

Section 4.1

Definition: The **Probability that an event will occur** is the relative frequency with which that event **can be expected** to occur.

TYPES OF PROBABILITY

1. **theoretical** denoted **P**
 2. **empirical** denoted **P'**
- As the sample size n increases, P' approaches P .

Section 4.2

Definition: **Sample space** – the set of all possible outcomes of an experiment. We generally denote a sample space with the letter **S**.

Definition: **Sample points** – the individual outcomes in a sample space.

Definition: **Event** – any subset of a sample space. We often denote an event with a capital letter.

Ex. A) Two boxes contain solid-colored marbles. Box 1 has marbles that are red, blue, purple, and green. Box 2 has marbles that are purple and white. One marble will be randomly selected from box 1, and then another marble will be randomly selected from box 2. **(a)** List the sample space of this experiment. **(b)** Find event A, the event that exactly one marble is purple. **(c)** Find event B, the event that at least one marble is purple.

If **each** sample point is **equally likely** to occur, we can find the probability of event A as follows:

$$P(A) = \frac{\text{number of outcomes in event A}}{\text{number of outcomes in the sample space}}$$

**** **NOTE:** With probabilities, we either reduce fractions, or give the answer as a decimal value to at least 3 decimal places. ****

Ex. B) In a statistics class, 32 students own a smart phone and 11 do not. If one of the students is randomly selected, find the probability that this student does not own a smart phone.

PROPERTIES OF PROBABILITY

1. $0 \leq P(A) \leq 1$
2. $\sum_{\text{all outcomes}} P(A) = 1$

(3.) [First we need a definition—we'll come back to this . . .]

Definition: If A is an event, \bar{A} (“**A complement**”, or “**not A**”) is the set of all sample points in the sample space that do **NOT** belong to A .

3. $P(\bar{A}) = 1 - P(A)$ also, $P(A) = 1 - P(\bar{A})$

Ex. C) A class of 49 students is surveyed, with the results below. (a) Let event M = the event that a student selected at random from the class is male. Find $P(M)$. (b) Let event C = the event that that a student selected at random from the class is wearing contacts. Find $P(C)$. (c) Find $P(M \text{ and } C)$. (d) Find $P(\bar{C})$.

EYE CORRECTION	MALES	FEMALES	Σ
GLASSES	4	6	
CONTACT LENSES	5	12	
LASER SURGERY	2	3	
NONE	10	7	
Σ			

PRACTICE PROBLEMS OVER SECTION 4.2

1. An experiment is made up of two events. First a coin is tossed and the result (heads/tails) is recorded. Then a die is rolled; the number appearing is recorded. List the sample space of this experiment.
2. In a national survey of business and leisure travelers, each was asked where they would most like “more space.” If a traveler is selected at random, find the **probability** (NOT percent!) that the individual:
 - (a) is a leisure traveler.
 - (b) would like more space in the airplane.
 - (c) does NOT need more room in the rental car.
 - (d) is a leisure traveler AND would like more rental car space.
 Make sure to be **showing your work**, as you will on quizzes and exams!

	AIRPLANE	HOTEL ROOM	RENTAL CAR	Σ
BUSINESS	370	90	37	497
LEISURE	280	155	68	503
Σ	650	245	105	1000

Section 4.3

Definition: Compound Event – any event made up of two or more simple events.

Definition: $P(A \text{ or } B)$ – the probability that **either** A or B occurs.

To find $P(A \text{ or } B)$: Find the total number of ways A can occur and the total number of ways B can occur, in such a way that no outcome is counted more than once. Then divide by the total number of possible outcomes in the sample space.

Definition: Events A and B are mutually exclusive or disjoint events if they cannot happen at the same time.

-- in other words, events A and B are mutually exclusive events if they have **no outcomes in common**. Visually, events A and B have **no overlap**.

Ex. D) Suppose that a box contains an assortment of shapes in a variety of colors as shown on the projector screen. One shape is selected at random from the box.

