## CHAPTER 6 EXAMPLES \& DEFINITIONS

## Section 6.2

Definition: The probability distribution associated with the standard score $z$ is called the standard normal distribution (or standard normal curve, or z-curve).

## Properties of the Standard Normal Distribution:

1. The total area under the normal curve is equal to 1 .
2. The distribution is mounded and symmetric; it extends indefinitely in both directions, approaching but never touching the horizontal axis.
3. The distribution has a mean of 0 and a standard deviation of 1 .
4. The mean divides the area in half, 0.50 on each side.
5. Nearly all the area is between $z=-3.00$ and $z=3.00$.

## -Finding a probability for a normal distribution using the calculator:

$2^{\text {nd }}$ DISTR [this is the VARS key]
normalcdf [option 2]
enter values for left boundary, right boundary, $\mu, \sigma$
ENTER

Ex. A) Draw a sketch, write in probability notation and find the area under the standard normal curve between:
(a) $z=0.85$ and $z=1.41$.
(b) $z=-2.51$ and $z=-0.05$.
(c) $z=3.16$ and $z=-0.99$.

## WHAT TO DO WHEN YOUR LOW OR HIGH VALUE IS -/+ INFINITY:

[Once you have stored these values, you should not need to do so again.]
Use -1 EE 99 for low value if low is $-\infty$.
You can store this in a letter: Ex. -1 EE 99 STO ALPHA L ENTER

Use 1 EE 99 for high value if high is $+\infty$
You can store value in a letter: Ex. 1 EE 99 STO ALPHA H ENTER

Ex. B) Suppose a thermometer is manufactured so that at water's freezing temperature, the thermometers have a mean reading of $0^{\circ} \mathrm{C}$, with a standard deviation of $1^{\circ} \mathrm{C}$. Thermometer readings are normally distributed. If a thermometer is placed in freezing water, draw a sketch, write in probability notation and find the following probabilities:
(a) the probability that the thermometer reads less than $1.5^{\circ} \mathrm{C}$.
(b) the probability that the thermometer reads over $-1^{\circ} \mathrm{C}$
(c) the probability that the thermometer reads under $-2^{\circ} \mathrm{C}$.

## -Finding a $z$-value when you know a probability for a standard normal distribution using the calculator:

$2^{\text {nd }}$ DISTR [this is the VARS key]
invNorm [option 3]
enter value for Area to the left of $\mathbf{z}, \mu, \sigma$ ENTER

Ex. C) Using thermometers as in the example above, find the temperature separating the lowest $15 \%$ of the readings from the rest of the readings.

Ex. D) What z -scores bound the middle $90 \%$ of a normal distribution?

Alpha notation: $\mathrm{z}_{\alpha}$ represents that z -value which has an area of $\alpha$ to its right.
Ex. E) (a) Find $z_{0.10 .}$
(b) Find $\mathrm{z}_{0.80}$.
(c) Find $\mathrm{z}_{0.025}$.

## PRACTICE PROBLEM OVER SECTION 6.2

1. Find the following values of $z$ :
(a) $z_{0.12}$ (b) $z_{0.28}$
(c) $\mathrm{Z}_{0.85}$
(d) $\mathrm{Z}_{0.05}$

## Section 6.3

Ex. F) The mean January temperature at Icefang, Alaska is $-1.5^{\circ} \mathrm{F}$ with a standard deviation of $6.9^{\circ} \mathrm{F}$. Temperatures are known to be normally distributed. Find the probability that a January temperature selected at random is greater than $-2^{\circ} \mathrm{F}$.

Ex. G) The random variable $x=$ stress resistance for a certain type of plastic sheet, measured in pounds per square inch (psi). If $x$ is normally distributed with $\mu=30 \mathrm{psi}$ and $\sigma=0.6 \mathrm{psi}$, write in probability notation and find the probability that a plastic sheet chosen at random has a resistance: (a) of less than 28.2 psi ,
(b) within 1 psi of the mean. On part (b), write a statement to interpret the answer.

## -Finding an x-value when you know a probability for a normal distribution using the calculator:

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2 nd DISTR [this is the VARS key]
invNorm [option 3]
enter value for Area to the left of x, \mu,\sigma
ENTER
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Ex. H) The Stanford-Benet IQ test is normally distributed with a mean of 100 and a standard deviation of 16. Mensa is an organization that only allows people to join if their IQs are in the top $2 \%$ of the population. What is the lowest IQ one can have and still be eligible to join Mensa?

When using an entire population: $\quad \mathbf{z}=\frac{\mathbf{X}-\mu}{\sigma}$
Ex. I) Waiting times at a large clinic are approximately normally distributed with a standard deviation of 14.8 minutes. If only $6 \%$ of the patients at this clinic wait less than 45 minutes to see a doctor, what is the mean waiting time?

## PRACTICE PROBLEMS OVER SECTION 6.3

1. Companies who design furniture for elementary school classrooms produce a variety of sizes for children of different ages. Suppose the heights of kindergarten children are normally distributed with a mean of 38.2 inches and a standard deviation of 1.8 inches. (a) What percentage of kindergarten children should the company expect to be under 3 feet tall? Write a statement to interpret the answer. (b) In what height interval should the company expect to find the middle $80 \%$ of kindergarteners? (c) At least how tall are the biggest $5 \%$ of kindergarteners?
2. The Graduate Record Exam verbal ability section has a mean of 497 and standard deviation 115. Assume that GRE scores are normally distributed. A graduate school program in English will only admit students with GRE verbal scores in the top $30 \%$. What is the lowest GRE score they will accept?

## >> continued <<

## PRACTICE PROBLEMS OVER SECTION 6.3, continued

3. The weights of ripe watermelons grown at Mr. Kerif's farm are normally distributed with a standard deviation of 2.8 pounds. Find the mean weight of the watermelons if only $3 \%$ weigh less than 15 pounds
4. The U.S. National Center for Health Statistics reports that males over 6 feet tall between 18 and 24 years of age have a mean weight of 175 pounds. Weights are normally distributed with a standard deviation of 14 pounds. Find the probability that the weight of such a randomly selected male is within 10 pounds of the population mean. Write a statement to interpret your answer.
5. The ages of farm operators in the United States of America are normally distributed with a mean of 50 years and a standard deviation of 9 years. Find the probability that the age of a U.S. farmer sampled at random differs from the population mean by more than 5 years.
