MATH 20043 CHAPTER 9 EXAMPLES AND DEFINITIONS

Section 9.1 Plane Figures

PART 1: Points, lines, & planes

Undefined terms: abstractions that we use without a purely mathematical, precise definition, but can only attempt to describe. Point, space, line, and plane are examples of undefined terms in geometry.

Point – a unique, exact location in space. It has no dimension, i.e. no length, width, height, volume, etc. A point only has **location**.

Space – the set of all points.

Line – a specific subset of points in space: a one-dimensional object that is formed of infinite points and has no endpoints. Since a line extends infinitely, it does not have a specific length; however a line segment has length, but no width.
- physical models – an edge, a clothesline, . . .

Plane – a specific subset of points in space: a two-dimensional group of points that goes on infinitely in all directions within those two dimensions.
- physical models – chalkboard, sheet of paper, wall, . . .

Three points are **collinear** if they lie on the same line. **Non-collinear** points do not all lie on the same line.

Properties of Lines

1. If two distinct lines intersect, they intersect in exactly one point.
2. Two or more lines are said to be concurrent if there is exactly one point common to all of them. These lines may be in the same plane, or they may be lines in space.
3. Exactly one line contains any two distinct points.

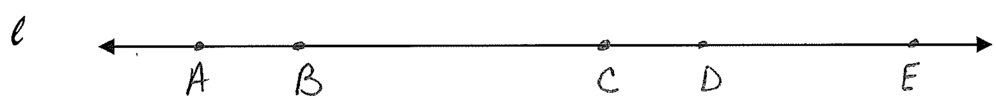
Two lines in a plane are **parallel** if and only if there is no point common to both lines. **Skew lines** are nonintersecting, nonparallel lines; they do **not** lie in a common plane.

Ex. A) Consider a line ℓ and a point K, not on ℓ. How many lines can be drawn through K, so that the new lines are parallel to line ℓ?

Subsets of lines: **1.** Half-line **2.** Ray **3.** Line segment

Ex. B. part 1) Above line ℓ below, mark each of the following:

(a) $\overrightarrow{AB}$ (b) $\overline{CE}$ (c) $\overline{ED}$ (d) $\overrightarrow{BA}$



Ex. B. part 2) Use line ℓ to find each of the following:

(a) $\overrightarrow{AC} \cap \overrightarrow{BD}$ (b) $\overline{BC} \cup \overrightarrow{CE}$

(c) $\overline{DB} \cap \overline{EC}$ (d) $\overline{BC} \cap \overrightarrow{CE}$

Properties of Planes

A plane can be uniquely determined by four combinations of points and/or lines:

1. Three noncollinear points.
2. A line and a point not on that line.
3. Two intersecting lines.
4. Two parallel lines.

A line separates a plane into three parts – the line and two **half-planes**. The line itself does **not** belong to either half-plane, but is called the **edge** or boundary of each half-plane.

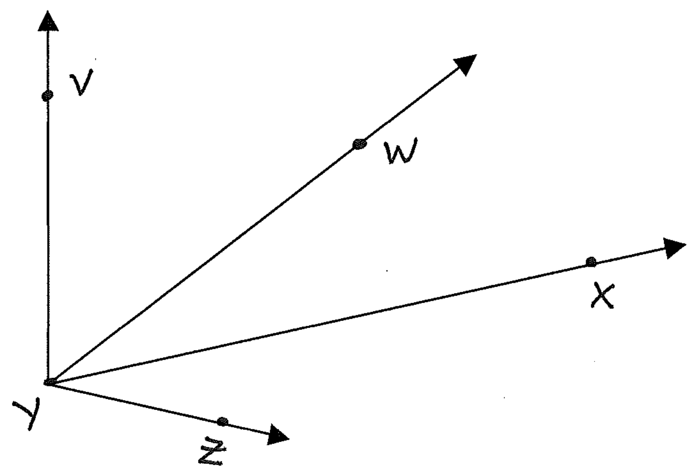
If two planes intersect, their intersection is a line. If two planes do not intersect (i.e., their intersection is empty), the planes are parallel.

Ex. C) What are the possible relationships between a line and a plane in space?

PART 2: Angles

An **angle** is the union of two rays that have the same endpoint. This common endpoint is called the **vertex**, and the rays are called the **sides** of the angle.