- (a) Are any of the events mutually exclusive? If so, name some.
- (b) Find $P(\text{blue and hexagon})$.
- (c) Find $P(\text{triangle and yellow})$.
- (d) Find $P(\text{blue or hexagon})$.
- (e) Find $P(\text{triangle or yellow})$.
- (f) Find $P(\text{square or red})$.

GENERAL ADDITION RULE If A and B are any two events, then:
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Ex. E) The table below categorizes a survey of college students by class and by favorite junk food. Use the table to find the probability that a college student selected at random (a) is a Freshman and prefers pizza, (b) does not prefer chips, and (c) is a Junior or prefers some 'other' junk food.

	Burgers	Chips	Chocolate	Fries	Ice Cream	Pizza	Other	Σ
Freshman	12	17	26	15	9	20	11	110
Sophomore	15	16	28	17	7	19	8	110
Junior	9	19	23	20	10	23	6	110
Senior	7	14	31	15	4	25	14	110
Σ	43	66	108	67	30	87	39	440

Ex. F) Suppose that $P(M) = 0.35$, $P(M \text{ and } N) = 0.07$, and $P(M \text{ or } N) = 0.67$. Find $P(N)$.

PRACTICE PROBLEMS OVER SECTION 4.3

1. In a national survey of business and leisure travelers, each was asked where they would most like “more space.” If a traveler is selected at random, find the probability that the individual:
(a) is a business traveler AND would like more rental car space. **(b)** is a business traveler OR would like more hotel room space. Make sure to be **showing your work**, as you will on quizzes and exams!

	AIRPLANE	HOTEL ROOM	RENTAL CAR	Σ
BUSINESS	370	90	37	497
LEISURE	280	155	68	503
Σ	650	245	105	1000

2. Suppose that $P(A \text{ or } B) = 0.59$, $P(A) = 0.41$, and $P(B) = 0.26$.
(a) Find $P(A \text{ and } B)$. **(b)** Are events A and B mutually exclusive? JUSTIFY YOUR ANSWER.
3. A newspaper reports that 60% of the workers at a facility belong to a union, 85% make over \$15 per hour, and 40% belong to the union and make over \$15 per hour.
(a) Do you believe these percentages? Justify your answer.
(b) Suppose that you have verified the first and third facts above. If you discover that 95% of the workers belong to the union OR make over \$15 per hour, what conclusion can you draw about the percent of workers making over \$15 per hour?
4. Suppose that $P(K) = 0.39$, $P(L) = 0.56$, and $P(K \text{ or } L) = 0.95$ **(a)** Find $P(K \text{ and } L)$. **(b)** Are events K and L mutually exclusive? JUSTIFY YOUR ANSWER.
5. Suppose that $P(M \text{ or } N) = 0.63$, $P(N) = 0.37$, and $P(M \text{ and } N) = 0.25$. Find $P(M)$.

