0.06 |
| 1 | 0.18 |
| 2 | 0.32 |
| 3 | 0.22 |
| 4 | 0.14 |
| 5 | 0.08 |

- 38.** According to a recent survey, 38% of Los Angeles drivers believe that driving on the shoulder of the road is acceptable driving behavior. Suppose five Los Angeles drivers are selected at random. Let the random variable x be the number of drivers who believe that driving on the shoulder of the road is acceptable. Construct a probability distribution for x .
- 39.** For a normal distribution of a random variable x with mean μ and standard deviation σ , find $P(\mu + \sigma < x < \mu + 3\sigma)$.
- 40.** Suppose that $P(A) = 0.19$, $P(B) = 0.30$, and $P(A | B) = 0.25$. Find $P(A \text{ or } B)$.

PART II: Here are some problems for more practice:

PROBLEMS 41 – 46 ARE MULTIPLE CHOICE:

- 41.** The area under the standard normal curve to the right of $z = 0.65$ is:
 (A) 0.258 (B) 0.385 (C) 0.637
 (D) 0.742 (E) None of these
- 42.** Find the area under the z curve between -0.58 and 3.04 .
 (A) 0.7178 (B) 0.4931 (C) 0.2798
 (D) 0.2822 (E) None of the above.

43. Find the following z value: $z_{0.06}$.
- (A) 0.0239 (B) 0.4761 (C) 1.55
(D) 1.56 (E) None of the above.
44. Given events A and B with $P(A) = 0.58$, $P(B) = 0.36$, and $P(A \text{ and } B) = 0.12$. Find $P(A \text{ or } B)$ to two decimal places.
- (A) 0.21 (B) 0.82 (C) 0.94
(D) 1.06 (E) None of these
45. Studies show that 19% of children entering kindergarten do not have all of their necessary immunizations. If random sample of 75 children entering kindergarten is selected, what is the expected number of children who do have all of their necessary immunizations?
- (A) About 3 children (B) About 12 children
(C) About 14 children (D) About 61 children (E) None of these
46. A survey reports that 6% of Americans who are hospitalized contract an infection while in the hospital. If 90 American hospital patients are selected at random, find the standard deviation of patients who contract an infection while in the hospital.
- (A) About 2 (B) About 5 (C) About 85
(D) Cannot be computed with this information (E) None of these
47. In a certain pediatrician's practice, 21% of babies under three months of age get colic. The pediatrician randomly selects 20 babies under three months of age.
- (a) Write in probability notation and find the probability that at least 3 of those babies get colic.
(b) Write a complete sentence to summarize your answer.
48. A parts store sells both new and used parts. Suppose that a part is selected at random. The probability that the part is used is 0.59. The probability that the part is both used and defective is 0.04. The probability that the part is either used or defective is 0.63. (a) Find the probability that the part is defective. (b) Given that the part is used, find the probability that the part is defective.
49. A cooler filled with ice and soft drinks contains 8 cans of cola and 5 cans of lemon-lime drink. When you get two drinks, one for yourself and one for a friend, you just push your hands into the ice to grab the cans, rather than looking at each can. (a) Find the probability that one of each type of drink is selected.
Now suppose that we are starting over. Once again, you pick two of the cans at random, one at a time without replacement. (b) Given that the first can chosen is a cola, what is the probability that the **second** can is lemon-lime?
50. Draw a sketch and find the two z-scores that bound the middle 73% of a normal distribution.
51. Using the standard normal distribution (mean of 0 and a standard deviation of 1), let $P(z < D) = 0.28$. Draw a sketch, and find the value of D.
52. Suppose that $P(M \text{ or } N) = 0.62$, $P(M) = 0.41$, and $P(M \text{ and } N) = 0.17$.
(a) Find $P(N)$. (b) Find $P(N | M)$. (c) Are M and N mutually exclusive events? Explain your answer.

53. The random variable x has the probability distribution given below. (a) Find the mean of the distribution. (b) Find the standard deviation of the distribution. (c) What is the probability that x is within two standard deviations of the mean?

| x | $P(x)$ |
|-----|--------|
| 0 | 0.03 |
| 1 | 0.05 |
| 2 | 0.26 |
| 3 | 0.41 |
| 4 | 0.17 |
| 5 | 0.07 |
| 6 | 0.01 |

54. Nine percent of men have the type of colorblindness in which they cannot distinguish between the colors red and green. If sixteen men are to be chosen at random, write in probability notation and find the probability that at most four have this type of colorblindness.

55. Using the standard normal distribution, let $P(-K < z < K) = 0.90$. Draw a sketch, and find the value of K .

56. A group of individuals were classified by their gender and their eye color, as shown in the table below. If an individual is selected at random, find the probability that the individual (a) is both male and has blue eyes, (b) does not have brown eyes, (c) is female or has green eyes. (d) Given that the individual is male, find the probability that his eye color is "other."

| | Brown | Blue | Green | Other | Σ |
|----------|-------|------|-------|-------|----------|
| Male | 74 | 34 | 12 | 6 | 126 |
| Female | 98 | 33 | 15 | 3 | 149 |
| Σ | 172 | 67 | 27 | 9 | 275 |

57. It has been reported that 32% of users of one particular credit-card pay their bills in full each month. A random sample of 15 cardholders is to be selected. (a) Write in probability notation and find the probability that none of these cardholders pay their bills in full each month? (b) Write a complete sentence to summarize your answer.

58. Draw a sketch and find $z_{0.25}$.

59. For a normally distributed random variable x with mean μ and standard deviation σ , find $P(\mu - 2\sigma < x < \mu + \sigma)$. [HINT: Draw a sketch.]