Ex. D) How many angles can you find in the figure below?



Degree – a unit of angular measure. A circle is made up of 360°.

A **straight angle** has a measure of 180°. The measure of a **right angle** is 90°. **Perpendicular** lines, rays, or segments intersect to form right angles.

An **acute angle** has a measure strictly between 0° and 90°. An **obtuse angle** has a measure strictly between 90° and 180°. The **interior** of an angle is always less than 180°. The **exterior** of an angle is always greater than 180°.

Adjacent angles – coplanar angles that have a common vertex and a common side and are disjoint (i.e., they do not overlap). These are also sometimes called **consecutive angles**.

Ex. E) Name the adjacent angles of the figure in example D.

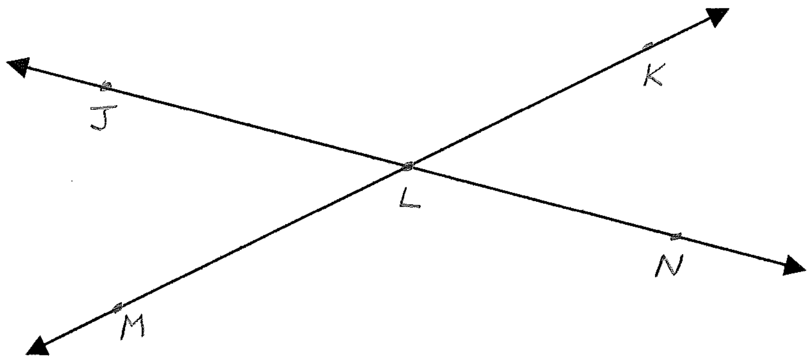
Two angles are **conjugate angles** if their measures add up to 360°.

Any two angles are **supplementary** if their measures add up to 180°. A **linear pair** of angles are adjacent angles that form a straight angle. .

Two angles are **complementary** if their measures add up to 90°.

Congruent angles have the same measure.

Ex. F) Suppose two lines $\overleftrightarrow{JN}$ and $\overleftrightarrow{KM}$ intersect as in the figure below. (a) Name the four angles with measures under 180°. (b) Which angles are congruent? Why are they congruent?

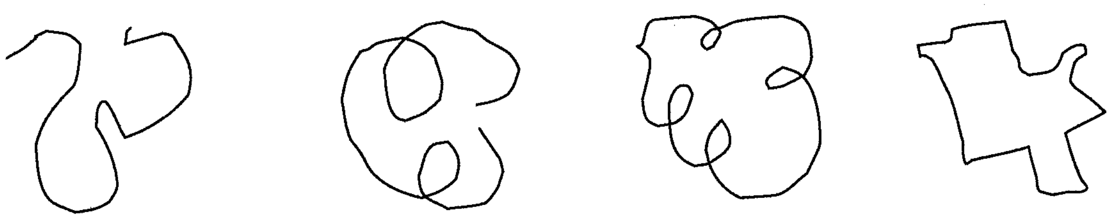


Vertical angles – the pairs of nonadjacent angles formed by intersecting lines. Vertical angles are congruent.

PART 3: Plane curves, circles, & polygons

A **plane curve** is a set of points that can be drawn without lifting the pencil. A curve is **simple** if no points are retraced, except possibly the endpoints. A curve is **closed** if drawn by starting and stopping at the same point.

Ex. G) Which of the following are simple curves? Which are closed curves?



Region – the union of a simple closed curve with its interior. A region is **convex** if a line segment joining any two points in the region is itself also in the region. A region that is not convex is called **concave** (or **non-convex**).

Circle – the set of all points in a plane at a given distance from a fixed point. This point is called the **center** of the circle. The given distance is called the **radius** of the circle. **Radius** is also defined as a line segment from the center to a point on the circle.

A **tangent** line to a circle is a line in the plane of the circle that contains exactly one point of the circle, i.e. it intersects the circle in exactly one point. A **secant** line to a circle is a line in the plane of the circle line that intersects the circle and is not tangent to the circle. A **chord** of a circle is the segment of a secant joining the two points of intersection with the circle. The **diameter** of a circle is a chord that contains the center of the circle. This means that the length of the diameter of a circle is twice the measurement of the radius of the circle.