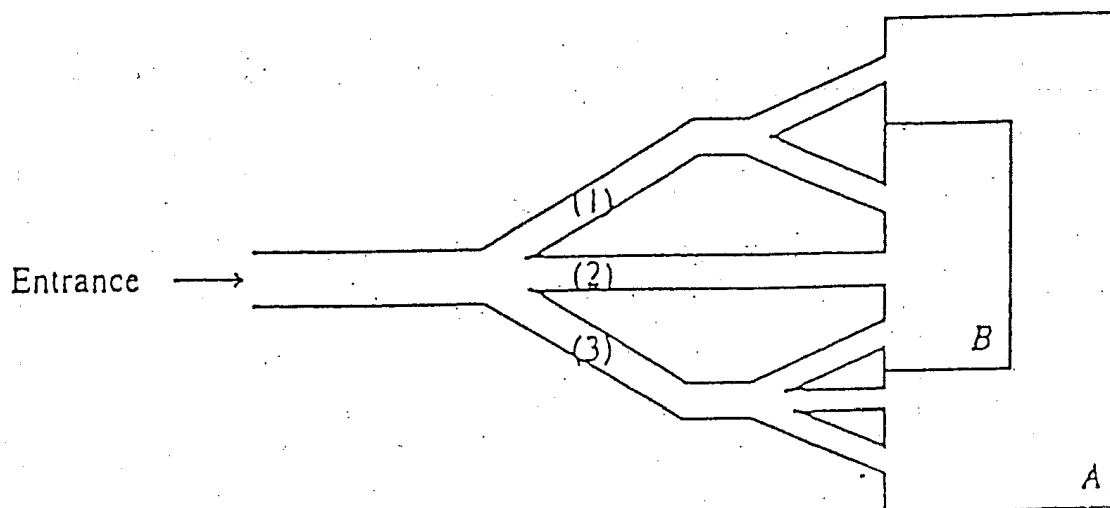
Section 4.4

Definition: P(A and B) – the probability that **both** A and B occur. If the events happen in sequence, $P(A \text{ and } B)$ is the probability that event A occurs followed by event B.

- We can draw samples with replacement or without replacement of sample points:
 - With replacement** -- After we take a sample, we **return** that sample point to the sample space. Thus, **every** sampling is done with the **same** sample space.
 - Without replacement** -- After we take a sample, we **do not return** that sample point to the sample space. Thus, **every** sampling is done with a **different** sample space.

- Ex. G)** A jar contains 3 red balls and 2 green balls. **(a)** Two balls are drawn out of the jar one at a time, and **are** returned to the jar. What kind of sampling is this? Find $P(\text{Green and Red})$. **(b)** Suppose the first ball WAS NOT returned before the second ball was drawn. What kind of sampling is this? Find $P(\text{Green and Red})$. **(c)** Find all probabilities involved with drawing two balls without replacement. **(d)** What is the probability of drawing at least one red ball?

Ex. H) Princess Carolyn fell in love with Ronald, a peasant. The king found out and ordered Ronald thrown to the lions. In response to his daughter's pleas, the king proposed a compromise. He designed a maze opening into two rooms, as shown in the figure. Carolyn was to choose one room in which to wait, and lions would be placed in the other. Ronald would walk through the maze until he entered a room. If Carolyn was in the room, they could be married; otherwise, the lions would get Ronald. If Ronald makes decisions in the maze at random, which room should Carolyn choose? (From *A Problem Solving Approach To Mathematics* by Billstein et al.)



Ex. I) Six purple ribbons and eight white ribbons are placed in a box. If two ribbons are selected at random, one at a time without replacement, what is the probability that one of each color is chosen?

PRACTICE PROBLEMS OVER SECTION 4.4

- Two balls are randomly selected without replacement from a jar containing five green balls and three red balls. Find the probability that **(a)** both are green. **(b)** both are red. **(c)** one of each color is selected.
- Two boxes contain solid-colored marbles. Box 1 has 4 marbles that are red, 6 that are blue, 8 that are purple, and 2 that are green. Box 2 has 7 marbles that are purple and 5 that are white. One ball will be randomly selected from box 1, and then another ball will be randomly selected from box 2. Using the tree diagram from Example A in your notes (on page 1 of this handout), add probabilities to the tree to answer the following questions: **(a)** What is the probability that exactly one ball is purple? **(b)** What is the probability that at least one ball is purple? **(c)** Thought question: Do you think this problem was easier to solve using the tree diagram, or would it have been easier to solve using a box diagram as we did for Example I? You might try the problem without a tree to decide which approach you prefer.
- Box 1 contains three orange balls and four white balls. Box 2 contains five orange balls and two white balls. One ball is randomly selected from Box 1 and placed into Box 2. Then one ball is randomly selected from Box 2. What is the probability that the ball selected from Box 2 is orange?

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PRACTICE PROBLEMS OVER SECTION 4.4, continued

4. The probability that a certain door is locked is 0.6. The key to the door is one of five unidentified keys hanging on a key rack. Two keys are randomly selected before approaching the door. What is the probability that the door can be opened without returning for another key?