SOLUTIONS

1. **[6.3] (a)** 0.259 There is a probability of 0.259 that a male college student selected at random is taller than 71 inches. **(b)** 0.371 There is a probability of 0.371 that the height of a male college student is within 1.5 inches of the population mean.
2. **[4.3]** Events A and B are not mutually exclusive because $P(A \text{ and } B) = 0.039$, not zero.
3. **[5.3] (a)** 0.936 There is a probability of 0.936 that at most 16 out of 20 gifted students will be bullied by the time they reach 8th grade. **(b)** $\mu = 13.4$ students **(c)** $\sigma = 2.1$ students
4. **[5.2] (a)** $P(x \geq 4) = 0.5$ **(b)** $P(1 \leq x \leq 3) = 0.41$ **(c)** expected value = $\mu = 3.33$
(d) $\sigma = 1.83$ **(e)** $P(1.50 < x < 5.16) = P(x = 2 \text{ or } 3 \text{ or } 4 \text{ or } 5) = 0.67$
5. **[4.5]** 0.15
6. **[6.3]** $\sigma = 0.12$ ounces
7. **[4.2]** $S = \{\text{fff, ffm, fmf, mff, fmm, mfm, mmf, mmm}\}$
8. **[6.2]** $z = 2.46$
9. **[5.2] (a)** $t =$ amount of time a TCU student studied this week **(b)** continuous
(c) $t = \{0, \dots, 0.5, \dots, 3, \dots, 9, \dots\}$ hours
10. **[6.3] (a)** 0.597 **(b)** 0.0008 **(c)** 0.438
11. **[4.3]** 0.55
12. **(a) [4.2]** $7/118$ or 0.059 **(b) [4.3]** $(10/118) \times (9/117) = 5/767$ or 0.007
13. **[5.2] (a)** $c =$ number of classes taken by a TCU student **(b)** discrete
(c) $c = \{1, 2, 3, 4, \dots\}$
14. **[4.2]** $16/33$ or 0.485
15. **[6.3]** $\sigma = 9$ minutes
16. **[4.3]** 0.56
17. **[5.3]** 0.959 There is a probability of 0.959 that at least two senior citizens of a randomly selected group of ten did some volunteer work last year.
18. **[6.3] (a)** $P(2696 < x < 2736) = 0.216$ There is a probability of 0.216 that that the calorie intake of a randomly selected U.S. male 20 to 39 years old will lie within 20 calories of the population mean. **(b)** $P(x < 2666 \text{ or } x > 2766) = 0.492$ There is a 49.2% chance that the calorie intake of a randomly sampled U.S. male 20 to 39 years old will differ from the population mean by more than 50 calories.
19. **(a) [4.2]** $7/50$ or 0.14 **(b) [4.2]** $11/15$ or 0.733 **(c) [4.4]** $193/300$ or 0.643

20. [4.5] (a) $58/162 = 29/81$ or 0.358 (b) $107/220$ or 0.486

21. [5.3] (a) (b) $\mu = 0.5, \sigma = 0.6$

| x | P(x) |
|---|-------|
| 0 | 0.593 |
| 1 | 0.339 |
| 2 | 0.065 |
| 3 | 0.004 |

22. [4.3] (a) 0.09 (b) No, because $P(D \text{ and } E) \neq 0$.

23. [6.2] (a) $z = \pm 1.15$ (b) $K = 0.74$ (c) $M = -2.05$

24. [4.2] c 25. [6.2] b 26. [6.2] c 27. [6.2] a

28. [6.2] d 29. [6.2] c 30. [5.3] b 31. [5.4] a

32. [4.2] d

33. [5.3] (a) (b) $\mu = 3.2, \sigma = 0.8$ (c) 0.8192

| x | P(x) |
|---|--------|
| 0 | 0.0016 |
| 1 | 0.0256 |
| 2 | 0.1536 |
| 3 | 0.4096 |
| 4 | 0.4096 |

34. [4.4] (a) $53/68$ or 0.779 (b) $15/68$ or 0.221

35. [5.3] 0.424 There is a probability of 0.424 that at most fifteen out of seventeen pizzas from Topper's pizza are delivered on time.

36. [6.3] $\mu = 63.7$ mph

37. [5.2] (a) $P(x \leq 3) = 0.78$ (b) $\mu = 2.44$ (c) $\sigma = 1.31$

38. [5.3]

| x | P(x) |
|---|-------|
| 0 | 0.092 |
| 1 | 0.281 |
| 2 | 0.344 |
| 3 | 0.211 |
| 4 | 0.065 |
| 5 | 0.008 |

39. [6.2] $P(\mu + \sigma < x < \mu + 3\sigma) = 0.157$ (or using the Empirical Rule, 0.1585)

40. [4.5] 0.415

PART II SOLUTIONS

41. [6.2] A **42. [6.2] A** **43. [6.2] C** **44. [4.3] B**

45. [5.4] D **46. [5.4] A**

47. 0.823 There is an 82.3% chance that a baby under three months of age from this pediatrician's practice will get colic.

48a. 0.08 **b. 0.068**

49a. 0.513 **b. 0.417**

50. $z = \pm 1.10$

51. $D = -0.58$

52a. 0.38 **b. 0.415** **c. No. $P(M \text{ and } N) \neq 0$.**

53a. $\mu = 2.89$ **b. $\sigma = 1.12$** **c. 0.96**

54. 0.999

55. $K = 1.64$

56a. 0.124 **b. 0.375** **c. 0.585** **d. 0.048**

57. 0.003 There is a 0.3% chance that none of the 15 cardholders sampled will pay their bills in full each month.

58. $z = 0.67$

59. 0.819