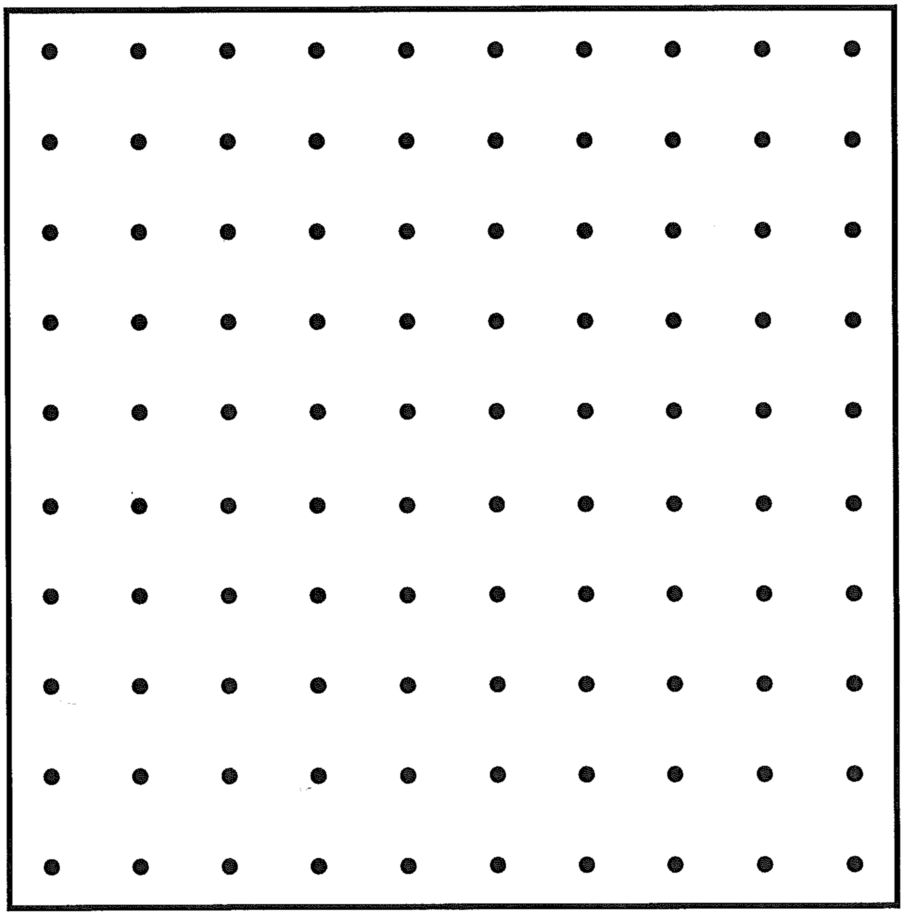
Polygon – a simple closed curve that is the union or three or more coplanar, distinct line segments. These line segments are called the **sides** or **edges** of the polygon. The endpoints of the line segments are called **vertices**. Adjacent vertices share a line segment.