[Problems 3 & 4 above adapted from *Elementary Statistics* by Johnson & Kuby, 8th edition.]

5. Dr. Goodnight tested 200 men and 300 women in his practice and found that 6% of the men and 4% of the women were colorblind. [HINT: a helpful way to approach this problem is to draw a tree diagram with the first part based on gender and the second based on color-blindness vs. non-color-blindness.]

(a) What is the probability that a person selected at random from this group was a man who was not colorblind? (b) What is the probability that a person selected at random from this group was colorblind?

Section 4.5

Definition: Conditional Probability – the probability that event A will occur **given** that event B has occurred. $P(A | B)$ – this is read “the probability of A, given B,”

Ex. J) Suppose that a box contains an assortment of shapes in a variety of colors as shown on the projector screen. One shape is selected at random from the box. Find the following probabilities:

- (a) $P(\text{Triangle} | \text{Yellow})$ (b) $P(\text{Yellow} | \text{triangle})$
 (c) $P(\text{Blue} | \text{Diamond})$ (d) $P(\text{Diamond} | \text{Blue})$.

Ex. K) The table below categorizes a survey of college students by class and by favorite junk food. Use the table to find the following probabilities for a college student selected at random: (a) given that a student is a Sophomore, find the probability that he or she prefers ice cream. (b) given that a student prefers burgers, find the probability that he or she is a Freshman. (c) given that a student does NOT prefer chocolate, find the probability that he or she is a Senior.

	Burgers	Chips	Chocolate	Fries	Ice Cream	Pizza	Other	Σ
Freshman	12	17	26	15	9	20	11	110
Sophomore	15	16	28	17	7	19	8	110
Junior	9	19	23	20	10	23	6	110
Senior	7	14	31	15	4	25	14	110
Σ	43	66	108	67	30	87	39	440

Conditional Probability Formula: $P(A | B) = \frac{P(A \text{ and } B)}{P(B)}$

Ex. L) Suppose that $P(M | N) = 0.46$, and $P(M \text{ and } N) = 0.33$. Find $P(N)$.

PRACTICE PROBLEMS OVER SECTION 4.5

1. A group of student records classifies the students by gender and by class, as shown below. Suppose a file is selected at random. **(a)** Find the probability that the individual is a female given that she is a sophomore. **(b)** Find the probability that the individual is a male given that he is NOT a freshman. Make sure to be **showing your work**, as you will on quizzes and exams!

	FR	SO	JR	SR	Σ
Male	21	20	18	16	75
Female	18	22	17	18	75
Σ	39	42	35	34	150

2. In a national survey of business and leisure travelers, each was asked where they would most like "more space." Suppose one of these people is selected at random. **(a)** Given that the individual is a leisure traveler, find the probability that the individual would like more hotel room space. **(b)** Given that the individual would like more hotel room space, find the probability that the individual is a leisure traveler. **(c)** Why are the values of (a) and (b) not the same?

	AIRPLANE	HOTEL ROOM	RENTAL CAR	Σ
BUSINESS	370	90	37	497
LEISURE	280	155	68	503
Σ	650	245	105	1000

- Suppose that $P(K) = 0.29$, $P(L) = 0.47$, and events K and L are mutually exclusive. Find $P(K | L)$.
- Suppose that $P(M) = 0.35$, $P(N) = 0.43$, and $P(N | M) = 0.21$. Find $P(M \text{ or } N)$. [HINT: 2-part problem.]
- The following probabilities are noted for a sample of gas station customers: $P(\text{customer checks oil}) = 0.10$, $P(\text{customer checks tire air pressure}) = 0.04$, and $P(\text{customer checks oil and air}) = 0.02$. Given that a customer checks the oil, find the probability that he/she checks the air.
- The dresser drawer in the cartoon below contains a jumble of 12 black socks, 16 blue socks, and 6 white socks. **(a)** A cat knocks one sock out of the drawer. What is the probability that the sock is not white? **(b)** A child reaches into the drawer without looking, and pulls out two socks. What is the probability that the socks are the same color?