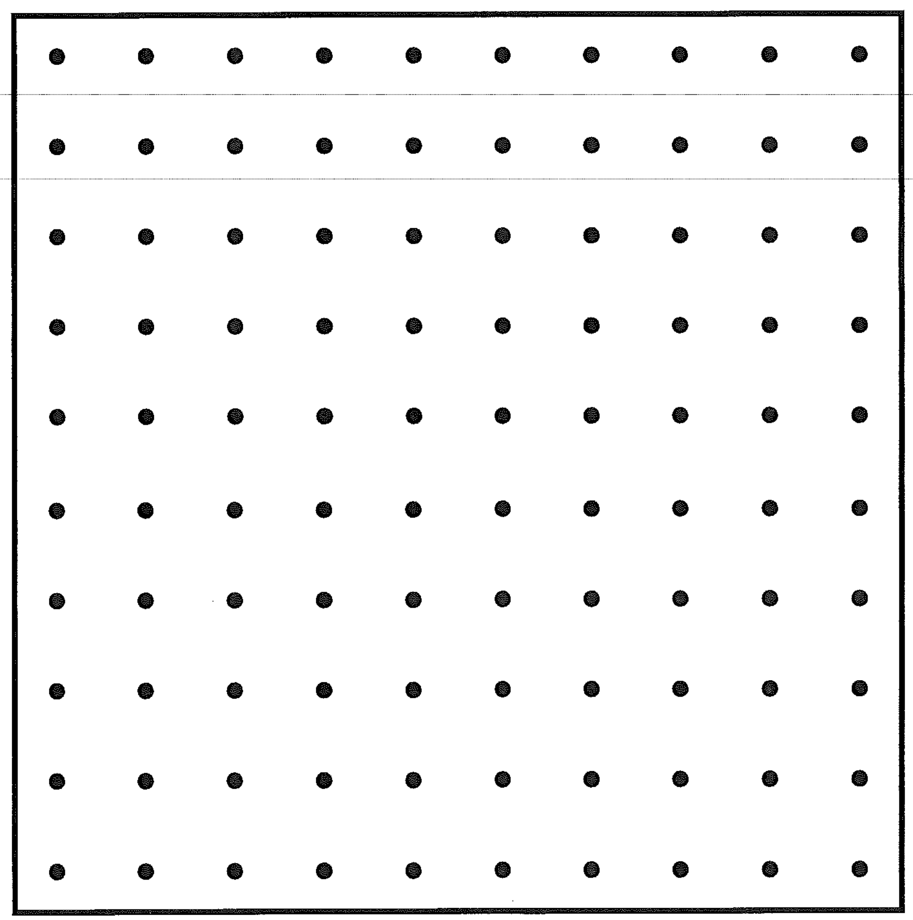
Diagonals of a polygon – the line segments joining nonadjacent vertices.

Ex. H) Form the following shapes on geoboard dot paper: (a) a pentagon that has exactly one pair of parallel sides. (b) a quadrilateral that has no parallel sides but has two pairs of congruent, adjacent sides.

Triangles can be characterized by their sides or by their angles:

Characteristics of sides	Characteristics of angles
Equilateral	**Right**
Isosceles	**Acute**
Scalene	**Obtuse**





Quadrilaterals:

Ex. I) Make a list of the names of quadrilaterals that you remember. How would you distinguish them from each other? (HINT: see how triangles were distinguished above).

Ex. J) Draw a diagram to express the relationships among quadrilaterals.

NOTE: Practice problems over section 9.1 are on a separate handout

Section 9.2 Polygons & Tesselations

Regular polygon – a simple, convex closed curve with all sides of equal length and all angles of equal measure.

Congruent polygons have the same size and shape.

The **interior angles**, or **vertex angles** of a polygon are the angles formed by adjacent sides of the polygon and lying within the polygon.

Ex. K) We know that the sum of the three interior angle measures of a triangle is 180°. What is the sum of the interior angle measures of (a) any convex quadrilateral? (b) any convex pentagon?

Central angle of a polygon – an angle whose vertex is the center of the polygon and whose sides go through adjacent vertices of the polygon.

Tessellation: a complete covering of a plane by shapes in a repeating pattern, without gaps or overlap.

Practice problems over section 9.2

1. Draw a sketch and find the sum of the interior angle measures of a convex hexagon.

2. What is the measure of each vertex angle of a regular hexagon?

Section 9.3 Space Figures

A **simple closed surface** is like a simple closed curve; instead of being in a plane, it is in space. It has no holes, and separates space into three disjoint sets of points – the interior, the exterior, and the surface itself.

Solid – the union of the surface and its interior; also called a space region.

Polyhedron – a simple closed surface in space whose boundary is composed of polygonal regions. These polygonal boundary regions are called the **faces** of the polyhedron. Plural of polyhedron: **polyhedra.**

Prism – a polyhedron formed by two congruent polygonal regions in parallel planes, along with three or more regions bounded by parallelograms joining the two polygons to form a closed surface. The polygonal regions are called the **bases**; the parallelograms are called **lateral faces**. The bases and lateral faces together make up the **faces** of the prism. The parallel edges joining the bases are called **lateral edges**.

Right prism – a prism whose lateral faces are rectangles and whose planes containing the lateral faces are perpendicular to the planes containing the bases. If a prism is not a right prism, it is an **oblique** prism.

Ex. L) A hexagonal prism has how many faces? Edges? Vertices?

Pyramid – a polyhedron formed by a simple closed polygonal region (the **base**), a point not in the plane of the region (the **apex**), and the triangular regions joining the point and the sides of the polygon.

Regular pyramid – a pyramid whose base is a regular polygon and whose lateral faces are all congruent triangles.

Altitude – a perpendicular segment from the apex to the base of a pyramid (or cone)

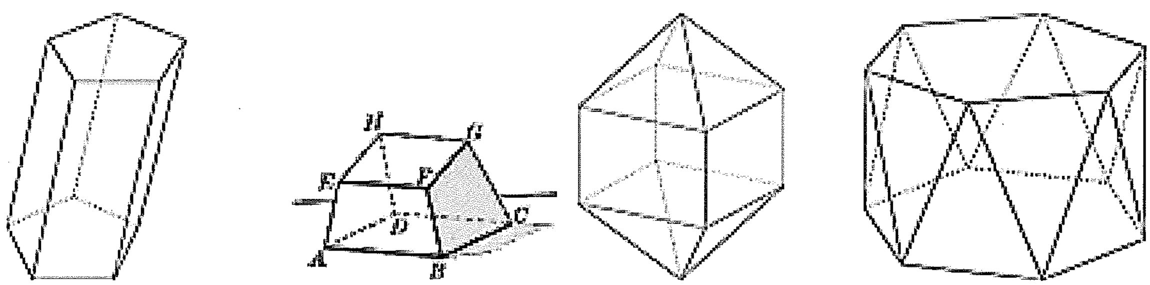
Regular polyhedron – if the faces of a convex polyhedron are congruent regular polygonal regions, **and** if each vertex is the intersection of the same number of edges, the polyhedron is regular. Also known as a **Platonic Solid.**

Euler's formula – Let **V** = the number of vertices, **E** = the number of edges, and **F** = the number of faces of any polyhedron.

Thus: $$\mathbf{V + F - E = 2}$$

Ex. M) Complete the table, indicating the number of vertices, edges, and faces, and confirming Euler's formula for each of the following polyhedra.

Polyhedron	**V**	**F**	**E**	**V + F – E**
Pentagonal prism				
Hexahedron				
Square dipyramid				
Hexagonal antiprism				



<u>Dihedral angle</u> – the union of two noncoplanar half planes and the line of intersection (the line forming the common edge).

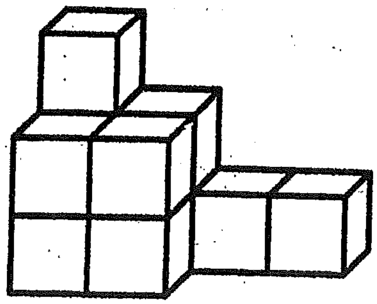
<u>Sphere</u> – the set of all points in *space* at a given distance from a fixed point. The <u>center</u> of a sphere is that fixed point. The <u>radius</u> of a sphere is a line segment from the center to any point on the surface of the sphere.

A **<u>cylinder</u>** is formed by two congruent *regions* (the **<u>bases</u>**) bounded by simple closed plane curves in parallel planes, connected by a lateral surface that rises from one base to the other. The bases of a **<u>circular cylinder</u>** are two congruent circles. The lateral surface of a **<u>right cylinder</u>** is perpendicular to its base.

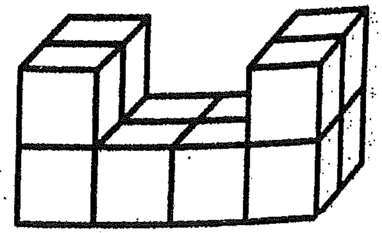
A **<u>cone</u>** is formed by the union of a simple closed curve (the **<u>base</u>**) and a lateral surface that slopes to a single point called the **<u>vertex</u>** or **<u>apex</u>** of the cone. The base of a **<u>circular cone</u>** is a circle. The vertex of a **<u>right circular cone</u>** is directly above the center of the base.

Ex. N) Draw the top view, front view, and right (side) view of each of the following solids:

(a)



(b)



<u>Section 9.4 Symmetric Figures; Geometry in Nature and Art</u>

Line of symmetry -- a line of reflection <u>through</u> a figure. It maps half of the image onto the other half. This is also called **reflection (plane) symmetry**.

Rotational symmetry -- has a central <u>point</u> of symmetry. If a figure has rotational symmetry, we can rotate the figure <u>less</u> than a full turn about that point and have the original figure <u>in the original position</u>.